4.04 *Inequalities in Two Variables*

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Solving a linear equation in one variable, like **3X + 6 = 12**, usually resulted in a single solution, like **X = 2**. The graph of this solution could be represented by a **single point** on a number line. A corresponding inequality, like **3X + 6 < 12**, required shading the number line to the left of the solution, not including the endpoint. The inequality **3X + 6 > 12** required shading to the right, not including the endpoint. The inequalities **3X + 6 ≤ 12** and **3X + 6 ≥ 12** also require shading, but the solutions to these include the endpoints.

In two dimensions, the equation **3X + Y = 12** represents a **line** that can be graphed in the **XY plane**. As the **point X = 2** divides the number line into two regions, the **line 3X + Y = 12** divides the **XY plane** into two regions, above and below (or right and left) of the line. It should be clear that **3X + Y < 12** represents the shading on one side of the line, while **3X + Y > 12** represents the shading on the other side. The inequalities **3X + Y < 12** and **3X + Y < 12 do not include the line** itself, and therefore are represented by **dotted lines**. The inequalities **3X + Y ≤ 12** and **3X + Y ≥ 12 do include the line** and are represented by **solid lines**.

The only question that remains is, "Which side of the the line should be shaded?" Probably the easiest way to decide is to solve the inequality for **Y**. For example, **3X + Y < 12** can be written as **Y < -3X + 12**. The values of Y are measured up and down the Y-axis, with values of **Y = -3X + 12** representing values that are on the line. Therefore, it seems reasonable that **Y < -3X + 12** represents values that are ***below the line*** **Y = -3X + 12,** while **Y > -3X + 12** represents values that are ***above the line***.

Remember that when graphing a line whose equation is in the form **Y = mX + b**, it is usually easiest to use the **Y-intercept** and **slope** to draw the line. If the equation is in **standard form, AX + BY = C**, then it is usually easiest to graph by finding the **X** and **Y-intercepts**.

When graphing a **linear inequality** there are three steps, summarized in the box on the next page.

STEPS TO GRAPH LINEAR INEQUALITIES

I. Change the inequality to an equation and graph the line.
- **A.** If **Y = mX + b**, then use **Y-intercept/slope** method.
- **B.** If **AX + BY = C** form, then use **intercepts** method.

II. Decide whether the line is included or not included.
- **A.** If "<" or ">", then use a **dotted line**.
- **B.** If "≤" or "≥", then use a **solid line**.

III. Decide whether to **shade above or below** the line.
- **A.** If the equation has a **positive Y-coefficient** and "<" or "≤", then **shade below the graph** of the line.
- **B.** If the equation has a **positive Y-coefficient** and ">" or "≥", then **shade above the graph** of the line.
- **C.** If the equation has a **negative Y-coefficient**, then multiply both sides of the inequality by **-1**, which reverses the direction of the inequality sign. Then shade above or below as indicated.

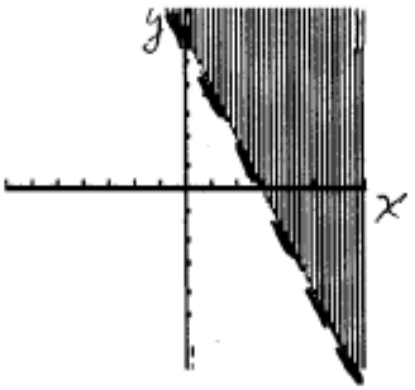
EXAMPLE 1: **Graph Y > -2X + 6**

Solution: **I.** Graph Y = -2X + 6

Y-int = (0,6); m = -2

II. Use a dotted line.

III. Since +Y > , shade above the line.



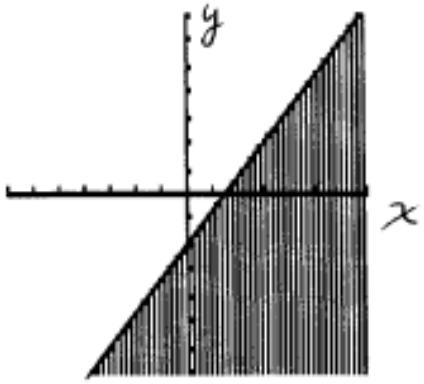
EXAMPLE 2: **Graph** $Y \leq \frac{4}{3}X - 2$

Solution: **I.** Graph $Y = \frac{4}{3}X - 2$

Y-int = (0, -2) $m = \frac{4}{3}$

II. Use a solid line.

III. Since +Y ≤, shade below the line.



[NOTE: In both of these examples, it is easiest to graph using the slope-intercept method!]

EXERCISES. Graph each of the following inequalities. Shade the appropriate areas.

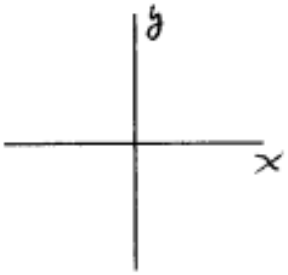
1. $Y < 3X + 2$

Y-int_____

m = _____

Type line ____

Shade_____



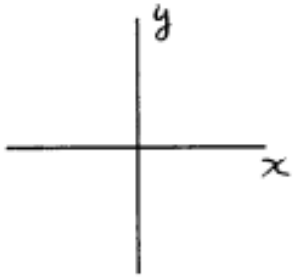
2. $Y > -2X + 4$

Y-int_____

m = _____

Type line_____

Shade_____



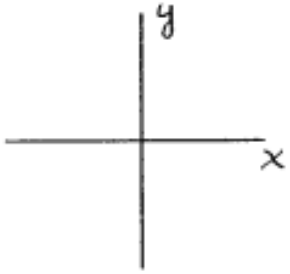
3. $Y \geq - X - 4$

Y-int_____

m = _____

Type line ____

Shade_____



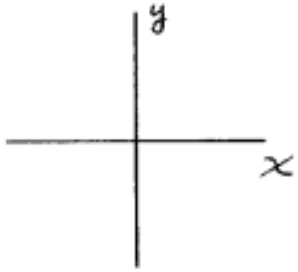
4. $Y \leq 2X - 4$

Y-int_____

m = _____

Type line_____

Shade_____



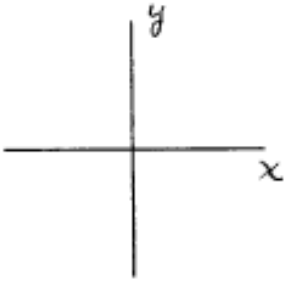
5. $Y < -\frac{4}{3} X + 2$

Y-int_____

m = _____

Type line ____

Shade_____



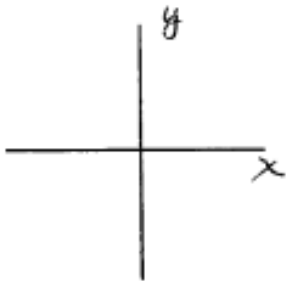
6. $Y \geq \frac{3}{2} X - 2$

Y-int_____

m = _____

Type line_____

Shade_____



EXAMPLE 3: Graph $2X + 3Y > -6$

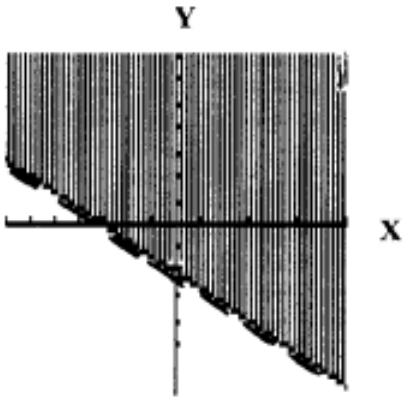
Solution: **I.** Graph $2X + 3Y = -6$

[NOTE: In both of these examples, it is easiest to graph using the two-intercepts method!]

X-int = (-3, 0)
Y-int = (0, -2)

II. Use a dotted line.

III. Since $+Y >$, shade above the line.



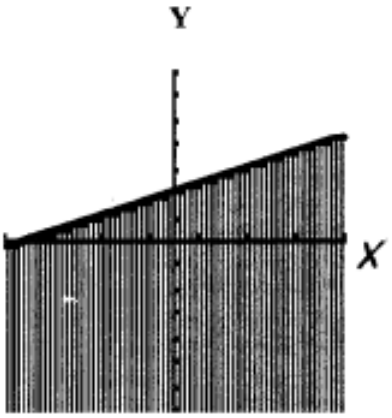
EXAMPLE 4: Graph $-X + 3Y \leq 6$

Solution: **I.** Graph $- X + 3Y = 6$

X-int = (-6, 0)
Y-int = (0, 2)

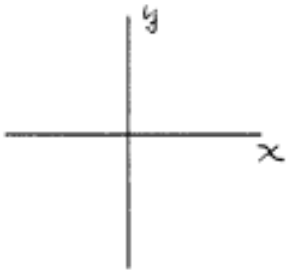
II. Use a solid line.

III. Since $+Y \leq$, shade below the line.



7. $-X + 3Y \geq 6$

X	Y
0	2
-6	0

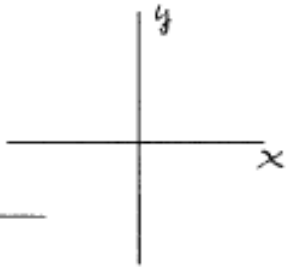


Type line: _____

Shade: _____

8. $3X + 2Y > -12$

X	Y
0	
	0



Type line: _____

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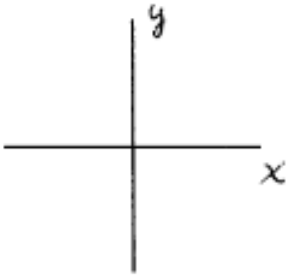
9. $3X + Y < -6$

X	Y
0	
	0

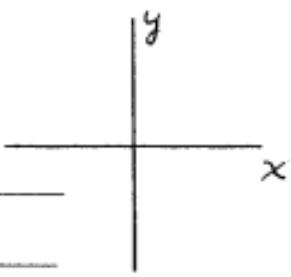


Type line: _____

Shade: _____

10. $-2X + Y \leq 8$

X	Y
0	
	0



Type line: _____

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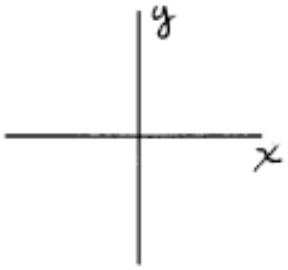
11. -3X + 4Y > -12

X	Y



Type line: _____

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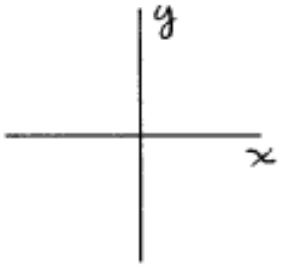
12. -3X + 2Y ≥ 12

X	Y



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Shade: _____

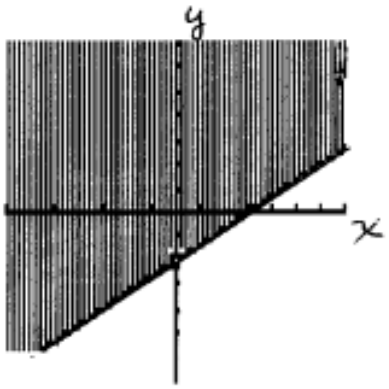
EXAMPLE 5: Graph 2X - 3Y ≤ 6

Solution:

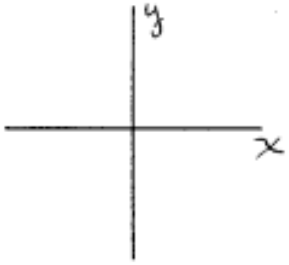
I. Graph 2X - 3Y = 6
Easiest to use two int. method
X-int = (3, 0); Y-int = (0,-2)

II. Use a solid line

III. Since -Y ≤, divide both sides by -1.
This means +Y ≥ , so shade above line.



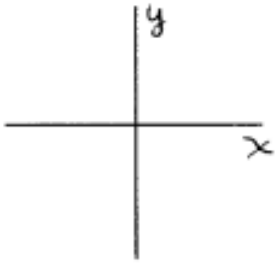
13. 3X - Y < -6



Type line: _____

Shade: _____

14 . -2X - Y ≤ 8



Type line: _____

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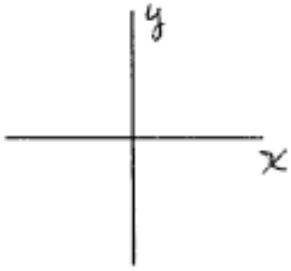
15. 3X - 4Y ≥ 12



Type line: _____

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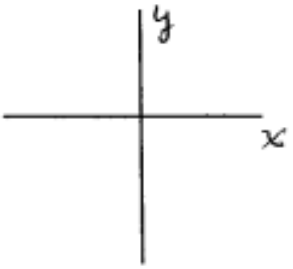
16. 3X - 2Y > -12



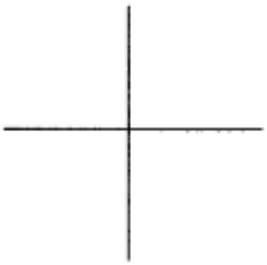
Type line: _____

Shade: _____

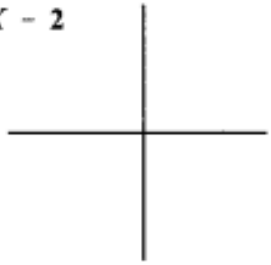
17. $Y > -3X - 4$

18. $Y \geq X$

19. $X - 3Y < -6$



20. $Y \geq -\frac{1}{4}X - 2$



21. $Y \geq \frac{3}{2}X + 5$

22. $Y < -3$

23. $X \leq 4$

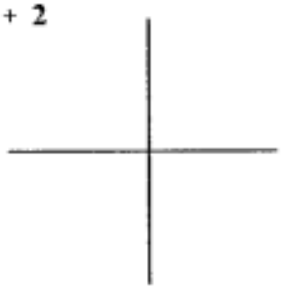
24. $X > -4$

25. $3X - Y < -3$

26. $X - 2Y > 6$

27. $3X + Y < 3$

28. $Y \geq \frac{3}{4} X + 2$



29. $Y \leq -\frac{2}{3} X + 5$

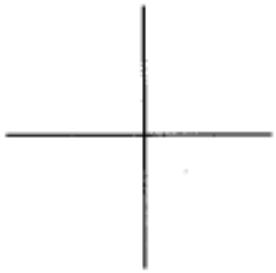
30. $2X - 3Y < -6$

31. $X - Y > 4$

32. $Y - X > -4$

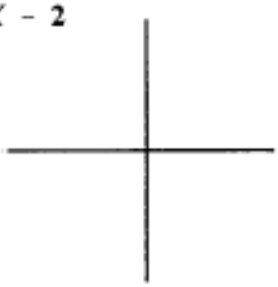
33. $Y \geq 2X - 6$

34. $Y \geq -3X + 6$

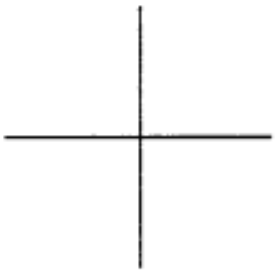


35. $X + 3Y < -6$

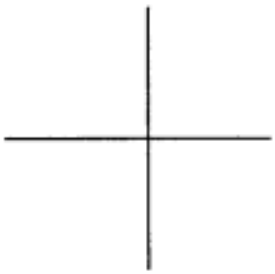
36. $Y \geq -\frac{1}{2}X - 2$



37. $X + 4 > 0$



38. $Y + 3 > 0$



39. $X - 2Y < 0$

40. $2Y - X > 0$

ANSWERS 4.04

p. 327 - 332:

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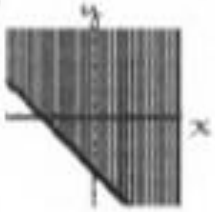

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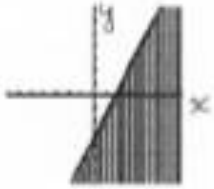

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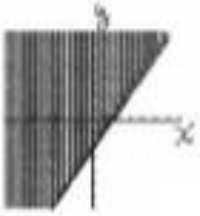

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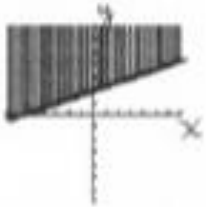

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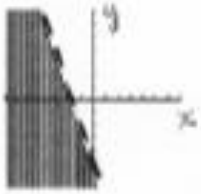

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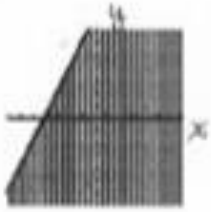

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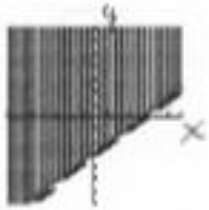

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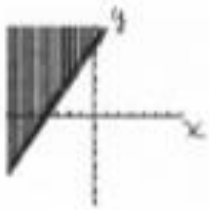

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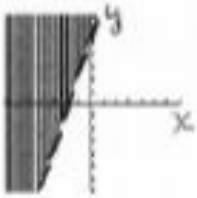

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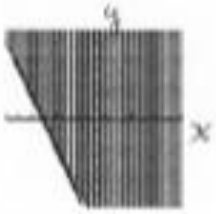

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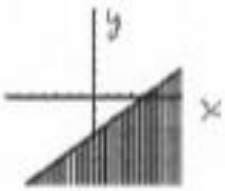

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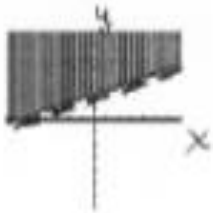

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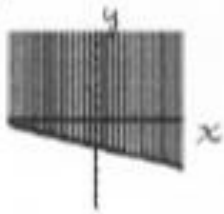
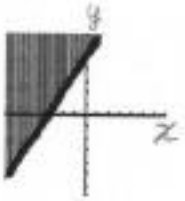
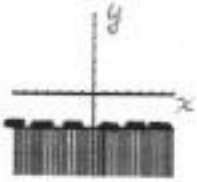
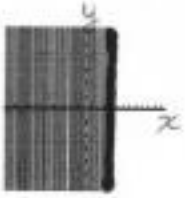


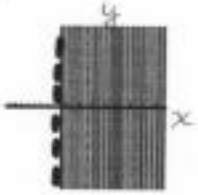
ANSWERS 4.04 (Continued)

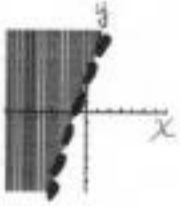
p. 327 - 332:

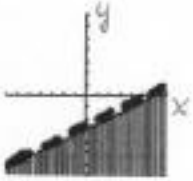
21. 

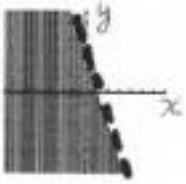
22. 

23. 

24. 

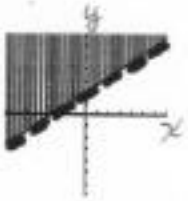
25. 

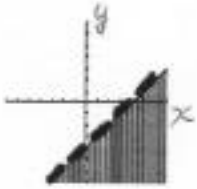
26. 

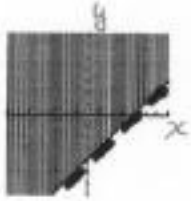
27. 

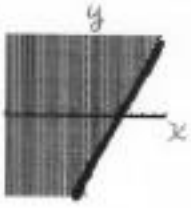
28. 

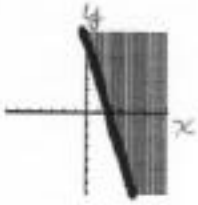
29. 

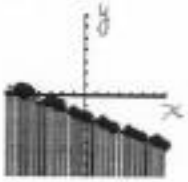
30. 

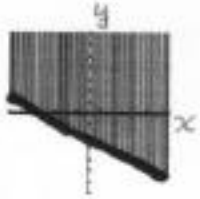
31. 

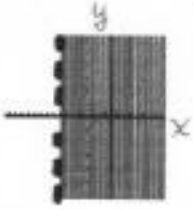
32. 

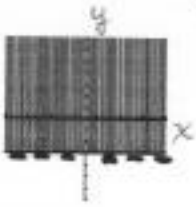
33. 

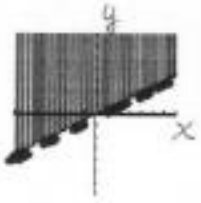
34. 

35. 

36. 

37. 

38. 

39. 

40. (Same as #39!)