4.06 *Equation of a Line*

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

In the last two sections, an equation in x and y was generally given to you. Given an equation, the problem was usually to find the slope, to find the x and y-intercepts, or perhaps to graph the line whose equation was given. Now in this lesson, we turn things around. As in the television game show *Jeopardy*, where they are given answers and the contestants are required to give the question, now you will be given the slope and a point on the line, and you will be required to find the **equation of a line**. (However, you must always remember to answer in the form of an <u>equation</u>!)

When giving the equation of a line, it is traditional to express the equation in either **standard form, Ax + By = C**, or in **slope-intercept form, y = mx + b**. With the increase in use of graphing calculators, which require the equation of a graph to be entered in the form of y = _____, it seems that the **slope-intercept form, y = mx + b** should be the equation form of choice. **In order to find the equation of a line, you must know the slope and a point on the line.**

The easiest way to accomplish this is to begin with the familiar equation **y = mx + b.**

If you know the slope and a point on the line,

you can find the equation of the line using

$$y = mx + b$$

This equation essentially has four unknowns, of which two are variables (x and y) and two are constants (m and b). If you know the slope m and a point (x, y) on the line, then you can substitute these three known values into the equation, and solve for the only remaining unknown, which is b. The next example illustrates this method.

[NOTE: The method taught and used by many textbooks and math instructors is called the "point-slope method." Either method can be used. The use of the slope-intercept formula seems simpler.]

EXAMPLE 1. Find the equation of the line with slope m=4 that passes through the point (-3,2).

Solution: Follow the following steps.

Step 1: Write the equation $y = mx + b$

Step 2: Substitute the numbers. You are given the value of m=4, and you know that x=-3 when y=2. Substitute these numbers into the equation

$$y = mx + b.$$
$$2 = 4(-3) + b$$

Step 3: Solve for b. $\quad$ 2 = -12 + b $\quad$ Add 12 to each side of the equation.

$$\underline{+12 \quad +12}$$
$$14 = b$$

Step 4: Write the equation: $y = mx + b$
$$y = 4x + 14 \text{ (Answer!)}$$

Step 5: $\quad$ **Check by substitution:** $\quad$ 2 = 4(-3) + 14
$$2 = -12 + 14$$

EXERCISES. In each of the following exercises, express the equation of the line in slope-intercept form.

1. $\qquad\qquad\qquad$ **m = 3, through (-2,4)**

Step 1: Write the equation: $y = mx + b$

Step 2: Substitute $\qquad$ m=3, x=-2, y=4
$$4 = 3(-2) + b$$

Step 3: Solve for b: $\qquad$ ___ = ___ + b
$$\text{___} = b$$

Step 4: Write the equation: $y = $___ x + ___ Answer

Step 5: Check by substitution:

2. m = -3 through (4,2) $\qquad\qquad\qquad$ **3.** m = -3 through (2,-4)

4. m = -2 through (-3,4)

5. m = 2 through (-3,-4)

6. m = 4 through (5,-3)

7. m = -4 through (3,-5)

8.   **m = 5/2, through (3,4)**

Step 1: Write the equation: **y = mx + b**

Step 2: Substitute m=$\frac{5}{2}$, x=3, y=4**:** $4 = \frac{5}{2}(3) + b$

Step 3: Solve for b. $4 = \frac{15}{2} + b$

    Multiply both sides by 2: $2(4) = 2(\frac{15}{2}) + 2(b)$

$$8 = 15 + 2b$$

$$\underline{\quad\quad} = 2b$$

$$b = \underline{\quad\quad}$$

Step 4: Write the equation:
Step 5: Check:

9. m = 3/4 through (-5,-2)

10. m = 3/2 through (-4,3)

11. m = -3/2 through (4,-3)

12. m = -5/3 through (6,-3)

13. m = 5/3 through (-6,3)

14. m = -7/2 through (4,1)

In order to find the equation of a line containing two points, think, what *two* elements are required? Answer: a **point** and a **slope.** Now, if you are given <u>two</u> points and <u>not</u> the slope**,** it shouldn't take a rocket scientist to see what is needed here!   If you are given two points, your first step must be to **find the slope m between the two points**. Use the formula as follows:

15. Find the equation of the line through (2, -4) and (-5, 6).

$$m = \frac{6 - (-4)}{-5 - 2} = \frac{10}{-7} = -\frac{10}{7}$$

y = mx + b

Now use **either** point, first (2, -4): $-4 = -\frac{10}{7}(2) + b$

Multiply both sides by 7: $7(-4) = 7(-\frac{20}{7}) + 7(b)$

_____ = _____ + 7b

b = _____

Write the equation: y = _____

16. As a check,

use **the other** point (-5, 6):

Multiply both sides by 7:

$$y = mx + b$$

$$6 = -\frac{10}{7}(-5) + b$$

$$7(6) = 7(\frac{50}{7}) + 7(b)$$

$$\underline{\quad\quad} = \underline{\quad\quad} + 7b$$

$$b = \underline{\quad\quad}$$

$$y = \underline{\quad\quad\quad}$$

Does this agree with answer on previous page? _____

17. **Find the equation of the line through (-4, 2) and (2, 14).**

18. **Find the equation of the line through (-4, 2) and (-2, -6).**

19. **Find the equation of the line through (-2, -3) and (2, 4).**

20. **Find the equation of the line through (2, 6) and (-2, 3).**

21. Find the equation of the line through (-3, 6) and (-1, 3).

22. Find the equation of the line through (4, -3) and (-1, 3).

23. Find the equation of the line through (-4, 5) and (1, 8).

24. Find the equation of the line through (3, -2) and (-4, 4).

25. Find the equation of the line with x-intercept 4 and y-intercept -3.

[Hint: (4, 0) and (0, -3)]

26. Find the equation of the line with x-intercept -2 and y-intercept -5.

27. Find the equation of the line
with x-intercept -4 and y-intercept 6.

28. Find the equation of the line
with x-intercept 6 and y-intercept 3.

In the next exercises, remember that if two lines are **parallel**, they have the same slope. If two lines are **perpendicular**, the slope of one is the **negative reciprocal** of the other.

29a) Find the equation of the line through
(4, -2) and parallel to y = -3x + 5.

m =   m$_{parallel}$=

b) Find the equation of the line through
(4,-2) and perpendicular to y = -3x + 5.

m = _____   m⊥ =

30a) Find the equation of the line through
(-3, 5) and parallel to y = -7x - 1.

m =   m$_{parallel}$=

b) Find the equation of the line through
(-3,5) and perpendicular to y = -7x - 1.

m = _____   m⊥ =

31a) Find the equation of the line through (4, 2) and parallel to $4x + 3y = 12$.

m =   m$_{parallel}$=

b) Find the equation of the line through (4, 2) and perpendicular to $4x + 3y = 12$.

m = _____   m⊥ =

32a) Find the equation of the line through (-3, 5) and parallel to $7x - 4y = 1$.

b) Find the equation of the line through (-3, 5) and perpendicular to $7x - 4y = 1$.

33a) Find the equation of the line through (4, -5) and parallel to $3x + 5y = -15$.

b) Find the equation of the line through (4,-5) and perpendicular to $3x + 5y = -15$.

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1.$y=3x+10$; **2.**$y=-3x+14$; **3.**$y=-3x+2$; **4.**$y=-2x-2$; **5.**$y=2x+2$;   **6.**$y=4x-23$;

7.$y=-4x+7$; **8.**$y=\frac{5}{2}x-\frac{7}{2}$; **9.**$y=\frac{3}{4}x+\frac{7}{4}$; **10.**$y=\frac{3}{2}x+9$; **11.**$y=-\frac{3}{2}x+3$; **12.**$y=-\frac{5}{3}x+7$;

13.$y=\frac{5}{3}x+13$; **14.**$y=-\frac{7}{2}x+15$; **15.**$y=-\frac{10}{7}x-\frac{8}{7}$; **16.**$y=-\frac{10}{7}x-\frac{8}{7}$; **17.**$y=2x+10$;

18. $y=-4x-14$; **19.** $y=\frac{7}{4}x+\frac{1}{2}$; **20.** $y=\frac{3}{4}x+\frac{9}{2}$; **21.** $y=-\frac{3}{2}x+\frac{3}{2}$; **22.** $y=-\frac{6}{5}x+\frac{9}{5}$;

23. $\mathbf{y=\frac{3}{5}x+\frac{37}{5}}$; **24.**$y=-\frac{6}{7}x+\frac{4}{7}$; **25.**$y=\frac{3}{4}x-3$; **26.**$y=-\frac{5}{2}x-5$; **27.**$y=\frac{3}{2}x+6$;

28.$y=-\frac{1}{2}x+3$;   **29a)**$y=-3x+10$, **b)**$y=\frac{1}{3}x-\frac{10}{3}$;   **30a)**$y=-7x-16$, **b)**$y=\frac{1}{7}x+\frac{38}{7}$;

31a)$y=-\frac{4}{3}x+\frac{22}{3}$, **31b)**$y=\frac{3}{4}x-1$;   **32a)**$y=\frac{7}{4}x+\frac{41}{4}$,   **b)**$y=-\frac{4}{7}x+\frac{23}{7}$;

33a)$y=-\frac{3}{5}x-\frac{13}{5}$, **b)**$y=\frac{5}{3}x-\frac{35}{3}$.

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