4.08 *Applications of Systems of Equations*

Dr. Robert J. Rapalje

ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

In Chapter 1 you learned to solve word problems by letting the variable x represent one of the unknown quantities and describing all other unknown quantities in terms of x. Then an equation was written (in terms of x), and the problem was to solve for x. In this chapter you have just learned to solve two equations with two unknowns. This opens a new strategy for solving applications. Now, instead of letting x represent one variable and describing everything in terms of x, you can let x and y represent two different variables, then write two equations, which you can solve by methods described in the last section of this chapter. As in Section 1.10, we have the same five steps to set up and solve word problems.

SOLVING WORD PROBLEMS WITH 2 VARIABLES

STEP 1: IDENTIFY THE VARIABLES. State exactly what it is that the variables represent. For example, "Let x = the number of dimes" and "y = the number of quarters."

STEP 2: WRITE THE EQUATIONS. Having completed Step 1, use this step in writing, not one, but two equations. There are usually two sentences or phrases given in the problem. Each of these translates into an equation.

STEP 3: SOLVE THE SYSTEM OF EQUATIONS. Use the elimination or substitution method.

STEP 4: ANSWER THE QUESTION. After solving for x and y, be sure you have answered the question.

STEP 5: CHECK. Check the answers in the worded problem itself and make sure the solution actually works. Reject any extraneous or "inappropriate" answers.

NUMBER PROBLEMS

The following examples illustrate the method of this chapter. Remember, you will need two equations for each exercise.

EXAMPLE 1. **The sum of two numbers is 37. The difference between the numbers is 13. Find the numbers.**

Solution:

Step 1: Let x = the first number; y = the second number

Step 2: Write the equations:

$$x + y = 37$$
$$x - y = 13$$

Step 3: Since both equations are given in standard form, use the elimination method. In this case, add the equations together.

$$x + y = 37$$
$$x - y = 13$$
$$2x = 50$$
$$x = 25$$

Substitute $x=25$ into the first equation:

$$x + y = 37$$
$$25 + y = 37$$
$$y = 12$$

Step 4: Answer the question: $x = 25$, the first number,
$y = 12$, the second number.

Step 5: Check The sum of the numbers $25 + 12$ is 37, and the difference $25-12$ is 13. It checks!

EXERCISES.

1. **The sum of two numbers is 18. The difference between the numbers is 24. Find the numbers.**

2. **The sum of two numbers is 120. The difference between the numbers is 24. Find the numbers.**

3. **The larger of two numbers is twice the smaller number. Their sum is 243. Find the numbers.**

EXAMPLE 2. **The larger of two numbers is twice the smaller number. Their sum is 60. Find the numbers.**

Solution:

Step 1: Let x = the smaller number; y = the larger number

Step 2 Write the equations:

$$y = 2x$$
$$x + y = 60$$

Step 3: Use the substitution method, substituting y = 2x into the second equation:

$$x + y = 60$$
$$x + (\quad) = 60$$
$$x + 2x = 60$$
$$3x = 60$$
$$x = 20$$

Substitute x=20 into first equation:

$$y = 2x$$
$$y = 2(20) \text{ or } 40$$

Step 4: Answer the question: x = 20, the smaller number,
y = 40, the larger number.

Step 5: Check The larger number 40 is twice the smaller number 20, and the sum of the numbers 20 + 40 is 60. It checks!

EXERCISES.

4. The sum of two numbers is 48. The second number is four less than three times the first number. Find the numbers.

5. The sum of two numbers is 18. Three times the first number plus twice the second number is 32. Find the numbers.

6. Two numbers are such that the larger is 16 less than twice the smaller number. The sum of the numbers is 50. Find the numbers.

7. The smaller of two numbers is 16 less than the larger. The sum of the numbers is 48. Find the numbers.

8. The smaller of two numbers is 30 less than twice the larger. The sum of the numbers is 48. Find the numbers.

COIN and MONEY PROBLEMS

In most of the remaining applications problems, especially in coin, money, interest, mixture, and motion problems, it may be helpful to organize the information of the problem in a chart as in the word problems in Chapter 1. As in Chapter 1, the charts always have three columns, and the number of rows, usually three, may vary from problem to problem. The way most charts are set up, you fill in the first two columns with variables and given information. Then multiply the first column times the second column to get the third column. The two equations will be in the first and third columns.

EXAMPLE 3: **Five hamburgers and 4 soft drinks together cost \$12.50. If 6 hamburgers and 2 drinks together cost \$11.50, find the cost of each hamburger and drink.**

Solution: (Note: The chart is not particularly helpful here!)

Step 1: Let x = the cost of each hamburger
y = the cost of each soft drink

Step 2: Write the equations:

$$5x + 4y = \$12.50$$
$$6x + 2y = \$11.50$$

Step 3: To use the elimination method, multiply the second equation by -2.

$$5x + 4y = 12.50$$
$$-2(6x + 2y) = -2(11.50)$$

$$5x + 4y = 12.50$$
$$\underline{-12x - 4y = -23.00}$$
$$-7x = -10.50$$

$$x = \frac{-10.50}{-7}$$ or \$1.50 per hamburger.

Substitute x = \$1.50 into the first equation:

$$5x + 4y = 12.50$$
$$5(1.50) + 4y = 12.50$$
$$7.50 + 4y = 12.50$$
$$4y = 5.00$$
$$y = \$1.25$$

Step 4: Answer the question: x = \$1.25, cost of a hamburger,
y = \$0.75, cost of soft drink.

Step 5: Check: 5(\$1.50) + 4(\$1.25) = \$12.50 and 6(\$1.50) + 2(\$1.25) = \$11.50
\$7.50 + \$5.00 = \$12.50 \$9.00 + \$2.50 = \$11.50 Checks!

9. **If 4 hamburgers and 5 hot dogs cost $13, and 8 hamburgers and 4 hot dogs cost $20, how much does each hamburger and hot dog cost?**

10. **If 6 small pizzas and 3 large pizzas can be purchased for $54, while the cost for 4 small pizzas and 5 large pizzas is $66. Find the cost of each size of pizza.**

11. **A man bought 5 boxes of cereal and 3 jars of peanut butter for a total of $21. If he had bought 3 boxes of cereal and 5 jars of peanut butter, then the cost would have been $19. Find the cost of the cereal and the peanut butter.**

12. **Tickets are sold to a barbeque, with adult tickets selling for $10 and children tickets for $5. If 100 tickets are sold for a total of $900, how many of each ticket were sold?**

EXAMPLE 4. A box contains 30 coins, in nickels and dimes, worth \$2.40. How many of each coin are there? (Note: A chart may or may not be necessary for this!)

Solution: Let x = number of nickels, then
y = number of dimes.

	No. Coins	Each (¢)	Values
Nickels	x	5	5(x)
Dimes	y	10	10(y)
	30	(Leave blank!)	240

Write the equations:

$$x + y = 30$$
$$5x + 10y = 240$$

Solve the equations: Multiply the first equation by -5.

$$-5(x + y) = -5(30)$$
$$5x + 10y = 240$$

$$-5x + -5y = -150$$
$$5x + 10y = 240$$

$$5y = 90$$
$$y = 18 \text{ Dimes}$$

Substitute $y = 18$ into the first equation.

$$x + y = 30$$
$$x + 18 = 30$$
$$x = 12 \text{ Nickels}$$

Answer the question: x = 12 Nickels
y = 18 Dimes

Check:
12(.05) = \$ .60
18(.10) = <u>1.80</u>
\$2.40

[Note: You may also let x = number of dimes, and y = number of nickels.]

EXERCISES.

13. **A box contains 20 coins in quarters and dimes worth $3.80. How many of each coin are there?**

14. **A box contains 35 coins in quarters and nickels worth $3.15. How many of each coin are there?**

15. **A certain number of quarters and three more dimes than quarters are worth $7.30. How many of each are there?**

16. **A certain number of dimes and three less pennies than dimes are worth $7.67. How many of each are there?**

EXAMPLE 5. **A woman invests a total of $55,000, some at 6% and the rest at 9% interest. If the total interest earned in one year is $4170, how much was invested at each rate?**

Solution: Use the familiar formula from Chapter 1: **Principle X Rate = Interest**

Identify the variables: **Let x = Principle invested at 6% (0.06)**
y = Principle invested at 9% (0.09)

	Principle	Rate	Interest
6%	x	0.06	0.06 x
9%	y	0.09	0.09 y
	$55,000	(Leave blank!)	$4170

Write the equations:

$$x + y = \$55000$$
$$0.06x + 0.09y = \$\ 4170$$

Solve the equations: **Multiply first equation by -0.06.**

$$-0.06(x + y) = -0.06\ (\$55{,}000)$$
$$0.06x + 0.09y = \$4170$$

$$-0.06x - 0.06y = -\$3300$$
$$0.06x + 0.09y = \$4170$$

$$0.03y = 870$$
$$y = \frac{870}{0.03} \text{ or } \$29{,}000 \ @\ 9\%$$

Substitute y = $29,000 into the first equation.

$$x + y = \$55000$$
$$x + \$29{,}000 = \$55{,}000$$
$$x = \$26{,}000 \ @\ 6\%$$

Answer the question:

x = $26,000 @ 6%
y = $29,000 @ 9%

Check:

$26,000	and	$26,000(0.06) = $ 1560.00
<u>$29,000</u>		$29,000(0.09) = <u>2610.00</u>
$54,000		$ 4170.00 Total

EXERCISES.

17. **A total of $1,000 was invested, some at 8% and the rest at 6% simple interest. The total interest earned for the year was $76. How much was invested at each rate?**

18. **A total of $10,000 was invested, some at 12% and the rest at 10% simple interest. The total interest earned for the year was $1060. How much was invested at each rate?**

19. **A man has $10,000 to invest, some in a relatively safe account earning 5% interest per year, and the rest in more speculative investments earning 12% per year. If the total interest earned for the year was $955, how much was invested at each rate?**

20. **A farmer had chickens and pigs. There were a total of 60 heads and 200 feet. How many chickens, and how many pigs did the farmer have?**

ANSWERS 4.08

p. 370 - 378:

1. 21, 3; 2. 72, 48; 3. 81, 162; 4. 13, 35; 5. -4, 22; 6. 22, 28; 7. 32, 16; 8. 26, 22;
9. Hamburg \$2, Hot dog \$1; 10. Sm \$4, Lg \$10; 11. Cereal \$3, Peanut butter \$2;
12. Child 20, Adult 80; 13. 12Q, 8D; 14. 7Q, 28N; 15. 20Q, 23D; 16. 70D, 67P;
17. \$800 @ 8%, \$200 @ 6%; 18. \$3000 @ 12%, \$7000 @ 10%;
19. \$3500 @ 5%, \$6500 @ 12%; 20. 40 pigs, 20 chickens.