4.09 *Functional Notation*

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

There is a special notation called functional notation that is frequently used in mathematics when one variable is described in terms of another. The notation $f(x)$ [read f of x] is often used to name a second variable. Instead of writing $y = 3x + 2$ you may write $f(x) = 3x + 2$ or $g(x) = 3x + 2$ or perhaps even $y(x) = 3x + 2$. Any letter may be used. This notation indicates that f or g or y is a function of the variable x, which means that it can be expressed in terms of x. To find the value of $f(2)$, just replace each x with the value 2. To find the value of $f(4)$, replace each x in the given formula with the value 4. To find the value of $f(-3)$, replace each x in the formula with the value -3. Notice that $f(x)$ does NOT mean to multiply f times x.

EXAMPLE 1. Given $f(x) = 2x + 3$. Find the values of a) $f(0)$, b) $f(7)$, c) $f(-5)$.

Solution:

a) $f(0)$ means that $x = 0$. Replace the x with the value of 0.

$f(x) = 2x + 3$
$f(0) = 2(0) + 3$
$= 3$

b) $f(7)$ means that $x = 7$. Replace the x with the value of 7.

$f(x) = 2x + 3$
$f(7) = 2(7) + 3$
$= 14 + 3$ or 17

c) $f(-5)$ means that $x = -5$. Replace the x with the value of -5.

$f(x) = 2x + 3$
$f(-5) = 2(-5) + 3$
$= -10 + 3$ or -7

EXAMPLE 2. Given $g(x) = -6x^2 + 3x - 5$. Find the values of a) $g(2)$, b) $g(-5)$.

Solution:

a) $g(2)$ means that $x = 2$. Replace each x with the value of 2.

$g(x) = -6x^2 + 3x - 5$
$g(2) = -6\cdot 2^2 + 3\cdot 2 - 5$
$= -6\cdot 4 + 6 - 5$
$= -24 + 1$ or -23

b) $g(7)$ means that $x = -5$.

$g(x) = -6x^2 + 3x - 5$
$g(-5) = -6\cdot(-5)^2 + 3\cdot(-5) - 5$
$= -6\cdot 25 - 15 - 5$
$= -150 - 20$ or -170

EXERCISES. **Complete the following.**

1. $f(x) = 3x + 2$

a) $f(0) = 3(\ 0\ ) + 2$

$= \underline{\qquad\qquad}$

b) $f(2) = 3(\ 2\ ) + 2$

$=$

c) $f(4) = 3(\qquad) + 2$

$=$

d) $f(-3) =$

$=$

e) $f(\$) = 3(\qquad) + 2$

f) $f(\ *\ ) = \underline{\qquad\qquad}$

g) $f(\#\#\#) = \underline{\qquad\qquad}$

h) $f(Junk) = \underline{\qquad\qquad}$

2. $g(x) = -3x + 5$

a) $g(0) = -3(\qquad) + 5$

$= \underline{\qquad\qquad}$

b) $g(2) = -3(\qquad) + 5$

$=$

c) $g(4) = -3(\qquad) + 5$

$=$

d) $g(-3) =$

$=$

e) $g(\$) = \underline{\qquad\qquad}$

f) $g(\ *\ ) = \underline{\qquad\qquad}$

g) $g(\#\#\#) = \underline{\qquad\qquad}$

h) $g(Junk) = \underline{\qquad\qquad}$

3. $h(x) = -2x - 4$

a) $h(0) = -2(\qquad) - 4$

$=$

b) $h(2) =$

$=$

c) $h(4) =$

$=$

d) $h(-3) =$

$=$

e) $h(Junk) =$

4. $y(x) = -4x + 6$

a) $y(0) = -4(\qquad) + 6$

$=$

b) $y(2) =$

$=$

c) $y(4) =$

$=$

d) $y(-3) =$

$=$

e) $y(Junk) =$

5. $f(x) = x^2 + 3x + 4$

a) $f(0) = (0)^2 + 3(0) + 4$

$=$

b) $f(4) = (4)^2 + 3(4) + 4$

$=$

$=$

c) $f(-5) = (\quad)^2 + 3(\quad) + 4$

$=$

$=$

d) $f(\$) =$

e) $f(\text{Junk}) =$

6. $g(x) = x^2 + 2x - 4$

a) $g(0) = (\quad)^2 + 2(\quad) - 4$

$=$

b) $g(4) = (\quad)^2 + 2(\quad) - 4$

$=$

$=$

c) $g(-5) = (\quad)^2 + 2(\quad) - 4$

$=$

$=$

d) $g(\$) =$

e) $g(\text{Junk}) =$

7. $f(x) = -x^2 + 4x - 3$

a) $f(0) = -(\quad)^2 + 4(\quad) - 3$

$=$

$=$

b) $f(2) = -(\quad)^2 + 4(\quad) - 3$

$=$

$=$

c) $f(4) = -(\quad)^2 + 4(\quad) - 3$

$=$

$=$

d) $f(-2) =$

$=$

$=$

e) $f(\#) =$

8. $g(x) = -2x^2 - 2x + 4$

a) $g(0) = -2(\quad)^2 - 2(\quad) + 4$

$=$

$=$

b) $g(2) = -2(\quad)^2 - 2(\quad) + 4$

$=$

$=$

c) $g(4) =$

$=$

$=$

d) $g(-2) =$

$=$

$=$

e) $g(\pi) =$

9. $f(x) = 2x - 5$

a) $f(\$) = 2(\quad) - 5$

b) $f(3x) = 2(3x) - 5$

$= \underline{\qquad\qquad}$

c) $f(5y) = 2(\qquad) - 5$

$= \underline{\qquad\qquad}$

d) $f(5y + 3) = 2(5y+3) - 5$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

e) $f(5x - 4) = \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

10. $g(x) = -4x + 6$

a) $g(\$) = \underline{\qquad\qquad}$

b) $g(3x) = -4(\qquad) + 6$

$= \underline{\qquad\qquad}$

c) $g(5y) = -4(\qquad) + 6$

$= \underline{\qquad\qquad}$

d) $g(5y + 3) = -4(\qquad\qquad) + 6$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

e) $g(5x - 4) = \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

11. $f(x) = 5x + 7$

a) $f(x + 2) = 5(x + 2) + 7$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

b) $f(-3x + 7) = 5(\qquad\qquad) + 7$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

c) $f(x^2 - 3) = \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

d) $f(4x + 3y) = \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

e) $f(7y^2 + 3y - 4) = \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

12. $g(x) = -4x - 5$

a) $g(x + 2) = -4(\qquad\qquad) - 5$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

b) $g(-3x + 7) = \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

c) $g(x^2 - 3) = \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

d) $g(4x + 3y) = \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

e) $g(7y^2 + 3y - 4) = \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

$= \underline{\qquad\qquad\qquad}$

In the next example and exercises, notice that $f(x)$ and $g(x)$ are involved in the same problems.

EXAMPLE 2. Given $f(x) = 5x - 2$ and $g(x) = -3x + 4$,

find a) $f[f(x)]$, b) $g[g(x)]$, c) $f[g(x)]$, and d) $g[f(x)]$.

Solution.

Remember? $f[(\text{Junk})] = 5(\quad\quad) - 2$ $\quad\quad$ $g[(\text{Junk})] = -3(\quad\quad) + 4$

a) $f[f(x)] = 5(\ f(x)\ ) - 2$
$= 5(\ 5x - 2\ ) - 2$
$= 25x - 10 - 2$
$= 25x - 12$

b) $g[g(x)] = -3(\ g(x)\ ) + 4$
$= -3(\ -3x + 4\ ) + 4$
$= 9x - 12 + 4$
$= 9x - 8$

c) $f[g(x)] = 5(\ g(x)\ ) - 2$
$= 5(\ -3x + 4\ ) - 2$
$= -15x + 20 - 2$
$= -15x + 18$

d) $g[f(x)] = -3(\ f(x)\ ) + 4$
$= -3(\ 2x - 3\ ) + 4$
$= -6x + 9 + 4$
$= -6x + 13$

EXERCISES.

13. $f(x) = 3x + 2$ $\quad\quad$ $g(x) = 2x - 3$

a) $f[(\text{Junk})] = 3(\quad\quad) + 2$

b) $g[(\text{Junk})] = 2(\quad\quad) - 3$

c) $f[f(x)] = 3(\ f(x)\ ) + 2$
$= 3(\quad\quad) + 2$
= ______________
= ______________

d) $g[g(x)] = 2(\ g(x)\ ) - 3$
$= 2(\quad\quad) - 3$
= ______________
= ______________

e) $f[g(x)] = 3(\ g(x)\ ) + 2$
$= 3(\quad\quad) + 2$
= ______________
= ______________

f) $g[f(x)] = 2(\ f(x)\ ) - 3$
$= 2(\quad\quad) - 3$
= ______________
= ______________

14. $f(x) = -5x + 2$ $g(x) = -7x - 8$

a) $f[\,(\text{Junk})]$ = ______________ b) $g[\,(\text{Junk})]$ = ___________

c) $f[\,f(x)\,]$ = d) $g[\,g(x)\,]$ =

e) $f[\,g(x)\,]$ = f) $g[\,f(x)\,]$ =

15. $f(x) = 3x + 2$ $g(x) = x^2 - 5x + 3$ $h(x) = \dfrac{3x - 2}{4x}$

a) $f[g(x)]$ = b) $g[f(x)]$ =

c) $h[f(x)]$ = d) $h[g(x)]$ =

16. $f(x) = 3x - 2$ $g(x) = x^2 + 5x - 3$ $h(x) = \dfrac{3x + 2}{4x}$

a) $f[g(x)]$ = b) $g[f(x)]$ =

c) $h[f(x)]$ = d) $h[g(x)]$ =

ANSWERS 4.09

p. 379 - 384:

1a) 2, b) 8, c) 14, d) -7, e) 3$+2, f) 3*+2, g) 3(###)+2, h) 3(Junk)+2;

2a) 5, b) -1, c) -7, d) 14, e) -3$+5, f) -3*+5, g) -3(###)+5, h) -3(Junk)+5;

3a) -4, b) -8, c) -12, d) 2, e) -2(Junk)-4; 4a) 6, b) -2, c) -10, d) 18, e) -4(Junk)+6;

5a) 4, b) 32, c) 14, d) $\$^2+3\$+4$, e) $(\text{Junk})^2+3(\text{Junk})+4$;

6a) -4, b) 20, c) 11, d) $\$^2+2\$-4$, e) $(\text{Junk})^2+2(\text{Junk})-4$;

7a) -3, b) 1, c) -3, d) -15, e) $-\#^2+4\#-3$; 8a) 4, b) -8, c) -36, d) 0, e) $-2\pi^2-2\pi+4$;

9a) 2$-5, b) 6x-5, c) 10y-5, d) 10y+1, e) 10x-13;

10a) -4$+6, b) -12x+6, c) -20y+6, d) -20y-6, e) -20x+22;

11a) 5x+17, b) -15x+42, c) $5x^2-8$, d) 20x+15y+7, e) $35y^2+15y-13$;

12a) -4x-13, b) 12x-33, c) $-4x^2+7$, d) -16x-12y-5, e) $-28y^2-12y+11$;

13a) 3(Junk)+2, b) 2(Junk)-3, c) 9x+8, d) 4x-9, e) 6x-7, f) 6x+1;

14a) -5(Junk)+2, b) -7(Junk)-8, c) 25x-8, d) 49x+48, e) 35x+42, f) 35x-22;

15a) $3x^2-15x+11$, b) $9x^2-3x-3$, c) $\frac{9x+4}{4(3x+2)}$, d) $\frac{3x^2-15x+7}{4(x^2-5x+3)}$;

16a) $3x^2+15x-11$, b) $9x^2+3x-9$, c) $\frac{9x-4}{4(3x-2)}$, d) $\frac{3x^2+15x-7}{4(x^2+5x-3)}$.