5.01 *Square Roots*

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Before beginning this section on square roots, remember what it means to **square a number**. Squaring a number is the operation of multiplying a number times itself. A **perfect square** is what you get when you square a number. In other words, a perfect square is simply the square of a whole number. Before you attempt to learn and understand square roots, you must be *very* familiar with the perfect squares. The first few examples are given in the following list.

$$\mathbf{0^2 = 0}$$
$$\mathbf{1^2 = 1}$$
$$\mathbf{2^2 = 4}$$
$$\mathbf{3^2 = 9}$$
$$\mathbf{4^2 = 16}$$
$$\mathbf{5^2 = 25}$$
$$\mathbf{6^2 = 36}$$
$$\mathbf{7^2 = 49}$$
$$\mathbf{8^2 = 64}$$
$$\mathbf{9^2 = 81}$$
$$\mathbf{10^2 = 100}$$
$$\mathbf{11^2 = 121}$$
$$\mathbf{12^2 = 144}$$
$$\mathbf{13^2 = 169.}$$

Of course, the list goes on, but these are the important ones, the ones you will need to know and recognize in order to understand this section.

Taking a **square root** of a number is actually the **inverse operation of squaring**. As subtraction is the opposite of addition, and division is the opposite of multiplication, taking a square root is the opposite of squaring. Did you ever do something and then wish you could undo it? Did you ever put on a jacket, then realize that it was too hot to be wearing a jacket? You probably un-did putting on the jacket by taking off the jacket. Did you ever do something on the computer that you didn't mean to do? Sometimes you can un-do what you did by hitting the un-do button. Did you ever square a number like 7 (which gives you 49), and then you thought to yourself, "I wish I hadn't done that!!" What could you do to get from 49 back to the 7 that you began with? The answer to this question is called **square root,** the opposite of squaring a number.

The symbol $\sqrt{49}$ is read "square root of 49," which means "what number would you square in order to get 49." In general, the expression $\sqrt{x}$, which could also be written $\sqrt[2]{x}$, means the **square root of x**, or in other words, **what squared would equal x?** The symbol $\sqrt{}$ is called a **square root sign** (or a **radical sign**). The quantity inside the square root sign is called the **radicand**, and the **2** is the **index of the radical**.

In this assignment, it is important to know the perfect squares through $\mathbf{13^2 = 169}$. See previous page.

EXAMPLE 1. Fill in the blanks below to find $\sqrt{25}$.

$\sqrt{25}$ = ______ because ( )^2 = _______.

Solution: $\sqrt{25}$ = __5__ because (5)^2 = __25__.

Therefore, the square root of 25 is 5.

EXERCISES. Fill in the blanks to find the square roots.

1. $\sqrt{49}$ = _______ because $7^2 = 49$.

2. $\sqrt{144}$ = _______ because ( )^2 = 144.

3. $\sqrt{9}$ = _______ because ( )^2 = _______.

4. $\sqrt{16}$ = _______ because ( )^2 = _______.

5. $\sqrt{64}$ = _______ because ( )^2 = _______.

6. $\sqrt{4}$ = _______ because ($)^2$ = _______.

7. $\sqrt{0}$ = _______ because ($)^2$ = _______.

8. $\sqrt{1}$ = _______ because ($)^2$ = _______.

9. $\sqrt{81}$ = _______. 10. $\sqrt{121}$ = _______. 11. $\sqrt{100}$ = _______.

12. $\sqrt{36}$ = _______. 13. $\sqrt{169}$ = _______.

In the next exercises, you will need to remember and use the law of exponents: $\mathbf{(x^m)^n = x^{mn}}$. When you raise an exponent to a power, you multiply the exponents. In particular, when you square an exponent, you multiply the exponent times 2.

EXAMPLE 2. Simplify $\mathbf{(x^3)^2}$

Solution: $\mathbf{(x^3)^2 = x^6}$

EXAMPLE 3. Simplify $\mathbf{(x^4)^2}$

Solution: $\mathbf{(x^4)^2 = x^8}$

EXAMPLE 4. Complete the blanks in order to find the $\sqrt{x^{10}}$.

$\sqrt{x^{10}}$ = _______ because **(** $\mathbf{)^2}$ = ________**.**

Solution: $\sqrt{x^{10}}$ = $\underline{\mathbf{x^5}}$ because **(** $\mathbf{x^5}$ $\mathbf{)^2}$ = $\underline{\mathbf{x^{10}}}$.

EXERCISES. Complete the blanks in order to find the square roots.

14. $\sqrt{x^4}$ = _____ because ($)^2$ = ________.

15. $\sqrt{x^6}$ = _____ because ($)^2$ = ________.

16. $\sqrt{x^8}$ = _____ because ($)^2$ = ________.

17. $\sqrt{x^{10}}$ = _____ because ($)^2$ = ________.

18. $\sqrt{x^{12}}$ = _____ because ($)^2$ = ________.

19. $\sqrt{x^{14}}$ = _____ because ($)^2$ = ________.

20. $\sqrt{x^{20}}$ = _____ because ($)^2$ = ________.

Did you notice that each time you take the square root of an exponent, you take half of the exponent, or you divide the exponent by 2? Doesn't this seem reasonable? If you raise a power to a power, then you multiply exponents. When you take a square root (the opposite of squaring) of an exponent, then you do the opposite of "multiply exponents", which is to "divide the exponents" by 2. Now that you have the idea, you don't need to show all the steps.

21. $\sqrt{x^{24}}$ = _____.   22. $\sqrt{x^{28}}$ = _____.   23. $\sqrt{x^{48}}$ = _____.

24. $\sqrt{x^{50}}$ = _____.   25. $\sqrt{x^{36}}$ = _____.   26. $\sqrt{x^{100}}$ = _____.

In the following examples and exercises, remember that you must take the square root of the number, and divide the exponents by 2!

EXAMPLE 5. Find the square root. $\sqrt{9x^{12}}$

Solution: $\sqrt{9x^{12}} = \mathbf{3x^6}$.

EXAMPLE 6. Find the square root. $\sqrt{16x^{16}}$

Solution: $\sqrt{16x^{16}} = \mathbf{4x^8}$.

EXERCISES. **Find the square roots.**

27. $\sqrt{49x^{6}}$ 28. $\sqrt{36x^{4}}$ 29. $\sqrt{25x^{12}}$ 30. $\sqrt{4x^{10}}$

31. $\sqrt{16x^{6}}$ 32. $\sqrt{49x^{12}}$ 33. $\sqrt{36x^{36}}$ 34. $\sqrt{25x^{100}}$

35. $\sqrt{9x^{18}}$ 36. $\sqrt{169x^{22}}$ 37. $\sqrt{36x^{14}}$ 38. $\sqrt{64x^{8}}$

39. $\sqrt{25x^{20}}$ 40. $\sqrt{16x^{80}}$ 41. $\sqrt{100x^{30}}$ 42. $\sqrt{121x^{2}}$

43. $\sqrt{81x^{16}}$ 44. $\sqrt{144x^{4}}$ 45. $\sqrt{64x^{100}}$ 46. $\sqrt{36x^{144}}$

If the root to be taken is not a **perfect power**, then sometimes it can be simplified by using the **product property of square roots.**

Product Property of Square Roots

$$\sqrt{a \bullet b} = \sqrt{a} \bullet \sqrt{b}$$

Because the product property of square roots is a property of **real numbers**, the **radicands** must be positive numbers. The property does not apply if both a and b are negative numbers.

To simplify a square root by this property, it may be helpful to think of the "radical two-step":

Radical Two-Step

Step 1: SORT
Step 2: SQRT

In the first step, you must **"sort"** the radical, placing the **"perfect squares"** in the first radical and the other "leftover" factors in the second radicals. In the second step, you take the **square root** of the perfect square, and just bring down the "leftover" radical. The square root is simplified completely when the quantity in the radical (i.e., the radicand) is minimized. The next examples will illustrate what it means to simplify a radical expression.

EXAMPLE 7. $\sqrt{45}$

Step 1. Find a perfect square factor of 45, and write 45 as a product. The perfect square is 9; the "left-over" factor is 5, and the product is 9·5.
$\sqrt{9} \bullet \sqrt{5}$

Step 2. Take the square root of 9, which is 3. Keep the square root sign on the 5.
$3 \bullet \sqrt{5}$ This is simplified, since radicand is minimized.

If you wanted to find the approximate value of $\sqrt{45}$, you could begin by thinking that $\sqrt{45}$ is slightly smaller than $\sqrt{49}$ which is 7. Therefore, $\sqrt{45}$ is somewhat smaller than 7. You may have a square root($\sqrt{x}$ or $\sqrt{\ }$) key on your calculator. If so, they try pressing [45], [$\sqrt{x}$]. You should get 6.7082039325 (the accuracy varies from calculator to calculator!). If this does not work (for graphing and other calculators), then try pressing [$\sqrt{x}$], then [45]. Rounding to the nearest hundredth, you should have 6.71, which as you can see is slightly less than 7.

EXAMPLE 8. Simplify $\sqrt{40}$

Step 1. Find a perfect square factor of 40, and write as a product. The perfect square is 4, the leftover factor is 10, and the product is 4·10.
$\sqrt{4} \bullet \sqrt{10}$

Step 2. Take square root of 4, which is 2. Keep the square root sign on the 10.
$2 \bullet \sqrt{10}$ or $2\sqrt{10}$

EXAMPLE 9. Simplify $\sqrt{72}$

Step 1. Find a perfect square factor of 72, and write as a product. If you said that the perfect square is 9, the leftover factor is 8, and the product is 9·8. However, this leaves a perfect square factor of 4 in the 8. You could also say the perfect square factor is 36, and the product is 36·2. If you have a choice as in this example, it is better to use the largest possible perfect square--that is, 36·2.
$\sqrt{36} \bullet \sqrt{2}$.

Step 2. Take square root of 36. Final answer: $6\sqrt{2}$.

EXERCISES. **Simplify the radicals completely**

47. $\sqrt{300}$

$\sqrt{100} \bullet \sqrt{3}$

____

48. $\sqrt{150}$

$\sqrt{25} \bullet \sqrt{\quad}$

____

49. $\sqrt{50}$

$\sqrt{\quad} \bullet \sqrt{\quad}$

____

50. $\sqrt{20}$

$\sqrt{\quad} \bullet \sqrt{\quad}$

____

51. $\sqrt{28}$

$\sqrt{\quad} \bullet \sqrt{\quad}$

____

52. $\sqrt{98}$

$\sqrt{\quad} \bullet \sqrt{\quad}$

____

53. $\sqrt{27}$

54. $\sqrt{8}$

55. $\sqrt{60}$

56. $\sqrt{44}$

57. $\sqrt{125}$

58. $\sqrt{24}$

59. $\sqrt{84}$

60. $\sqrt{75}$

61. $\sqrt{90}$

62. $\sqrt{52}$

63. $\sqrt{175}$

64. $\sqrt{54}$

65. $\sqrt{200}$ 66. $\sqrt{32}$ 67. $\sqrt{80}$

68. $\sqrt{48}$ 69. $\sqrt{120}$ 70. $\sqrt{72}$

Sometimes there are variables in the square root. As you may have noticed, when you take the square root of a variable raised to a power, you must divide the exponent by 2. If the power is even, then this is no problem. However, if the variable is raised to an odd power, then the procedure is explained in the next examples.

EXAMPLE 10. **Simplify** $\sqrt{x^3}$

Solution:

Step 1. The perfect square factor is x^2, the leftover factor is x, and you can write it as $x^2 \cdot x$.

$$\sqrt{x^2} \bullet \sqrt{x}$$

Step 2. Take the square root of x^2, which is x.

$$x\sqrt{x}$$

EXAMPLE 11. **Simplify** $\sqrt{x^5}$

Solution:

Step 1. The perfect square factors are x^2 and x^4. Use the largest perfect square x^4, which leaves a left-over factor of x. Write x^5 as the product $x^4 \cdot x$.

$$\sqrt{x^4} \bullet \sqrt{x}$$

Step 2. Take the square root of x^4, which is x^2.

$$x^2\sqrt{x}\ .$$

EXAMPLE 12. **Simplify** $\sqrt{50x^9}$

Solution:

Step 1. The perfect square factors are 25 and the largest of the factors x^2, x^4, x^6, x^8. Use 25 x^8 as the perfect square, which leaves a left-over factor of 2 x. Write 50 x^9 as the product 25 $x^8 \cdot 2x$.

$$\sqrt{25x^8} \bullet \sqrt{2x}$$

Step 2. Take the square root of 25 x^8, which is 5 x^4.

$$5x^4\sqrt{2x}$$

EXAMPLE 13. **Simplify** $\sqrt{250x^8y^5}$

Solution:

Step 1. First the perfect square factors are 25, x^8, and since y is raised to an odd power, use one less than 5, y^4. Using 25 x^8y^4 as the perfect square, the left-over factor is 10y. Write 250 x^8y^5 as 25 $x^8y^4 \cdot 10y$.

$$\sqrt{25x^8y^4} \bullet \sqrt{10y}$$

Step 2. Take square root of 25 x^8y^4. which is 5 x^4y^2.

$$5x^4y^2\sqrt{10y}$$

EXERCISES. **Simplify the radicals completely.**

71. $\sqrt{75x^4}$

$\sqrt{25x^4} \bullet \sqrt{3}$

____

72. $\sqrt{75x^5}$

$\sqrt{25x^4} \bullet \sqrt{3x}$

____

73. $\sqrt{50x^7}$

$\sqrt{25x^6} \bullet \sqrt{\quad}$

____

74. $\sqrt{28x^7}$

$\sqrt{4x^6} \bullet \sqrt{\quad}$

____

75. $\sqrt{18x^8}$

$\sqrt{9x^8} \bullet \sqrt{\quad}$

____

76. $\sqrt{18x^9}$

$\sqrt{\quad} \bullet \sqrt{\quad}$

____

77. $\sqrt{98x^9}$

78. $\sqrt{99x^{10}}$

79. $\sqrt{60x^2}$

80. $\sqrt{150x^{13}}$

81. $\sqrt{72x^9}$

82. $\sqrt{32x^6}$

83. $\sqrt{75x^8y^9} =$

84. $\sqrt{40x^{11}y^6}$

85. $\sqrt{98x^7y^{13}}$

86. $\sqrt{63x^{15}y^8}$

87. $\sqrt{40x^6y^{16}}$

88. $\sqrt{175x^{13}y^9}$

89. $\sqrt{72x^9y^{16}}$

90. $\sqrt{48x^4y^{25}}$

91. $\sqrt{32x^4y^{16}}$

92. $\sqrt{144x^9y^{144}}$

93. $\sqrt{98x^7y^{13}}$

94. $\sqrt{300x^{15}y^{25}}$

ANSWERS 5.01

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1. 7; 2. 12; 3. 3; 4. 4; 5. 8; 6. 2; 7. 0; 8. 1; 9. 9; 10. 11; 11. 10; 12. 6; 13. 13; 14. x^2; 15. x^3; 16. x^4; 17. x^5; 18. x^6; 19. x^7; 20. x^{10}; 21. x^{12}; 22. x^{14}; 23. x^{24}; 24. x^{25}; 25. x^{18}; 26. x^{50}; 27. $7x^3$; 28. $6x^2$; 29. $5x^6$; 30. $2x^5$; 31. $4x^3$; 32. $7x^6$; 33. $6x^{18}$; 34. $5x^{50}$; 35. $3x^9$; 36. $13x^{11}$; 37. $6x^7$; 38. $8x^4$; 39. $5x^{10}$; 40. $4x^{40}$; 41. $10x^{15}$; 42. $11x$; 43. $9x^8$; 44. $12x^2$;

45. $8x^{50}$; 46. $6x^{72}$; 47. $10\sqrt{3}$; 48. $5\sqrt{6}$; 49. $5\sqrt{2}$; 50. $2\sqrt{5}$; 51. $2\sqrt{7}$;

52. $7\sqrt{2}$; 53. $3\sqrt{3}$; 54. $2\sqrt{2}$; 55. $2\sqrt{15}$; 56. $2\sqrt{11}$; 57. $5\sqrt{5}$; 58. $2\sqrt{6}$;

59. $2\sqrt{21}$; 60. $5\sqrt{3}$; 61. $3\sqrt{10}$; 62. $2\sqrt{13}$; 63. $5\sqrt{7}$; 64. $3\sqrt{6}$; 65. $10\sqrt{2}$;

66. $4\sqrt{2}$; 67. $4\sqrt{5}$; 68. $4\sqrt{3}$; 69. $2\sqrt{30}$; 70. $6\sqrt{2}$; 71. $5x^2\sqrt{3}$; 72. $5x^2\sqrt{3x}$;

73. $5x^3\sqrt{2x}$; 74. $2x^3\sqrt{7x}$; 75. $3x^4\sqrt{2}$; 76. $3x^4\sqrt{2x}$; 77. $7x^4\sqrt{2x}$; 78. $3x^5\sqrt{11}$;

79. $2x\sqrt{15}$; 80. $5x^6\sqrt{6x}$; 81. $6x^4\sqrt{2x}$; 82. $4x^3\sqrt{2}$; 83. $5x^4y^4\sqrt{3y}$; 84. $2x^5y^3\sqrt{10x}$;

85. $7x^3y^6\sqrt{2xy}$; 86. $3x^7y^4\sqrt{7x}$; 87. $2x^3y^5\sqrt{10}$; 88. $5x^6y^4\sqrt{7xy}$; 89. $6x^4y^8\sqrt{2x}$;

90. $4x^3y^{11}\sqrt{3y}$; 91. $4x^2y^8\sqrt{2}$; 92. $12x^4y^{72}\sqrt{x}$; 93. $7x^3y^6\sqrt{2xy}$; 94. $10x^7y^{12}\sqrt{3xy}$.