5.04 Multiplying Square Roots

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PRODUCTS OF MONOMIAL EXPRESSIONS

When multiplying square roots, use the **product property of square roots, written in reverse order**: $\sqrt{a}\bullet\sqrt{b}=\sqrt{ab}$. Remember as before, that this is a property of real numbers, so a and b must not be negative numbers. The property also applies for cube roots, fourth roots, etc., so the rule can be written in general for radicals: $\sqrt[n]{a}\bullet\sqrt[n]{b}=\sqrt[n]{ab}$ where "**a**" and "**b**" are non-negative quantities, and "**n**" represents the **index** or the **order of the radical**.

EXAMPLE 1. $\sqrt{5}\bullet\sqrt{7}$.

Solution: $\sqrt{35}$ This radical cannot be simplified.

EXAMPLE 2. $\sqrt{10}\bullet\sqrt{6}$.

Solution: $\sqrt{60}$ This radical can and must be simplified.

$\sqrt{4}\,\sqrt{15}$

$2\sqrt{15}$

EXAMPLE 3. Simplify $4\sqrt{5}\bullet 3\sqrt{7}$ **and give the decimal approximation.**

Solution: This is actually the product of four numbers:

$4\bullet\sqrt{5}\bullet 3\bullet\sqrt{7}$ You can multiply in any order, so multiply 4 times 3

$(4\bullet 3)\bullet\sqrt{5}\bullet\sqrt{7}$ and $\sqrt{5}$ times $\sqrt{7}$.

$12\sqrt{35}$.

As a check, calculate the value of $4\sqrt{5}\bullet 3\sqrt{7}$ and the value of $12\sqrt{35}$. The decimal values you obtain in these calculations should be exactly the same. Rounded to nearest hundredth = **70.99**.

EXAMPLE 4. Simplify $4\sqrt{15} \bullet 3\sqrt{10}$, and give the decimal approximation.

Solution:

$$4 \bullet \sqrt{15} \bullet 3 \bullet \sqrt{10}$$

$$(4 \bullet 3) \bullet \sqrt{15} \bullet \sqrt{10}$$

$$12\sqrt{150}$$

$$12\sqrt{25}\,\sqrt{6}$$

$$12 \bullet 5\sqrt{6} \text{ or } 60\sqrt{6}.$$

The decimal approximation, rounded to the nearest hundredth, is **146.97**.

EXERCISES. Simplify the radical expressions. Give decimal approximations.

1. $\sqrt{5} \bullet \sqrt{3}$

$\sqrt{\quad}$

2. $\sqrt{7} \bullet \sqrt{2}$

$\sqrt{\quad}$

3. $\sqrt{10} \bullet \sqrt{3}$

4. $\sqrt{6} \bullet \sqrt{7}$

5. $\sqrt{6} \bullet \sqrt{3}$

$\sqrt{\quad}$

$\sqrt{\quad}\,\sqrt{\quad}$

___ $\sqrt{\quad}$

6. $\sqrt{14} \bullet \sqrt{2}$

$\sqrt{\quad}$

$\sqrt{\quad}\,\sqrt{\quad}$

___ $\sqrt{\quad}$

7. $\sqrt{15} \bullet \sqrt{3}$

8. $\sqrt{10} \bullet \sqrt{5}$

9. $2\sqrt{5} \bullet 7\sqrt{3}$

___ $\sqrt{\quad}$

10. $5\sqrt{7} \bullet 3\sqrt{2}$

___ $\sqrt{\quad}$

11. $20\sqrt{10} \bullet 5\sqrt{3}$

12. $9\sqrt{6} \bullet 4\sqrt{7}$

13. $3\sqrt{6} \bullet 5\sqrt{2}$

___ $\sqrt{\quad}$

___ $\sqrt{\quad}\,\sqrt{\quad}$

____

____

14. $6\sqrt{14} \bullet 5\sqrt{2}$

___ $\sqrt{\quad}$

___ $\sqrt{\quad}\,\sqrt{\quad}$

____

____

15. $5\sqrt{10} \bullet 9\sqrt{6}$

16. $4\sqrt{14} \bullet 9\sqrt{7}$

EXAMPLE 5. Simplify $\sqrt{46} \bullet \sqrt{69}$, and check by calculating decimal approximations.

Solution #1 **(The hard way!):** $\sqrt{\mathbf{3174}}$ Multiply the radicands **46 · 69 = 3174**.

The problem is how do you simplify $\sqrt{\mathbf{3174}}$? (Answer: See Solution #2 below!)

Solution #2 **(The EASY way!):** $\sqrt{2 \bullet 23}\,\sqrt{3 \bullet 23}$ Factor the numbers in a single radical.

$\sqrt{2 \bullet 23 \bullet 3 \bullet 23}$ Form two radicals, sorting perfect squares.

$\sqrt{23^2} \bullet \sqrt{2 \bullet 3}$

$23\sqrt{6}$

Calculator: $\sqrt{46} \bullet \sqrt{69} =$ 56.338264084

$23\sqrt{6}$ = 56.338264084. Rounded to nearest hundredth: **56.34**.

Notice that in this example and in the exercises that follow, it is probably easier to break the numbers down instead of multiplying them out. In the process, you are looking for "pairs" of numbers. Remember, "if you ain't got no pair, then you ain't got no square!!"

EXERCISES. **Simplify the radical expressions. Give decimal approximations.**

17. $\sqrt{35} \bullet \sqrt{77}$

$\sqrt{\qquad}$

$\sqrt{\quad}\ \sqrt{\quad}$

$\_\_\ \sqrt{\quad}$

18. $\sqrt{35} \bullet \sqrt{55}$

$\sqrt{\qquad}$

$\sqrt{\quad}\ \sqrt{\quad}$

$\_\_\ \sqrt{\quad}$

19. $\sqrt{77} \bullet \sqrt{55}$

20. $\sqrt{38} \bullet \sqrt{95}$

21. $\sqrt{85} \bullet \sqrt{34}$

22. $\sqrt{39} \bullet \sqrt{65}$

23. $\sqrt{85} \bullet \sqrt{20}$

24. $\sqrt{39} \bullet \sqrt{52}$

25. $4\sqrt{3} \cdot 6\sqrt{15}$ **26.** $2\sqrt{6} \cdot 5\sqrt{10}$ **27.** $4\sqrt{15} \cdot 11\sqrt{21}$ **28.** $5\sqrt{85} \cdot 3\sqrt{51}$

29. $3\sqrt{38} \cdot 2\sqrt{57}$ **30.** $10\sqrt{18} \cdot 3\sqrt{15}$ **31.** $6\sqrt{35} \cdot 8\sqrt{45}$ **32.** $6\sqrt{155} \cdot 5\sqrt{124}$

THE DISTRIBUTIVE PROPERTY

Frequently, products of radical expressions involve the **distributive property**: **a(b+c) = ab + ac.** The same techniques that were used in Chapter 1 will be used again here.

EXAMPLE 6. **Use the distributive property to multiply**

$$6(2\sqrt{3} + 5\sqrt{7})$$

Solution:

$$6 \cdot 2\sqrt{3} + 6 \cdot 5\sqrt{7}$$

$$12\sqrt{3} + 30\sqrt{7}$$

NOTE: Remember when multiplying square roots and coefficients, multiply the coefficients together, to obtain the coefficient, and multiply the radicands together to obtain the radicands. In other words, multiply the numbers that are outside the radical and keep them outside, and multiply the numbers that are inside the radicals and keep them inside the radical.

EXAMPLE 7. **Use the distributive property to multiply**

$\sqrt{6}\,(2\sqrt{5} + 3\sqrt{7})$

Solution: $\sqrt{6} \bullet 2\sqrt{5} + \sqrt{6} \bullet 3\sqrt{7}$

$2\sqrt{30} + 3\sqrt{42}$

EXAMPLE 8. **Use the distributive property to multiply**

$4\sqrt{6}\,(2\sqrt{5} - 3\sqrt{7})$

Solution: $4\sqrt{6} \bullet 2\sqrt{5} - 4\sqrt{6} \bullet 3\sqrt{7}$

$8\sqrt{30} - 12\sqrt{42}$

EXAMPLE 9. **Use the distributive property to multiply**

$4\sqrt{6}\,(2\sqrt{15} + 3\sqrt{10})$

Solution: $4\sqrt{6} \bullet 2\sqrt{15} + 4\sqrt{6} \bullet 3\sqrt{10}$

$8\sqrt{90} + 12\sqrt{60}$ Watch out! These radicals simplify!

$8\sqrt{9}\,\sqrt{10} + 12\sqrt{4}\,\sqrt{15}$

$8 \bullet 3\sqrt{10} + 12 \bullet 2\sqrt{15}$

$24\sqrt{10} + 24\sqrt{15}$

EXERCISES. **Use the distributive property to multiply. Simplify completely.**

33. $5\,(2\sqrt{7} + 4\sqrt{3})$

34. $8\,(3\sqrt{2} - 7\sqrt{5})$

35. $\sqrt{5}\,(2\sqrt{7} - 4\sqrt{3})$

36. $\sqrt{7}\,(3\sqrt{2} - 6\sqrt{5})$

37. $6\sqrt{5}\,(2\sqrt{7} + 4\sqrt{3})$

38. $8\sqrt{7}\,(3\sqrt{2} - 6\sqrt{5})$

39. $5\sqrt{6}\,(7\sqrt{2} - 4\sqrt{3})$

40. $8\sqrt{10}\,(3\sqrt{6} + 7\sqrt{5})$

41. $8\sqrt{10}\,(2\sqrt{6} - 3\sqrt{2})$

42. $2\sqrt{6}\,(4\sqrt{3} + 5\sqrt{2})$

43. $4\sqrt{15}\,(3\sqrt{30} + 2\sqrt{10})$

44. $2\sqrt{21}\,(3\sqrt{3} + 5\sqrt{14})$

45. $3\sqrt{20}\,(2\sqrt{5} - \sqrt{15})$

46. $5\sqrt{10}\,(3\sqrt{15} + 8\sqrt{30})$

PRODUCTS OF BINOMIALS -- "F OI L"

Sometimes the radical expressions are in the form of binomials. To take the product of two binomial expressions with radicals involved, just use the "F OI L" method as in Chapter 2. Do you remember?

Product of Two Binomials: F OI L*

$$(a + b)(c + d) = \overset{F}{ac} + \overset{O}{ad} + \overset{I}{bc} + \overset{L}{bd}$$

*** First, Outer, Inner, Last**

EXAMPLE 10. **Find the product by "F OI L." Check by calculating the problem and the answer, and round to the nearest hundredth.**

$(4+\sqrt{6})(2+\sqrt{6})$

Solution:

$$\overset{F}{8} + \overset{O}{4\sqrt{6}} + \overset{I}{2\sqrt{6}} + \overset{L}{\sqrt{6}\sqrt{6}}$$

$$8 + 4\sqrt{6} + 2\sqrt{6} + \sqrt{36}$$

$$8 + 4\sqrt{6} + 2\sqrt{6} + 6$$

$$14 + 6\sqrt{6}$$

Calculate: $(4+\sqrt{6})(2+\sqrt{6}) = 28.6969384567$

$14 + 6\sqrt{6} = 28.6969384567$. Rounded to nearest hundredth: **28.70**

EXAMPLE 11. **Find the product by "F OI L." Check by calculating the problem and the answer, and round to the nearest hundredth.**

$(\mathbf{4\sqrt{2} + 7\sqrt{3})(2\sqrt{2} - 5\sqrt{3}})$

Solution:

$$\begin{array}{cccc} \mathbf{F} & \mathbf{O} & \mathbf{I} & \mathbf{L} \end{array}$$

$$\mathbf{4\sqrt{2} \bullet 2\sqrt{2} - 4\sqrt{2} \bullet 5\sqrt{3} + 7\sqrt{3} \bullet 2\sqrt{2} - 7\sqrt{3} \bullet 5\sqrt{3}}$$

$$\mathbf{8 \bullet 2 \quad - \quad 20\sqrt{6} \quad + \quad 14\sqrt{6} \quad - \quad 35 \bullet 3}$$

$$\mathbf{16 \quad - \quad 6\sqrt{6} \quad - \quad 105}$$

$$\mathbf{-89 \; - \; 6\sqrt{6}}$$

Calculate: $(\mathbf{4\sqrt{2} + 7\sqrt{3})(2\sqrt{2} - 5\sqrt{3}}) = -103.696938457$

$\mathbf{-89 \; - \; 6\sqrt{6}} = -103.696938457$. Nearest hundredth: $\mathbf{-103.70}$

EXAMPLE 12. **Find the product by "F OI L." Check by calculating the problem and the answer, and round to the nearest hundredth.**

$(\mathbf{4\sqrt{6} - 7\sqrt{3}})^2$

Solution: Rewrite as $(\mathbf{4\sqrt{6} - 7\sqrt{3})(4\sqrt{6} - 7\sqrt{3}})$

$$\begin{array}{cccc} \mathbf{F} & \mathbf{O} & \mathbf{I} & \mathbf{L} \end{array}$$

$$\mathbf{4\sqrt{6} \bullet 4\sqrt{6} - 4\sqrt{6} \bullet 7\sqrt{3} - 4\sqrt{6} \bullet 7\sqrt{3} + 7\sqrt{3} \bullet 7\sqrt{3}}$$

$$\mathbf{16 \bullet 6 \quad - \quad 28\sqrt{18} \quad - \quad 28\sqrt{18} \quad + \quad 49 \bullet 3}$$

$$\mathbf{96 \quad - \quad 56\sqrt{18} \quad + \quad 147}$$

$$\mathbf{243 \quad - \quad 56\sqrt{9}\sqrt{2}}$$

$$\mathbf{243 \quad - \quad 56 \bullet 3\sqrt{2}}$$

$$\mathbf{243 \quad - \quad 168\sqrt{2}}$$

Calculate: $(\mathbf{4\sqrt{6} - 7\sqrt{3}})^2 = 5.41212152132$

$\mathbf{243 - 168\sqrt{2} =} \; 5.41212152132$. Nearest hundredth: **5.41**

EXERCISES. **Find the products of the binomials. Check by calculating the problem and the answer, and round to the nearest hundredth.**

47. $(6+\sqrt{2})(4+\sqrt{2})$

F O I L

48. $(6-\sqrt{2})(4-\sqrt{2})$

F O I L

49. $(8-\sqrt{3})(6+\sqrt{3})$

50. $(5-\sqrt{2})(7+\sqrt{2})$

51. $(\sqrt{2}+\sqrt{10})(\sqrt{2}+\sqrt{6})$

52. $(\sqrt{6}-\sqrt{3})(\sqrt{2}-\sqrt{3})$

53. $(\sqrt{5}-\sqrt{2})(\sqrt{10}-\sqrt{6})$

54. $(\sqrt{5}-\sqrt{2})(\sqrt{10}+\sqrt{6})$

55. $(\sqrt{10} - \sqrt{6})(\sqrt{10} + \sqrt{6})$

56. $(\sqrt{5} - \sqrt{2})(\sqrt{5} + \sqrt{2})$

57. $(4\sqrt{5} - 3\sqrt{6})(4\sqrt{5} + 3\sqrt{6})$

58. $(5\sqrt{6} - \sqrt{2})(5\sqrt{6} + \sqrt{2})$

59. $(5\sqrt{6} - 2\sqrt{3})(5\sqrt{6} + 2\sqrt{3})$

60. $(2\sqrt{3} - 5\sqrt{6})(2\sqrt{3} + 5\sqrt{6})$

61. $(6\sqrt{2} + 4\sqrt{3})(3\sqrt{2} + 8\sqrt{3})$

62. $(6\sqrt{2} - 4\sqrt{3})(3\sqrt{2} - 8\sqrt{3})$

63. $(2\sqrt{6} - 3\sqrt{10})(3\sqrt{2} - 4\sqrt{3})$

64. $(4\sqrt{5} + 8\sqrt{2})(3\sqrt{10} + 4\sqrt{5})$

65. $(3\sqrt{6} - 2\sqrt{3})(5\sqrt{6} + 4\sqrt{3})$

66. $(6\sqrt{3} - 5\sqrt{6})(2\sqrt{3} - 8\sqrt{6})$

67. $(6 - \sqrt{3})^2$

68. $(\sqrt{2} + 6)^2$

69. $(\sqrt{6} + \sqrt{3})^2$

70. $(\sqrt{6} - \sqrt{3})^2$

71. $(5\sqrt{2} + 2\sqrt{3})^2$

72. $(5\sqrt{2} - 2\sqrt{5})^2$

73. $(5\sqrt{2} - 2\sqrt{6})^2$

74. $(7\sqrt{3} + 4\sqrt{6})^2$

75. $(4\sqrt{5} - 2\sqrt{8})^2$

76. $(6\sqrt{5} - 2\sqrt{15})^2$

77. $(6\sqrt{12} + 10\sqrt{6})^2$

78. $(6\sqrt{3} - 5\sqrt{15})^2$

ANSWERS 5.04

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1. $\sqrt{15}$, 3.87; **2.** $\sqrt{14}$, 3.74; **3.** $\sqrt{30}$, 5.48; **4.** $\sqrt{42}$, 6.48; **5.** $3\sqrt{2}$, 4.24;

6. $2\sqrt{7}$, 5.29; **7.** $3\sqrt{5}$, 6.71; **8.** $5\sqrt{2}$, 7.07; **9.** $14\sqrt{15}$, 54.22; **10.** $15\sqrt{14}$, 56.12;

11. $100\sqrt{30}$, 547.72; **12.** $36\sqrt{42}$, 233.31; **13.** $30\sqrt{3}$, 51.96; **14.** $60\sqrt{7}$, 158.75;

15. $90\sqrt{15}$, 348.57; **16.** $252\sqrt{2}$, 356.38; **17.** $7\sqrt{55}$, 51.91; **18.** $5\sqrt{77}$, 43.87;

19. $11\sqrt{35}$, 65.08; **20.** $19\sqrt{10}$, 60.08; **21.** $17\sqrt{10}$, 53.76; **22.** $13\sqrt{15}$, 50.35;

23. $10\sqrt{17}$, 41.23; **24.** $26\sqrt{3}$, 45.03; **25.** $72\sqrt{5}$, 161.00; **26.** $20\sqrt{15}$, 77.46;

27. $132\sqrt{35}$, 780.92; **28.** $255\sqrt{15}$, 987.61; **29.** $114\sqrt{6}$, 279.24; **30.** $90\sqrt{30}$, 492.95;

31. $720\sqrt{7}$, 1904.94; **32.** $1860\sqrt{5}$, 4159.09; **33.** $10\sqrt{7} + 20\sqrt{3}$; **34.** $24\sqrt{2} - 56\sqrt{5}$;

35. $2\sqrt{35} - 4\sqrt{15}$; **36.** $3\sqrt{14} - 6\sqrt{35}$; **37.** $12\sqrt{35} + 24\sqrt{15}$; **38.** $24\sqrt{14} - 48\sqrt{35}$;

39. $70\sqrt{3} - 60\sqrt{2}$; **40.** $48\sqrt{15} + 280\sqrt{2}$; **41.** $32\sqrt{15} - 48\sqrt{5}$; **42.** $24\sqrt{2} + 20\sqrt{3}$;

43. $180\sqrt{2} + 40\sqrt{6}$; **44.** $18\sqrt{7} + 70\sqrt{6}$; **45.** $60 - 30\sqrt{3}$; **46.** $75\sqrt{6} + 400\sqrt{3}$;

47. $26 + 10\sqrt{2}$, 40.14 ; **48.** $26 - 10\sqrt{2}$, 11.86 ; **49.** $45 + 2\sqrt{3}$, 48.46;

50. $33 - 2\sqrt{2}$, 30.17; **51.** $2 + 2\sqrt{3} + 2\sqrt{5} + 2\sqrt{15}$, 17.68; **52.** $2\sqrt{3} - 3\sqrt{2} - \sqrt{6} + 3$, -0.23;

53. $5\sqrt{2} - \sqrt{30} - 2\sqrt{5} + 2\sqrt{3}$, 0.59; **54.** $5\sqrt{2} + \sqrt{30} - 2\sqrt{5} - 2\sqrt{3}$, 4.61; **55.** 4; **56.** 3;

57. 26; **58.** 148; **59.** 138; **60.** -138; **61.** $132 + 60\sqrt{6}$, 278.97; **62.** $132 - 60\sqrt{6}$, -14.97;

63. $12\sqrt{3} - 24\sqrt{2} - 18\sqrt{5} + 12\sqrt{30}$, 12.32; **64.** $60\sqrt{2} + 80 + 48\sqrt{5} + 32\sqrt{10}$, 373.38;

65. $66 + 6\sqrt{2}$, 74.49; **66.** $276 - 174\sqrt{2}$, 29.93; **67.** $39 - 12\sqrt{3}$, 18.22;

68. $38 + 12\sqrt{2}$, 54.97; **69.** $9 + 6\sqrt{2}$, 17.49; **70.** $9 - 6\sqrt{2}$, 0.51; **71.** $62 + 20\sqrt{6}$, 110.99;

72. $70 - 20\sqrt{10}$, 6.75; **73.** $74 - 40\sqrt{3}$, 4.72; **74.** $243 + 168\sqrt{2}$, 480.59;

75. $112 - 32\sqrt{10}$, 10.81; **76.** $240 - 120\sqrt{3}$, 32.15; **77.** $1032 + 720\sqrt{2}$, 2050.23;

78. $483 - 180\sqrt{5}$, 80.51.

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