5.03 *Adding and Subtracting Square Roots*

After simplification of radicals, the next step is **operations with radicals**--that is, addition, subtraction, multiplication, and division of radicals. Addition and subtraction of radicals is just like combining like terms. Even as $3x + 4x = 7x$, so it is true that $3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$. It is also true that $3\sqrt[3]{2} + 4\sqrt[3]{2} = 7\sqrt[3]{2}$. The expression $3\sqrt{2} + 4\sqrt{3}$ cannot be combined because $\sqrt{2}$ and $\sqrt{3}$ are unlike radicals. Similarly, and $3\sqrt[3]{5} + 4\sqrt{5}$ cannot be combined since $\sqrt[3]{5}$ and $\sqrt{5}$ are unlike radicals.

EXERCISES. Combine like radicals.

1. $5\sqrt{2} + 3\sqrt{2}$ 2. $3\sqrt{3} + 2\sqrt{3}$ 3. $7\sqrt{2} + 5\sqrt{3}$

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4. $6\sqrt{5} - 5\sqrt{5}$ 5. $7\sqrt{3} - 3\sqrt{7}$ 6. $7\sqrt{3} - 3\sqrt{3}$

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7. $8\sqrt{2} + 3\sqrt{3} + 2\sqrt{2}$ 8. $8\sqrt{3} - 11\sqrt{3} + 2\sqrt{2}$

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9. $8\sqrt{3} - 3\sqrt{3} + 2\sqrt{3}$ 10. $8\sqrt{3} + 3\sqrt{2} + 2\sqrt{3}$

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11. $8\sqrt{5} - 11\sqrt{3} - 13\sqrt{5}$

12. $8\sqrt{5} - 11\sqrt{5} - 13\sqrt{5}$

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13. $2\sqrt[3]{5} + 7\sqrt[3]{5}$

14. $3\sqrt[3]{2} + 2\sqrt[3]{3}$

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15. $8\sqrt[3]{3} - 11\sqrt[3]{3}$

16. $2\sqrt{2} + 2\sqrt[3]{3}$

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17. $5\sqrt[3]{3} - 6\sqrt[3]{3}$

18. $7\sqrt[3]{5} + 22\sqrt[3]{5}$

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19. $5\sqrt[3]{3} + 7\sqrt[3]{2} - 4\sqrt[3]{3} - 8\sqrt[3]{2}$

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20. $5\sqrt[3]{3} + 7\sqrt[3]{2} - 4\sqrt{3} - 8\sqrt{2} + 7\sqrt{5} - 7\sqrt[3]{5}$

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21. $8\sqrt{2} + 3\sqrt{2} - 14\sqrt{2}$

22. $\sqrt{3} - 19\sqrt{3} + 13\sqrt{3}$

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23. $8\sqrt{2} + 3\sqrt{2} - 7\sqrt{2}$

24. $5\sqrt{3} - 6\sqrt{3} + 5\sqrt{5}$

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QUESTION: Can $6 - 4\sqrt{2}$ be simplified to $2\sqrt{2}$?

Answer: NO! This is a very common error! Even as $6 - 4x$ cannot be combined, neither can $6 - 4\sqrt{2}$. It is possible to factor the common factor of 2 from $6 - 4\sqrt{2}$ and write $2(3 - 2\sqrt{2})$. At least for now, there is no particular reason to do this.

QUESTION: Can $3\sqrt{2} + 4\sqrt{8}$ be combined?

Answer: At first glance, it appears that $\sqrt{2}$ and $\sqrt{8}$ are unlike radicals. However, since $\sqrt{8}$ simplifies to $2\sqrt{2}$, the expression <u>can</u> and will be combined in the next example!

EXAMPLE 1. Combine like terms if possible for $3\sqrt{2} + 4\sqrt{8}$.

Solution:

$3\sqrt{2} + 4\sqrt{8}$ Simplify radicals .

$3\sqrt{2} + 4\sqrt{4}\sqrt{2}$

$3\sqrt{2} + 4 \cdot 2\sqrt{2}$ Multiply 4 times 2

$3\sqrt{2} + 8\sqrt{2}$ Combine like radicals

$11\sqrt{2}$

EXAMPLE 2: **Combine like terms if possible for $6\sqrt{3} + 8\sqrt{27} - 5\sqrt{12}$.**

Solution:

$6\sqrt{3} + 8\sqrt{27} - 5\sqrt{12}$ Simplify the radicals.

$6\sqrt{3} + 8\sqrt{9}\sqrt{3} - 5\sqrt{4}\sqrt{3}$

$6\sqrt{3} + 8 \cdot 3\sqrt{3} - 5 \cdot 2\sqrt{3}$ Multiply numbers.

$6\sqrt{3} + 24\sqrt{3} - 10\sqrt{3}$ Combine like radicals.

$20\sqrt{3}$

EXAMPLE 3: **Simplify and combine like terms if possible** $8\sqrt{125} - 10\sqrt{72} - 8\sqrt{20}$.

Solution: $8\sqrt{125} - 10\sqrt{72} - 8\sqrt{20}$ Simplify radicals.

$8\sqrt{25}\sqrt{5} - 10\sqrt{36}\sqrt{2} - 8\sqrt{4}\sqrt{5}$

$8\cdot5\sqrt{5} - 10\cdot6\sqrt{2} - 8\cdot2\sqrt{5}$ Multiply numbers.

$40\sqrt{5} - 60\sqrt{2} - 16\sqrt{5}$ Combine like radicals.

$24\sqrt{5} - 60\sqrt{2}$

EXERCISES.   **Simplify and combine like terms if possible.**

25. $\sqrt{2} + \sqrt{8}$ **26.** $\sqrt{27} + \sqrt{12}$ **27.** $\sqrt{27} + \sqrt{18}$

$\sqrt{2} + \sqrt{\ }\sqrt{\ }$ $\sqrt{\ }\sqrt{3} + \sqrt{\ }\sqrt{3}$ $\sqrt{\ }\sqrt{3} + \sqrt{\ }\sqrt{2}$

$\sqrt{2} + \underline{\ \ }\sqrt{\ }$ $\underline{\ }\sqrt{3} + \underline{\ }\sqrt{3}$ _____

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28. $\sqrt{125} + \sqrt{50}$ **29.** $\sqrt{72} + \sqrt{50}$ **30.** $\sqrt{75} + \sqrt{48}$

31. $8\sqrt{2} + 2\sqrt{8}$ **32.** $8\sqrt{12} + 3\sqrt{75}$

$8\sqrt{2} + \underline{\ }\sqrt{\ }\sqrt{\ }$ $8\sqrt{\ }\sqrt{3} + 3\sqrt{\ }\sqrt{3}$

$8\sqrt{2} + \underline{\ }\cdot\underline{\ }\sqrt{\ }$ $8\underline{\ }\sqrt{3} + 3\underline{\ }\sqrt{3}$

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33. $7\sqrt{20} - 2\sqrt{45}$

$7\sqrt{}\sqrt{5} - 2\sqrt{}\sqrt{5}$

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34. $5\sqrt{27} + 6\sqrt{12}$

$5\sqrt{}\sqrt{} + 6\sqrt{}\sqrt{3}$

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35. $3\sqrt{75} + 4\sqrt{48} - 6\sqrt{3}$

$3\sqrt{}\sqrt{} + 4\sqrt{}\sqrt{} - 6\sqrt{3}$

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36. $3\sqrt{20} - 8\sqrt{125} + 6\sqrt{45}$

$3\sqrt{}\sqrt{} - 8\sqrt{}\sqrt{} + 6\sqrt{}\sqrt{}$

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37. $3\sqrt{75} - 4\sqrt{48} - 8\sqrt{8}$

38. $4\sqrt{72} - 8\sqrt{50} + 3\sqrt{98}$

39. $3\sqrt{80} - 5\sqrt{20} - 5\sqrt{12}$

40. $3\sqrt{24} - 2\sqrt{125} - 5\sqrt{54}$

41. $5\sqrt{63} + 7\sqrt{28} - 8\sqrt{175}$

42. $3\sqrt{32} - 9\sqrt{108} + 4\sqrt{98}$

EXTRA CHALLENGE

43. $3\sqrt{300x^3} + 2x\sqrt{75x}$

44. $4x\sqrt{50x} - 5\sqrt{72x^3}$

45. $5x\sqrt{20x^7} - 4\sqrt{45x^9}$

46. $7x^2\sqrt{24x} + 8\sqrt{54x^5}$

47. $\sqrt[3]{16} + \sqrt[3]{54} + \sqrt[3]{250}$

48. $5\sqrt[3]{16} + 2\sqrt[3]{54} - 2\sqrt[3]{250}$

49. $2\sqrt[3]{81} - 3\sqrt[3]{375} + \sqrt[3]{24}$

50. $7\sqrt[3]{40} + 3\sqrt[3]{320} - 4\sqrt[3]{5}$

p. 413 - 418:

1. $8\sqrt{2}$; 2. $5\sqrt{3}$; 3. $7\sqrt{2} + 5\sqrt{3}$; 4. $\sqrt{5}$; 5. $7\sqrt{3} - 3\sqrt{7}$; 6. $4\sqrt{3}$; 7. $10\sqrt{2} + 3\sqrt{3}$;

8. $-3\sqrt{3} + 2\sqrt{2}$; 9. $7\sqrt{3}$; 10. $10\sqrt{3} + 3\sqrt{2}$; 11. $-5\sqrt{5} - 11\sqrt{3}$; 12. $-16\sqrt{5}$;

13. $9\sqrt[3]{5}$; 14. $3\sqrt[3]{2} + 2\sqrt[3]{3}$; 15. $-3\sqrt[3]{3}$; 16. $2\sqrt{2} + 2\sqrt[3]{3}$; 17. $-\sqrt[3]{3}$; 18. $29\sqrt[3]{5}$;

19. $\sqrt[3]{3} - \sqrt[3]{2}$; 20. $5\sqrt[3]{3} + 7\sqrt[3]{2} - 4\sqrt{3} - 8\sqrt{2} + 7\sqrt{5} - 7\sqrt[3]{5}$; 21. $-3\sqrt{2}$; 22. $-5\sqrt{3}$;

23. $4\sqrt{2}$; 24. $-\sqrt{3} + 5\sqrt{5}$; 25. $3\sqrt{2}$; 26. $5\sqrt{3}$; 27. $3\sqrt{3} + 3\sqrt{2}$; 28. $5\sqrt{5} + 5\sqrt{2}$;

29. $11\sqrt{2}$; 30. $9\sqrt{3}$; 31. $12\sqrt{2}$; 32. $31\sqrt{3}$; 33. $8\sqrt{5}$; 34. $27\sqrt{3}$; 35. $25\sqrt{3}$;

36. $-16\sqrt{5}$; 37. $-\sqrt{3} - 16\sqrt{2}$; 38. $5\sqrt{2}$; 39. $2\sqrt{5} - 10\sqrt{3}$; 40. $-9\sqrt{6} - 10\sqrt{5}$;

41. $-11\sqrt{7}$; 42. $40\sqrt{2} - 54\sqrt{3}$; 43. $40x\sqrt{3x}$; 44. $-10x\sqrt{2x}$; 45. $-2x^4\sqrt{5x}$;

46. $38x^2\sqrt{6x}$; 47. $10\sqrt[3]{2}$; 48. $6\sqrt[3]{2}$; 49. $-7\sqrt[3]{3}$; 50. $22\sqrt[3]{5}$.

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419