

5.07 Quadratic Formula

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

In Chapter 2, certain quadratic equations (equations involving a variable raised to the second power) were solved by factoring. In that chapter, one important fact about quadratic equations was neglected. That is, not all quadratic equations can be solved by factoring. Since all quadratic equations that cannot be factored involve radicals, it was necessary to save this topic until now. ALL quadratic equations can be solved using what is known as the **quadratic formula**. This formula will be derived in a higher algebra course by a method called **completing the square**. In this first level of algebra, the formula is presented and used without proof.

The general form of the quadratic equation is $ax^2+bx+c=0$, where “a”, “b”, and “c” represent any real numbers. Notice that “a” is always the coefficient of x^2 , “b” is always the coefficient of x , and “c” is always the **constant coefficient**. The solution to this equation is given by the

World Famous Quadratic Formula!!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If this is the first time you have ever seen this formula, it can be rather intimidating. However, it is not nearly as bad as it looks. (It is really as easy as “a, b, c!”) If you are wondering, “Do I have to memorize this formula?”, the answer is, “No! You will learn it very well just by doing all the homework.” If you are not going to do the homework, then “Yes! You need to memorize it!”

The quadratic formula may be used to solve any quadratic equation, even if the equation can be factored. However, it is usually best to use factoring if the equation factors and use the quadratic formula otherwise. *Please note that, while the use of the quadratic formula on the next pages is not as easy as the factoring method, it does work on all problems, and it is much easier than it may at first appear.* Just follow the outline that is given on the next page.

GUIDELINES FOR USING QUADRATIC FORMULA

1. The equation should always be set equal to zero.
2. Arrange the terms in descending powers of the variable.
3. Identify the “a”, “b”, and “c”: The coefficient of x^2 is “a”, the coefficient of x is “b”, and the constant term is “c”.
4. It is preferred (not required!) that “a” be positive.
5. Write down the formula.
6. Substitute “a”, “b”, and “c”.
7. Simplify the radical, simplify and reduce the fraction if possible.

EXAMPLE 1. Solve for x : $x^2 + 5x - 2 = 0$. **EXAMPLE 2.** Solve for x : $x^2 + 5 = 6x$.

Solution: Equation is already set = 0.

$$a = 1 \quad b = 5 \quad c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{5^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-5 \pm \sqrt{25 + 8}}{2}$$

$$x = \frac{-5 \pm \sqrt{33}}{2}$$

Example 1: Fraction cannot be reduced!

Solution: Set equal to zero: $x^2 - 6x + 5 = 0$.

$$a = 1 \quad b = -6 \quad c = 5$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{6 + \sqrt{36 - 20}}{2}$$

$$x = \frac{6 \pm \sqrt{16}}{2}$$

$$x = \frac{6 \pm 4}{2}$$

$$x = \frac{6 + 4}{2} \quad \text{or} \quad x = \frac{6 - 4}{2}$$

$$x = \frac{10}{2} = 5 \quad \text{or} \quad x = \frac{2}{2} = 1$$

Notice that in Example 1, the answer contains a radical, whereas in Example 2, you obtained the square root of a perfect square. This perfect square in Example 2 means that it could have been solved by factoring, whereas Example 1 (not a perfect square) could not be solved by factoring.

EXERCISES. Solve the following quadratic equations using the quadratic formula.

1. $x^2 + 3x - 2 = 0$

Equation is already set = 0.

a=____ b=____ c=____

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$

2. $x^2 + 5 = 5x$

Set equal to zero: x^2 ____x + ____ = 0

a=____ b=____ c=____

Write formula: $x =$ _____

$$x = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$

3. $x^2 - 3x = 5$

Set = 0: _____ = 0

a=____ b=____ c=____

Formula:

Substitute:

4. $x^2 = 7 + 5x$

Set = 0: _____ = 0

a=____ b=____ c=____

Formula:

Substitute:

5. $x^2 + 3x - 4 = 0$
a=_____ b=_____ c=_____

6. $x^2 - 5x - 50 = 0$
a=_____ b=_____ c=_____

7. $x^2 - 4x = 21$
Set = 0:
a=_____ b=_____ c=_____

8. $x^2 + 50 = 12x + 30$
a=_____ b=_____ c=_____

9. $x^2 - 5x = 0$ (No constant term!)
a=_____ b=_____ c=_____

10. $x^2 - 25 = 0$ (No x term!)
a=_____ b=_____ c=_____

EXAMPLE 3. Solve for x: $x^2 + 6x - 2 = 0$

Solution: Equation is already set = 0

$$a = 1 \quad b = 6 \quad c = -2$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 + 8}}{2}$$

$$x = \frac{-6 \pm \sqrt{44}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{11}}{2}$$

$$x = \frac{2(-3 \pm \sqrt{11})}{2}$$

$$x = -3 \pm \sqrt{11}$$

EXAMPLE 4. Solve for x: $x^2 + 25 = 6x$

Set equal to zero: $x^2 - 6x + 25 = 0$

$$a = 1 \quad b = -6 \quad c = 25$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(25)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - 100}}{2}$$

$$x = \frac{6 \pm \sqrt{-64}}{2}$$

Negative in the radical!!

No Real Solution!

Notice that in Example 3 the radical needed to be simplified, and then the fraction reduced. Pay attention to these steps--traditionally they are difficult for students and a predictable source of errors.

Notice that in Example 4, there was a negative in the radical. This negative in the radical means that there were no real solutions.

EXERCISES. Solve the quadratic equations.

11. $x^2 - 4x - 6 = 0$

12. $x^2 + 4x + 2 = 0$

13. $x^2 - 6x - 6 = 0$

14. $x^2 = 4x + 8$

15. $2x(x - 4) = -7$

16. $2x(x - 4) = 7$

17. $3x^2 + 2(3 + x) = 4 - 6x$

18. $3x(x - 4) = 7 - 8x$

ANSWERS 5.07

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1. $\frac{-3 \pm \sqrt{17}}{2}$; 2. $\frac{5 \pm \sqrt{5}}{2}$; 3. $\frac{3 \pm \sqrt{29}}{2}$; 4. $\frac{5 \pm \sqrt{53}}{2}$; 5. -4, 1; 6. 10, -5; 7. 7, -3;
8. 10, 2; 9. 0, 5; 10. 5, -5; 11. $2 \pm \sqrt{10}$; 12. $-2 \pm \sqrt{2}$; 13. $3 \pm \sqrt{15}$; 14. $2 \pm 2\sqrt{3}$;
15. $\frac{4 \pm \sqrt{2}}{2}$; 16. $\frac{4 \pm \sqrt{30}}{2}$; 17. $\frac{-4 \pm \sqrt{10}}{3}$; 18. $7/3, -1$.

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