### 1.02 Order of Operations

Basic Algebra: One Step at a Time. Page 7-12: \#36, 37, 42, 44, 48. Extra Problems: 39, 51, 53, 55, 63, 67, 69, 70

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## ORDER OF OPERATIONS

LEVEL 1: Simplify within symbols of grouping:
Parentheses ( ), brackets [ ], set braces \{ \}, above or below fraction bars, and absolute value bars | | (see next section).

LEVEL 2: Raise to the power (i.e. exponents).
LEVEL 3: Multiply or divide in order from left to right.
LEVEL 4: Add or subtract in order from left to right.
p. 11: 36. $\quad 6+6^{2} \div 3 \cdot 2$

According to the order of operations, since there are NO parentheses here, the next step is to Raise to the Power.

$$
6+36 \div 3 \cdot 2
$$

The next step is to Multiply or Divide from Left to Right. This means to divide by 3 first, and then multiply by 2 . [NOTE: DO NOT multiply $3 \cdot 2!!$ ]

$$
\begin{aligned}
& 6+36 \div 3 \cdot 2 \\
& 6+12 \cdot 2 \\
& 6+24
\end{aligned}
$$

The last step is to add, so the final answer is 30 .
37.

$$
\left(16+2^{2}\right) \div 2 \cdot 2
$$

Since there ARE parentheses, do what is within Parentheses first! This would be the $2^{2}=4$, and then add 16 .

$$
\begin{aligned}
& (16+4) \div 2 \cdot 2 \\
& 20 \div 2 \cdot 2
\end{aligned}
$$

The next step is to Multiply or Divide from Left to Right. This means to divide by 2 first, and then multiply by 2.

$$
\begin{aligned}
& 20 \div 2 \cdot 2 \\
& 10 \cdot 2
\end{aligned}
$$

Final Answer: 20
42. $24-12 \div 2 \cdot 3+6 \cdot 2^{3}$

According to the order of operations, since there are NO parentheses here, the next step is to Raise to the Power.

$$
24-12 \div 2 \cdot 3+6 \bullet 8
$$

The next step is to Multiply or Divide from Left to Right. Be sure to do the operations from Left to Right. First divide by 2, and then multiply by 3. [NOTE:
DO NOT multiply $2 \cdot 3!!]$

$$
\begin{aligned}
& 24-12 \div 2 \cdot 3+6 \bullet 8 \\
& 24-6 \cdot 3+6 \bullet 8 \\
& 24-18+6 \bullet 8
\end{aligned}
$$

Multiply $6 \cdot 8=48$ :

$$
24-18+48
$$

Finally subtract, and then add from left to right:

$$
6+48
$$

Final Answer: 54
44.

$$
35-20 \div 5+7^{2} \cdot 2-6 \cdot 3+9+10 \div 2
$$

Again, since there are NO parentheses here, the next step is to Raise to the Power.

$$
35-20 \div 5+49 \cdot 2-6 \cdot 3+9+10 \div 2
$$

The next step is to Multiply or Divide from Left to Right. Be sure to do the operations from Left to Right.

$$
\begin{aligned}
& 35-20 \div 5+49 \cdot 2-6 \cdot 3+9+10 \div 2 \\
& 35-4+49 \cdot 2-6 \cdot 3+9+10 \div 2 \\
& 35-4+49 \cdot 2-6 \cdot 3+9+10 \div 2 \\
& 35-4+98-6 \cdot 3+9+10 \div 2 \\
& 35-4+98-18+9+10 \div 2 \\
& 35-4+98-18+9+5
\end{aligned}
$$

Finally, do the addition and subtraction, of course from left to right:

$$
\begin{gathered}
31+98-18+9+5 \\
129-18+9+5 \\
111+9+5
\end{gathered}
$$

Final Answer:
125
48. $\frac{(6 \div 2 \cdot 3)^{2}+2 \cdot 3^{2}}{(5+2)^{2}-4 \cdot 2^{2}}+\frac{(20+5) \cdot 2^{2}}{(20-5 \cdot 2)^{2}}$

The first step is Parentheses. Since there are sometimes two operations within a set of parentheses, it will take two steps to do this.

$$
\begin{aligned}
& \frac{(3 \cdot 3)^{2}+2 \cdot 3^{2}}{(7)^{2}-4 \cdot 2^{2}}+\frac{(25) \cdot 2^{2}}{(20-10)^{2}} \\
& \frac{(9)^{2}+2 \cdot 3^{2}}{(7)^{2}-4 \cdot 2^{2}}+\frac{(25) \cdot 2^{2}}{(10)^{2}}
\end{aligned}
$$

48. (Continued) The next step is to Raise to the Power.

$$
\frac{81+2 \cdot 9}{49-4 \cdot 4}+\frac{(25) \bullet 4}{100}
$$

The next step is to Multiply or Divide from Left to Right. Be sure to do the operations from Left to Right.

$$
\frac{81+18}{49-16}+\frac{100}{100}
$$

Finally, do the Addition and Subtraction, of course from left to right:

$$
\begin{gathered}
\frac{99}{33}+\frac{100}{100} \\
3+1
\end{gathered}
$$

Final Answer:
4
[By the way, you should show as many steps as you need to be comfortable with your work. You would not be expected to show ALL of these steps!!]

## Extra Problems: \#49, 51, 53, 55, 63, 67, 69, 70

49. $\frac{7+2}{5^{2}-4^{2}}$

First, do what is above and below the fraction bar:

$$
\frac{9}{5^{2}-4^{2}}
$$

Now, below the fraction bar, you must do the exponents first!

$$
\begin{gathered}
\frac{9}{25-16} \\
\frac{9}{9} \\
1
\end{gathered}
$$

51. $8(-7)+|6(-5)|$

Multiply the numbers first.

$$
-56+|-30|
$$

You know that $|-30|=30$, so

$$
\begin{gathered}
-56+30 \\
-26
\end{gathered}
$$

53. $\frac{(-2)^{3}+4^{2}}{3-5^{2}+3 \cdot 6}$

Do the exponents first!

$$
\frac{-8+16}{3-25+3 \cdot 6}
$$

Next, do the multiplication in the denominator.

$$
\frac{-8+16}{3-25+18}
$$

Now, do the numerator and denominator from left to right.

$$
\begin{gathered}
\frac{8}{-22+18} \\
\frac{8}{-4} \\
-2
\end{gathered}
$$

55. $\frac{27-2 \cdot 3^{2}}{8 \div 2^{2}-(-2)^{2}}$

Do the exponents first!

$$
\frac{27-2 \bullet 9}{8 \div 4-4}
$$

Next, do the multiplication in the numerator and division in denominator.

$$
\frac{27-18}{2-4}
$$

Now, do the subtractions.

$$
\frac{9}{-2} \text { or }-\frac{9}{2}
$$

63. $\frac{5 x}{15 x^{2}}$ for $x=3$

Note: While there are probably easier ways to do this, this method uses only the use of order of operations explained in this section, beginning with "Plug it in, plug it in!" Of course, other methods are also entirely acceptable here!

$$
\frac{5 \cdot 3}{15 \cdot 3^{2}}
$$

Do the exponent first.

$$
\begin{aligned}
& \frac{5 \bullet 3}{15 \bullet 9} \\
& \frac{15}{15 \bullet 9}
\end{aligned}
$$

Divide out the 15 .

$$
\frac{1 反 1}{15.9} \text { or } \frac{1}{9}
$$

67. $-x^{2}-5 x$ for $x=-3$

NOTE: I have always used the tradition of placing parentheses around a negative number! This tradition will never let you down, and it saves a lot of errors! So let's get started with the other tradition: "PLUG IT IN!!" $-(-3)^{2}-5(-3)$

Do the exponents first.

$$
-9-5(-3)
$$

Next, multiply! Remember, a negative times a negative is a positive!

$$
-9+15
$$

6
69. $\frac{3 a-4 a^{2}}{a^{2}-20}$ for $a=5$

$$
\frac{3 \cdot 5-4 \cdot 5^{2}}{5^{2}-20}
$$

Do exponents first.

$$
\frac{3 \cdot 5-4 \cdot 25}{25-20}
$$

Multiply!

$$
\frac{15-100}{25-20}
$$

## Subtract!

$$
\frac{-85}{5} \text { or }-17
$$

70. $\frac{a^{3}-4 a}{a(a-3)}$ for $a=-2 \quad$ NOTE: Use $a=(-2)$

$$
\frac{(-2)^{3}-4(-2)}{(-2)((-2)-3)}
$$

Do inside the gold parentheses first.

$$
\frac{(-2)^{3}-4(-2)}{(-2)(-5)}
$$

Do exponent next.

$$
\frac{-8-4(-2)}{(-2)(-5)}
$$

Multiply! Again, remember that a negative times a negative is a positive!

$$
\begin{aligned}
& \frac{-8+8}{10} \\
& \frac{0}{10} \text { or } 0
\end{aligned}
$$

