

1.10 Word Problems

Basic Algebra: One Step at a Time, Page 65-92:

3, 6, 7, 19, 20, 29, 31, 32, 35, 37, 38, 48, 49, 51, 57, 59, 61, 62, 64, 65.

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In this section, the following "categories" of word problems will be considered:

I. Number problems

II. Consecutive number problems

III. Perimeter problems

IV. Coin problems

V. Mixture problems (Optional)

Before describing these categories of word problems, it will be helpful to identify five steps in setting up and solving word problems:

STEP 1: IDENTIFY THE VARIABLE. State exactly what it is that the variable represents. For example, "Let x = the number" or "Let x = the smaller of two numbers" or "Let x = the width of a rectangle" or "Let x = the number of dimes" Then express all other quantities to be used in the problem in terms of x . This is the most important, often the most difficult, and usually the most overlooked step of the problem.

STEP 2: WRITE THE EQUATION. Having completed Step 1, use this step in writing the equation. This is often no more than translating a sentence of the problem into an equation. Read the problem carefully, paraphrasing as necessary.

STEP 3: SOLVE THE EQUATION. This is usually the easy part!

STEP 4: ANSWER THE QUESTION. After solving for x , there may be other quantities to be determined. Be sure you have answered the question before going on to the next exercise.

STEP 5: CHECK. Check the answers in the worded problem itself and make sure the solution actually works. Reject any **extraneous** (i.e., inappropriate) answers.

3. Three times a number plus 8 is equal to five times the number. Find the number.

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ the number (since this is the only thing you need to find!)

STEP 2: Write the equation (just translate the following sentence into math symbols.

Three times a number plus 8 is equal to five times the number.

$$3 * (x) + (8) = 5 * (x)$$

STEP 3: Solve the equation.

$$3x + 8 = 5x$$

Subtract $3x$ from each side:

$$3x - 3x + 8 = 5x - 3x$$

$$8 = 2x$$

Divide both sides by 2:

$$4 = x$$

STEP 4: Answer the question.

The unknown number is 4.

STEP 5: Check.

Check: $3 * (x) + (8) = 5 * (x)$

$$3 * (4) + (8) = 5 * (4)$$

$$12 + 8 = 20 \text{ It checks!!}$$

6. Five more than four times a number is equal to 35 less than twice a number. Find the number.

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ the number (since this is the only thing you need to find!)

STEP 2: Write the equation (just translate the following sentence into math symbols).

Five more than four times a number is equal to 35 less than twice a number

$$4 * (x) + 5 = 2 * (x) - 35$$

STEP 3: Solve the equation.

$$4x + 5 = 2x - 35$$

Subtract $2x$ from each side:

$$4x - 2x + 5 = 2x - 2x - 35$$

$$2x + 5 = -35$$

Subtract 5 from each side:

$$2x + 5 - 5 = -35 - 5$$

$$2x = -40$$

Divide both sides by -2 :

$$x = -20$$

STEP 4: Answer the question.

The unknown number is -20 .

STEP 5: Check.

$$\begin{aligned} \text{Check: } 4 * (x) + 5 &= 2 * (x) - 35 \\ 4 * (-20) + 5 &= 2 * (-20) - 35 \\ -80 + 5 &= -40 - 35 \\ -75 &= -75 \quad \text{It checks!!} \end{aligned}$$

7. Five more than four times a number is equal to 35 less twice a number. Find the number. [NOTE: This looks like # 6, but read the problem carefully. One word makes all the difference in these two problems!]

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ the number (since this is the only thing you need to find!)

STEP 2: Write the equation (just translate the following sentence into math symbols.

Five more than four times a number is equal to 35 less twice a number
 $4 * (x) + 5 = 35 - 2 * (x)$

STEP 3: Solve the equation.

$$4x + 5 = 35 - 2x$$

Add $+2x$ to each side:

$$4x + 5 + 2x = 35 - 2x + 2x$$
$$6x + 5 = 35$$

Subtract 5 from each side:

$$6x + 5 - 5 = 35 - 5$$
$$6x = 30$$

Divide both sides by 6:

$$x = 5$$

STEP 4: Answer the question.

The unknown number is 5.

STEP 5: Check.

Check: $4 * (x) + 5 = 35 - 2 * (x)$
 $4 * (5) + 5 = 35 - 2 * (5)$
 $20 + 5 = 35 - 10$
 $25 = 25$ It checks!!

19. Three numbers are such that the second number is 4 more than the first, and the third number is equal to the sum of the first two numbers. The sum of the three numbers is 256. Find the numbers.

SOLUTION:

STEP 1: Let $x = \underline{\hspace{1cm}}$.

Let $x =$ first number (since this is at the end of the first phrase)

$x+4 =$ second number

$2x+4 =$ third number (sum of the first two numbers)

STEP 2: Write the equation.

$$\begin{aligned} \text{First} + \text{Second} + \text{Third} &= 256 \\ (x) + (x+4) + (2x+4) &= 256 \end{aligned}$$

STEP 3: Solve the equation.

$$\begin{aligned} x + x + 4 + 2x + 4 &= 256 \\ 4x + 8 &= 256 \end{aligned}$$

Subtract 8 from each side:

$$4x + 8 - 8 = 256 - 8$$

$$4x = 248$$

Divide both sides by 4:

$$x = 62$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 62 && \text{First number} \\ x + 4 &= 62 + 4 = 66 && \text{Second number} \\ 2x + 4 &= 2(62) + 4 = \underline{128} && \text{Third number} \\ &&& \mathbf{256} \end{aligned}$$

STEP 5: Check.

Check: Sum of the numbers = 256
See above! It checks!!

20. Three numbers are such that the second number is 6 less than twice the first, and the third number is 5 more than the sum of the first two numbers. The sum of the numbers is 293. Find the numbers.

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ first number (since this is at the end of the first phrase)
 $2x-6 =$ second number
 $3x-6+5 =$ third number (sum of the first two numbers plus 5 more)
 $3x - 1 =$ third number

STEP 2: Write the equation.

$$\begin{array}{rclcl} \text{First} & + & \text{Second} & + & \text{Third} & = & 293 \\ (x) & + & (2x - 6) & + & (3x - 1) & = & 293 \end{array}$$

STEP 3: Solve the equation.

$$\begin{array}{r} x + 2x - 6 + 3x - 1 = 293 \\ 6x - 7 = 293 \end{array}$$

Add +7 to each side:

$$6x - 7 + 7 = 293 + 7$$

$$6x = 300$$

Divide both sides by 6:

$$x = 50$$

STEP 4: Answer the question. (YOU FINISH IT!!)

$$\begin{array}{rcl} x = 50 & \text{First number} \\ \underline{\hspace{2cm}} = \underline{\hspace{2cm}} & \text{Second number} \\ \underline{\hspace{2cm}} = \underline{\hspace{2cm}} & \text{Third number} \\ & \mathbf{293} \end{array}$$

STEP 5: Check.

Check: Sum of the numbers = 293
See above! Does it check?

29. Two consecutive even integers are such that twice the first, plus three times the second, is equal to 156. Find the numbers.

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ first number

$x+2 =$ second number (since they are consecutive EVEN numbers!)

STEP 2: Write the equation.

$$\begin{aligned} 2 * \text{First} + 3 * \text{Second} &= 156 \\ 2*(x) + 3*(x+2) &= 156 \end{aligned}$$

STEP 3: Solve the equation.

$$\begin{aligned} 2x + 3x + 6 &= 156 \\ 5x + 6 &= 156 \end{aligned}$$

Subtract 6 from each side:

$$5x + 6 - 6 = 156 - 6$$

$$5x = 150$$

Divide both sides by 5:

$$x = 30$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 30 && \text{First number} \\ x + 2 &= 32 && \text{Second number} \end{aligned}$$

STEP 5: Check.

$$\begin{aligned} \text{Check: } 2*30 + 3*32 \\ 60 + 96 &= 156 && \text{It checks!!} \end{aligned}$$

31. Three consecutive odd integers are such that the sum of the integers is 7 less than four times the smallest. Find the numbers.

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ first integer

$x+2 =$ second integer (since these are consecutive ODD integers!)

$x+4 =$ third integer

$3x+6 =$ the sum

STEP 2: Write the equation.

The sum of the integers is 7 less than four times the smallest

$$3x+6 = 4x - 7$$

STEP 3: Solve the equation.

$$3x+6 = 4x - 7$$

Subtract $3x$ from each side:

$$\begin{aligned} 3x+6 - 3x &= 4x - 7 - 3x \\ 6 &= x - 7 \end{aligned}$$

Add $+7$ to each side:

$$\begin{aligned} 6 + 7 &= x - 7 + 7 \\ 13 &= x \end{aligned}$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 13 && \text{First number} \\ x + 2 &= 15 && \text{Second number} \\ x + 4 &= 17 && \text{Third number} \end{aligned}$$

STEP 5: Check.

Check: Sum of the numbers equals 7 less than 4 times smallest number.

$$\begin{aligned} 45 &= 4 * 13 - 7 \\ 45 &= 52 - 7 && \text{It checks!!} \end{aligned}$$

32. Three consecutive integers are such that the first, plus twice the second, plus three times the third is equal to 200. Find the numbers.

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ first number

$x+1 =$ second number (since these are consecutive integers!)

$x+2 =$ third number

STEP 2: Write the equation.

$$\begin{aligned} \text{First} + 2 * \text{Second} + 3 * \text{Third} &= 200 \\ (x) + 2 * (x+1) + 3 * (x+2) &= 200 \end{aligned}$$

STEP 3: Solve the equation.

$$\begin{aligned} x + 2x + 2 + 3x + 6 &= 200 \\ 6x + 8 &= 200 \end{aligned}$$

Subtract 8 from each side:

$$6x + 8 - 8 = 200 - 8$$

$$6x = 192$$

Divide both sides by 6:

$$x = 192/6 = 32$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 32 && \text{First number} \\ x + 1 &= 33 && \text{Second number} \\ x + 2 &= 34 && \text{Third number} \end{aligned}$$

STEP 5: Check.

$$\begin{aligned} \text{Check: } (x) + 2 * (x+1) + 3 * (x+2) &= 200 \\ 32 + 2 * 33 + 3 * 34 & \\ 32 + 66 + 102 &= 200 \quad \text{It checks!!} \end{aligned}$$

35. The second side of a triangle is four more the first side, and the third side is 10 less than the second side. The perimeter of the triangle is 70 feet. Find the sides of the triangle.

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ first side (since this is at the end of the first phrase)

$x+4 =$ second side

$x+4 - 10 =$ third side

$x - 6 =$ third side

STEP 2: Write the equation.

First + Second + Third = Perimeter

$$(x) + (x+4) + (x-6) = 70$$

STEP 3: Solve the equation.

$$x + x+4 + x - 6 = 70$$

$$3x - 2 = 70$$

Add +2 to each side:

$$3x - 2 + 2 = 70 + 2$$

$$3x = 72$$

Divide both sides by 3:

$$x = 24$$

STEP 4: Answer the question.

$$x = 24 \quad \text{First side}$$

$$x+4 = 28 \quad \text{Second side}$$

$$x+4 - 10 = \underline{18} \quad \text{Third side}$$

STEP 5: Check. **70** **It does check!!**

37. The third side of a triangle is 6 less the first side, and the second side is twice the third side. The perimeter of the triangle is 82 feet. Find the sides of the triangle.

SOLUTION:

STEP 1: Let $x =$ _____.

Let $x =$ first side (since this is at the end of the first phrase)

$x - 6 =$ third side

$2(x - 6) =$ second side

STEP 2: Write the equation.

$$\begin{array}{r} \text{First} + \text{Third} + \text{Second} = \text{Perimeter} \\ (x) + (x-6) + 2(x-6) = 82 \end{array}$$

STEP 3: Solve the equation.

$$\begin{array}{r} x + x - 6 + 2x - 12 = 82 \\ 4x - 18 = 82 \end{array}$$

Add +18 to each side:

$$\begin{array}{r} 4x - 18 + 18 = 82 + 18 \\ 4x = 100 \end{array}$$

Divide both sides by 4:

$$x = 25$$

STEP 4: Answer the question.

$$\begin{array}{r} x = 25 \quad \text{First side} \\ x - 6 = 19 \quad \text{Third side} \\ 2(x - 6) = \underline{38} \quad \text{Second side} \end{array}$$

STEP 5: Check. **82** **It does check!!**

38. The third side of a triangle is twice the first side, and the second side is 10 less than the third side. The perimeter of the triangle is 70 feet. Find the sides of the triangle.

SOLUTION:

STEP 1: Let $x =$ _____.

Let $x =$ first side (since this is at the end of the first phrase)

$2x =$ third side

$2x - 10 =$ third number

STEP 2: Write the equation.

$$\begin{aligned} \text{First} + \text{Second} + \text{Third} &= \text{Perimeter} \\ (x) + (2x) + (2x - 10) &= 70 \end{aligned}$$

STEP 3: Solve the equation.

$$\begin{aligned} x + 2x + 2x - 10 &= 70 \\ 5x - 10 &= 70 \end{aligned}$$

Add +10 to each side:

$$5x - 10 + 10 = 70 + 10$$

$$5x = 80$$

Divide both sides by 5:

$$x = 16$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 16 && \text{First side} \\ 2x &= 32 && \text{Second side} \\ 2x - 10 &= \underline{22} && \text{Third number} \end{aligned}$$

STEP 5: Check. **70** It does check!!

46. The width of a rectangle is 50 feet less than the **length**. If the perimeter is 400 feet, find the length and width of the rectangle.

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let x = length of the rectangle
 $x - 50$ = width of the rectangle

STEP 2: Write the equation.

$$2(\quad) + 2(\quad) = \text{Perimeter}$$
$$2(x) + 2(x-50) = 400 \quad \text{or} \quad 2(x-50) + 2(x) = 400$$

STEP 3: Solve the equation.

$$2x + 2x - 100 = 400$$
$$4x - 100 = 400$$

Add +100 to each side:

$$4x - 100 + 100 = 400 + 100$$
$$4x = 500$$

Divide both sides by 4:

$$x = 125$$

STEP 4: Answer the question.

$$x = 125 \text{ Length}$$
$$x - 50 = 75 \text{ Width}$$

STEP 5: Check.

Check: $2W + 2L = P$

$$2(75) + 2(125) = 400$$
$$150 + 250 = 400 \quad \text{It checks!!}$$

- 47.** The length of a rectangle is 3 more than twice the width. The perimeter is 56 meters. Find the dimensions of the rectangle. (Note: the length and/or width do not have to come out even! Express the answer in fractional form or as a repeating decimal.)

SOLUTION:

STEP 1: Let $x =$ _____.

Let $x =$ width of the rectangle
 $2x+3 =$ length of the rectangle
 $56 =$ Perimeter

STEP 2: Write the equation.

$$2(\quad) + 2(\quad) = \text{Perimeter}$$

$$2(x) + 2(2x+3) = 56$$

STEP 3: Solve the equation.

$$2x + 4x + 6 = 56$$

$$6x + 6 = 56$$

Subtract 6 from each side:

$$6x + 6 - 6 = 56 - 6$$

$$6x = 50$$

Divide both sides by 6:

$$\frac{6x}{6} = \frac{50}{6}$$

$$x = \frac{25}{3} \text{ or } 8\frac{1}{3}$$

STEP 4: Answer the question.

$$x = 8\frac{1}{3} \text{ Width}$$

$$2x + 3 = 2\left(8\frac{1}{3}\right) + 3$$

$$= 16\frac{2}{3} + 3 = 19\frac{2}{3} \text{ Length}$$

STEP 5: Check. Check: $2(W + L) = P$

$$2\left(8\frac{1}{3} + 19\frac{2}{3}\right) = 56$$

$$2(28) = 56 \text{ It checks!!}$$

48. The length of a rectangle is three less than five times the width. The perimeter is ten times the width. Find the dimensions and perimeter of the rectangle.

SOLUTION:

STEP 1: Let $x =$ _____.

Let $x =$ width of the rectangle
 $5x-3 =$ length of the rectangle
 $10x =$ Perimeter

STEP 2: Write the equation.

$$\begin{aligned} 2(\quad) + 2(\quad) &= \text{Perimeter} \\ 2(x) + 2(5x-3) &= 10x \end{aligned}$$

STEP 3: Solve the equation.

$$\begin{aligned} 2x + 10x - 6 &= 10x \\ 12x - 6 &= 10x \end{aligned}$$

Subtract $12x$ from each side:

$$12x - 12x - 6 = 10x - 12x$$

$$-6 = -2x$$

Divide both sides by -2 :

$$x = 3$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 3 \text{ Width} \\ 5x - 3 &= 5(3) - 3 = 12 \text{ Length} \\ 10x &= 30 \text{ Perimeter} \end{aligned}$$

STEP 5: Check.

$$\begin{aligned} \text{Check: } 2W + 2L &= P \\ 2(3) + 2(12) &= 30 \\ 6 + 24 &= 30 \quad \text{It checks!!} \end{aligned}$$

49. The perimeter of a rectangle is 46. Twice the length is 4 more than five times the width. Find the length and width of the rectangle.

SOLUTION:

STEP 1: Let $x =$ _____.

Let $x =$ width of the rectangle
 $5x+4 =$ two lengths of the rectangle

STEP 2: Write the equation.

$$\begin{aligned} 2(W) + 2(L) &= \text{Perimeter} \\ 2(x) + 5x+4 &= 46 \end{aligned}$$

STEP 3: Solve the equation.

$$2x + 5x + 4 = 46$$

$$7x + 4 = 46$$

Subtract 4 from each side:

$$7x + 4 - 4 = 46 - 4$$

$$7x = 42$$

Divide both sides by 7:

$$x = 6$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 6 \text{ Width} \\ 5x + 4 &= 5(6) + 4 = 34 = 2 \cdot \text{Length} \\ 17 &= \text{Length} \end{aligned}$$

STEP 5: Check.

$$\text{Check: } 2W + 2L = P$$

$$2(6) + 2(17) = 46$$

$$12 + 34 = 46 \quad \text{It checks!!}$$

51. A certain number of quarters and three more dimes than quarters are worth \$7.30. How many of each are there?

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ number of quarters
 $x+3 =$ number of dimes

Type	No Coins	Each	Values
Q	x	25	$25(x)$
D	$x+3$	10	$10(x+3)$
			$730¢$

STEP 2: Write the equation from the last column of the chart above.

$$25(x) + 10(x+3) = 730$$

STEP 3: Solve the equation.

$$\begin{aligned} 25x + 10x + 30 &= 730 \\ 35x + 30 &= 730 \\ 35x &= 700 \\ x &= 20 \end{aligned}$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 20 \text{ Quarters} \\ x+3 &= 23 \text{ Dimes} \end{aligned}$$

STEP 5: Check.

$$\begin{array}{r} 20 \text{ Quarters } \$5.00 \\ 23 \text{ Dimes } \quad \underline{2.30} \\ \text{TOTAL: } \$7.30 \text{ It checks!!} \end{array}$$

53. A certain number of nickels and some dimes are worth **\$7.20**. The number of dimes is three less than twice the number of nickels. How many of each are there?

SOLUTION:

STEP 1: Let $x = \underline{\hspace{1cm}}$.

Let $x =$ number of nickels

$2x - 3 =$ number of dimes

Type	No Coins	Each	Values
N	x	5	$5(x)$
D	$2x - 3$	10	$10(2x - 3)$
			720¢

STEP 2: Write the equation from the last column of the chart above.

$$5(x) + 10(2x - 3) = 720$$

STEP 3: Solve the equation.

$$\begin{aligned} 5x + 20x - 30 &= 720 \\ 25x - 30 &= 720 \\ 25x &= 750 \\ x &= 30 \end{aligned}$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 30 \text{ Nickels} \\ 2x - 3 &= 2(30) - 3 = 57 \text{ Dimes} \end{aligned}$$

STEP 5: Check.

Check:	30 Nickels	\$1.50	
	57 Dimes	<u>5.70</u>	
	TOTAL:	\$7.20	It checks!!

57. A certain number of quarters, four times as many pennies as quarters, and 6 more dimes than pennies are worth **\$3.36**. How many of each coin are there?

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ number of quarters

$4x =$ number of pennies

$4x+6 =$ number of dimes

Type	No Coins	Each	Values
Q	x	25	$25(x)$
P	$4x$	1	$1(4x)$
D	$4x+6$	10	$10(4x+6)$
			336¢

STEP 2: Write the equation from the last column of the chart above.

$$25(x) + 1(4x) + 10(4x+6) = 336$$

STEP 3: Solve the equation.

$$\begin{aligned} 25x + 4x + 40x + 60 &= 336 \\ 69x + 60 &= 336 \\ 69x &= 276 \\ x &= 4 \end{aligned}$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 4 \text{ Quarters} \\ 4x &= 4(4) = 16 \text{ Pennies} \\ 4x+6 &= 16+6 = 22 \text{ Dimes} \end{aligned}$$

STEP 5: Check.

$$\begin{array}{r} 4 \text{ Quarters } \$1.00 \\ 16 \text{ Pennies } \quad .16 \\ 22 \text{ Dimes } \quad \underline{2.20} \\ \text{TOTAL: } \quad \$3.36 \text{ It checks!!} \end{array}$$

59. A box contains \$6.60 in nickels, dimes, and quarters. There are three times as many nickels as quarters, and the number of dimes is 4 less than the number of nickels. How many of each coin are there?

SOLUTION:

STEP 1: Let $x =$ _____.

Let $x =$ number of quarters

$3x =$ number of nickels

$3x-4 =$ number of dimes

Type	No Coins	Each	Values
Q	x	25	$25(x)$
N	$3x$	5	$5(3x)$
D	$3x-4$	10	$10(3x-4)$
			660¢

STEP 2: Write the equation from the last column of the chart above.

$$25(x) + 5(3x) + 10(3x-4) = 660$$

STEP 3: Solve the equation.

$$25x + 15x + 30x - 40 = 660$$

$$70x - 40 = 660$$

$$70x = 700$$

$$x = 10$$

STEP 4: Answer the question.

$$x = 10 \text{ Quarters}$$

$$3x = 3(10) = 30 \text{ Nickels}$$

$$3x-4 = 30-4 = 26 \text{ Dimes}$$

STEP 5: Check.

$$10 \text{ Quarters } \$2.50$$

$$30 \text{ Nickels } 1.50$$

$$26 \text{ Dimes } \underline{2.60}$$

$$\text{TOTAL: } \$6.60 \text{ It checks!!}$$

61. A certain number of pennies, four times as many dimes as pennies, and a number of quarters which is 16 less than twice the number of dimes, are worth **\$24.92**. How many of each coin are there?

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ number of pennies
 $4x =$ number of dimes
 $2(4x) - 16 =$ number of quarters
 $8x - 16 =$ number of quarters

Type	No Coins	Each	Values
P	x	1	$1(x)$
D	$4x$	10	$10(4x)$
Q	$8x - 16$	25	$25(8x - 16)$
			2492¢

STEP 2: Write the equation from the last column of the chart above.

$$1(x) + 10(4x) + 25(8x - 16) = 2492$$

STEP 3: Solve the equation.

$$\begin{aligned} 1x + 40x + 200x - 400 &= 2492 \\ 241x - 400 &= 2492 \\ \frac{241x}{241} &= \frac{2892}{241} \\ x &= 12 \end{aligned}$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 12 \text{ Pennies} \\ 4x &= 4(12) = 48 \text{ Dimes} \\ 8x - 16 &= 96 - 16 = 80 \text{ Quarters} \end{aligned}$$

STEP 5: Check.

12	Pennies	\$ 0.12	
48	Dimes	4.80	
80	Quarters	<u>20.00</u>	
	TOTAL:	\$24.92	It checks!!

62. A sum of money consists of nickels, dimes, and quarters amounting to **\$1.90**. If there are twice as many nickels as quarters and three less dimes than nickels, how many of each coin are there?

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ number of quarters

$2x =$ number of nickels

$2x-3 =$ number of dimes

Type	No Coins	Eq	Values
Q	x	25	$25(x)$
N	$2x$	5	$5(2x)$
D	$2x-3$	10	$10(2x-3)$
			190¢

STEP 2: Write the equation from the last column of the chart above.

$$25(x) + 5(2x) + 10(2x-3) = 190$$

STEP 3: Solve the equation.

$$\begin{aligned}
 25x + 10x + 20x - 30 &= 190 \\
 55x - 30 &= 190 \\
 55x &= 220 \\
 \frac{55x}{55} &= \frac{220}{55} \\
 x &= 4
 \end{aligned}$$

STEP 4: Answer the question.

$$x = 4 \text{ Quarters}$$

$$2x = 2(4) = 8 \text{ Nickels}$$

$$2x-3 = 8 - 3 = 5 \text{ Dimes}$$

STEP 5: Check.

4	Quarters	\$ 1.00	
8	Nickels	.40	
5	Dimes	<u>.50</u>	
	TOTAL:	\$ 1.90	It checks!!

63. A box contains nickels, dimes, and quarters worth **\$12.60**. The number of dimes is 2 less than three times the number of nickels, and the number of quarters is 4 less than twice the number of dimes. How many of each coin are there?

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ number of nickels
 $3x - 2 =$ number of dimes
 $2(3x - 2) - 4 =$ number of quarters
 $6x - 4 - 4 =$ number of quarters
 $6x - 8 =$ number of quarters

Type	No Coins	Eq	Values
N	x	5	$5(x)$
D	$3x - 2$	10	$10(3x - 2)$
Q	$6x - 8$	25	$25(6x - 8)$
			$1260¢$

STEP 2: Write the equation from the last column of the chart above.

$$5(x) + 10(3x - 2) + 25(6x - 8) = 1260$$

STEP 3: Solve the equation.

$$\begin{aligned} 5x + 30x - 20 + 150x - 200 &= 1260 \\ 185x - 220 &= 1260 \\ 185x &= 1480 \\ \frac{185x}{185} &= \frac{1480}{185} \\ x &= 8 \end{aligned}$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 8 && \text{Nickels} \\ 3x - 2 &= 3(8) - 2 = 22 && \text{Dimes} \\ 2(3x - 2) - 4 &= 2(22) - 4 = 40 && \text{Quarters} \end{aligned}$$

STEP 5: Check.

$$\begin{array}{r} 8 \text{ Nickels} \quad \$.40 \\ 22 \text{ Dimes} \quad \quad 2.20 \\ 40 \text{ Quarters} \quad \underline{10.00} \\ \text{TOTAL:} \quad \underline{\$12.60} \quad \text{It checks!!} \end{array}$$

64. A box contains nickels, dimes, and quarters worth **\$69.50**. The number of nickels is 10 more than twice the number of dimes. There are as many quarters as nickels and dimes combined. How many of each coin are there?

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ number of dimes
 $2x+10 =$ number of nickels
 $x + 2x+10 =$ number of quarters
 $3x + 10 =$ number of quarters

Type	No Coins	Each	Values
D	x	10	$10(x)$
N	$2x+10$	5	$5(2x+10)$
Q	$3x+10$	25	$25(3x+10)$
			6950¢

STEP 2: Write the equation from the last column of the chart above.

$$10(x) + 5(2x+10) + 25(3x+10) = 6950$$

STEP 3: Solve the equation.

$$\begin{aligned} 10x + 10x + 50 + 75x + 250 &= 6950 \\ 95x + 300 &= 6950 \\ 95x &= 6650 \\ \frac{95x}{95} &= \frac{6650}{95} \\ x &= 70 \end{aligned}$$

STEP 4: Answer the question.

$$\begin{aligned} x &= 70 && \text{Dimes} \\ 2x+10 &= 2(70) + 10 = 150 && \text{Nickels} \\ x+2x+10 &= 70 + 150 = 220 && \text{Quarters} \end{aligned}$$

STEP 5: Check.

70	Dimes	\$ 7.00	
150	Nickels	7.50	
220	Quarters	<u>55.00</u>	
	TOTAL:	\$69.50	It checks!!

65. A merchant mixes some candy worth \$3.50 per pound with cheap stuff worth \$1.00 per pound. There are 10 more pounds of the cheap stuff than the more expensive candy. If the total value of the mixture is \$28, how many pounds of each are there?

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x =$ number of pounds of expensive candy
 $x+10 =$ number of cheap stuff

Type	No Pounds	Eq	Values in \$\$
Expensive	x	3.50	$3.50(x)$
Cheap	$x+10$	1	$1(x+10)$
			28

STEP 2: Write the equation from the last column of the chart above.

$$3.50(x) + 1(x+10) = 28$$

STEP 3: Solve the equation.

$$\begin{aligned}
 3.5x + 1.0x + 10 &= 28 \\
 4.5x + 10 &= 28 \\
 4.5x &= 18 \\
 \frac{4.5x}{4.5} &= \frac{18}{4.5} \\
 x &= \frac{180}{45} \\
 x &= 4
 \end{aligned}$$

STEP 4: Answer the question.

$$\begin{aligned}
 x &= 4 \text{ pounds Expensive Candy} \\
 x+10 &= 4 + 10 = 14 \text{ pounds Cheap Stuff}
 \end{aligned}$$

STEP 5: Check.

$$\begin{aligned}
 4 \text{ Expensive} & @ \$3.50 = \$ 14.00 \\
 14 \text{ Cheap Stuff} & @ \$1.00 = 14.00 \\
 \text{TOTAL:} & \$ 8.00 \text{ It checks!!}
 \end{aligned}$$

Age Problem #1.

Caylyne is 5 years older than Cassie. The sum of their ages is 17.
How old is Cassie, and how old is Caylyne?

SOLUTION:

STEP 1: Let $x =$ _____.

Let $x =$ Cassie's age
 $x+5 =$ Caylyne's age

STEP 2: Write the equation from the last sentence in the problem above.

“ The **SUM** of their ages is **17**.” (That is, you have to **ADD** the ages together!)

$$x + x+5 = 17$$

STEP 3: Solve the equation.

$$2x + 5 = 17$$

Subtract 5 from each side of the equation:

$$\begin{array}{r} 2x + 5 = 17 \\ -5 \quad -5 \\ \hline 2x = 12 \end{array}$$

Divide both sides by 2:

$$x = 6$$

STEP 4: Answer the question.

$$\begin{array}{l} x = 6 \text{ Cassie's Age} \\ x+5 = 6 + 5 = 11 \text{ Caylyne's Age} \end{array}$$

STEP 5: Check.

Check: The sum of their ages is $6 + 11 = 17$. It checks!!

Age Problem #2.

The age of Caylyne is one year less than twice the age of Cassie. Daddy's age is three times the age of Caylyne. The sum of all of their ages is 50. How old is each?

SOLUTION:

STEP 1: Let $x = \underline{\hspace{2cm}}$.

Let $x = \text{Cassie's age}$
 $2x-1 = \text{Caylyne's age}$
 $3(2x-1) = \text{Daddy's age}$

STEP 2: Write the equation from the last sentence in the problem above.

"The **SUM** of all their ages is **50**." (That is, you have to **ADD** all the ages together!)

$$\begin{array}{r} \text{Cassie} + \text{Caylyne} + \text{Daddy} = 50 \\ x + 2x-1 + 3(2x-1) = 50 \end{array}$$

STEP 3: Solve the equation.

$$\begin{array}{r} x + 2x-1 + 3(2x-1) = 50 \\ x + 2x-1 + 6x-3 = 50 \\ 9x - 4 = 50 \end{array}$$

Add $+4$ to each side of the equation:

$$\begin{array}{r} 9x - 4 = 50 \\ \quad + 4 \quad + 4 \\ \hline 9x = 54 \end{array}$$

Divide both sides by 9: $\frac{9x}{9} = \frac{54}{9}$
 $x = 6$

STEP 4: Answer the question.

$$\begin{array}{r} \text{Let } x = 6 \quad \text{Cassie's age} \\ 2x-1 = 2(6) - 1 = 11 \quad \text{Caylyne's age} \\ 3(2x-1) = 3(11) = 33 \quad \text{Daddy's age} \end{array}$$

STEP 5: Check. The sum of their ages is $6 + 11 + 33 = 50$. It checks!!