2.05 Factoring Difference of Squares Perfect Square Trinomials

Basic Algebra: One Step at a Time. **Pages 157-163: 67, 72, 74, 75, 77**

Extra Problems: 49, 63, 67, 69, 71, 81

Dr. Robert J. Rapalje, Retired
Central Florida, USA

p. 162: 67. $x^4 - 16$ Notice that x^4 and 16 are both perfect squares.
The First times First must be x^4: x^2 times x^2

$$(x^2 \qquad)(x^2 \qquad)$$

The Last times Last must be 16: 4 times 4.

$$(x^2 \quad 4)(x^2 \quad 4)$$

Because the 16 is negative, use opposite signs.

$$(x^2 - 4)(x^2 + 4)$$

The factor $(x^2 - 4)$ is itself a difference of squares, and so it must be re-factored. However, the factor $(x^2 + 4)$ is the SUM of squares. It does not re-factor, and it must be left as it is in the final answer.

$$(x^2 - 4)(x^2 + 4)$$

$(x - 2)(x + 2)(x^2 + 4)$ Final Answer!!

p. 162: 72. $16x^4 - 81$ Notice that $16x^4$ and 81 are both perfect squares.

The First times First must be $16x^4$: $4x^2$ times $4x^2$

$(4x^2 \quad)(4x^2 \quad)$

The Last times Last must be 81: 9 times 9.

$(4x^2 \quad 9)(4x^2 \quad 9)$

Because the 81 is negative, use opposite signs.

$(4x^2 - 9)(4x^2 + 9)$

The factor $(4x^2 - 9)$ is itself a difference of squares, and so it must be re-factored. However, the factor $(4x^2 + 9)$ is the SUM of squares. It does not re-factor, and it must be left as it is in the final answer.

$(4x^2 - 9)(4x^2 + 9)$

$(2x - 3)(2x + 3)(4x^2 + 9)$ Final Answer!!

74.

$x^4 + 13x^2 + 36$ Notice that First times First must be x^4: x^2 times x^2

$(x^2 \quad)(x^2 \quad)$

The Last times Last must be 36 and the OI term must add up to $13x^2$ (Try 9 • 4, both positive)

$(x^2 \quad 9)(x^2 \quad 4)$

$(x^2 + 9)(x^2 + 4)$ O term is $4x^2$, and the I term is $9x^2$, for a total of $13x^2$,

The factors $(x^2 + 9)$ and $(x^2 + 4)$ are both sums of squares!
They cannot be NOT re-factored so this is the final answer.

p. 163: 75.

$x^4 - 13x^2 + 36$ Notice that First times First must be x^4: x^2 times x^2

$(x^2 \quad)(x^2 \quad)$

The Last times Last must be 36 and the OI term must add up to $13x^2$ (Try 9 • 4, both negative)

$(x^2 \quad 9)(x^2 \quad 4)$

$(x^2 - 9)(x^2 - 4)$ O term is $-4x^2$, and the I term is $-9x^2$, for a total of $-13x^2$

The factors $(x^2 - 9)$ and $(x^2 - 4)$ are both difference of squares. Each must be re-factored so this is NOT the final answer.

$(x^2 - 9)(x^2 - 4)$

$(x - 3)(x + 3)(x - 2)(x + 2)$ Final Answer!!

p. 163: 77.

$x^4 - 29x^2 + 100$ The First times First must be x^4: x^2 times x^2 .

$(x^2 \quad)(x^2 \quad)$

The Last times Last must be 100 and the OI term must add up to $-29x^2$ (Try 25 • 4!!)

$(x^2 \quad 25)(x^2 \quad 4)$

$(x^2 - 25)(x^2 - 4)$ O term is $-4x^2$, and the I term is $-25x^2$, for a total of $-29x^2$

$(x^2 - 25)(x^2 - 4)$

The factors $(x^2 - 25)$ and $(x^2 - 4)$ are each difference of squares. Each must be re-factored.

$(x^2 - 25)(x^2 - 4)$

$(x - 5)(x + 5)(x - 2)(x + 2)$ Final Answer!!

Extra Problems: #49, 63, 67, 69, 71, 81

49. $-32s^2 + 80st - 50t^2$ In ANY factoring problem, the first step is to take out the common factor. Remember FCFF: Factor the Common Factor First!! There is a common factor of 2 to all three terms. It will help to take out the negative as well, so take out the –2. This changes all the signs within the parentheses

$-32s^2 + 80st - 50t^2$

$-2(\_\_\_ - \_\_\_ + \_\_\_)$

$-2(16s^2 - 40st + 25t^2)$

Notice that this is a trinomial , but even more importantly, do you see the PERFECT SQUARES? The First and Last terms are both PERFECT SQUARES!! And the problem factors accordingly!

$-2(4s - \_\_\_)(4s - \_\_\_)$

$-2(4s - 5t)(4s - 5t)$

Can it really be this easy?? Check the OUTER times OUTER (-20st) and the INNER times INNER (-20st), and see that it adds up to -40st ! Can you believe it ??

The FINAL ANSWER can be written: $-2(4s - 5t)^2$

63. $6a^2 - 54$ In ANY factoring problem, the first step is to take out the common factor. Remember FCFF: Factor the Common Factor First!! Be sure to get ALL the common factors. In this case, take out the 6.

$6(a^2 - 9)$

What remains is a DIFFERENCE OF TWO SQUARES!!

$6(a - \_\_\_)(a + \_\_\_)$

$6(a - 3)(a + 3)$

Can it really be this easy??

67. $200 - 2t^2$ In ANY factoring problem, the first step is to take out the common factor. Remember FCFF: Factor the Common Factor First!! Be sure to get ALL the common factors. In this case, take out the 2.

$2(100 - t^2)$

What remains is a DIFFERENCE OF TWO SQUARES !!

$2(10 - \underline{\quad})(10 + \underline{\quad})$

$2(10 - t)(10 + t)$

69. $-80a^2 + 45$ In ANY factoring problem, the first step is to take out the common factor. Remember FCFF: Factor the Common Factor First!! Notice that there is a common factor to both terms, and that common factor is 5. What works even better than taking out the 5 is to take out the –5 .

$-80a^2 + 45$

$-5(\underline{\quad} - \underline{\quad})$

$-5(16a^2 - 9)$

Notice that this is a difference of squares which factors:

$-5(4a - \underline{\quad})(4a + \underline{\quad})$

$-5(4a - 3)(4a + 3)$

71. $5t^2 - 80$ In ANY factoring problem, the first step is to take out the common factor. Remember FCFF: Factor the Common Factor First!! Be sure to get ALL the common factors. In this case, take out the 5.

$5(t^2 - 16)$

What remains is a DIFFERENCE OF TWO SQUARES!! $5(t - \underline{\quad})(t + \underline{\quad})$

$5(t - 4)(t + 4)$

81. $-3x^3 + 24x^2 - 48x$ **In ANY factoring problem, the first step is to take out the common factor. Remember FCFF: Factor the Common Factor First!! There is a common factor of $3x$ to all three terms. It will help to take out the negative as well, so take out the $-3x$. This changes all the signs within the parentheses. Note: Be sure to get ALL the common factors, both the -3 and the x factors!**

$-3x^3 + 24x^2 - 48x$

$-3x(\_\_\_ - \_\_\_ + \_\_\_)$

$-3x(x^2 - 8x + 16)$

Notice that this is a trinomial which factors:

$-3x(x - \_\_\_)(x - \_\_\_)$

$-3x(x - 4)(x - 4)$ **or** $-3x(x - 4)^2$