2.09 Quadratic Equations by Factoring

Basic Algebra: One Step at a Time. Pages 185-192: #21, 22, 23, 25, 26, 27, 28, 30, 34, 37, 38, 42, 46, 55, 56

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21. $3(2x-9) = -x^2$

Solution: The first step will be to remove parentheses by the Distributive Property.

$$6x - 27 = -x^2$$

Next, you must set the equation equal to zero. The easiest way to do this is to add $+x^2$ to each side of the equation.

$$\begin{array}{r} 6x - 27 = -x^2 \\ +x^2 \qquad\qquad +x^2 \\ \hline x^2 + 6x - 27 = 0 \end{array}$$

Now, factor the trinomial, using 9 and 3, with opposite signs.

$$(x+9)(x-3) = 0$$

Set each factor equal to zero.

$$x + 9 = 0 \quad \text{or} \quad x - 3 = 0$$

Final Answer: $x = -9$ or $x = 3$

22. $7x = 18 - x^2$

Solution: The first step is to set the equation equal to zero. To do this, you can add $+x^2$ and subtract 18 from each side:

$$\begin{array}{rcl} 7x & = & 18 - x^2 \\ +x^2 - 18 & & -18 + x^2 \end{array}$$

$$x^2 + 7x - 18 = 0$$

Now, factor the trinomial, using 9 and 2, with opposite signs.

$$(x + 9)(x - 2) = 0$$

Set each factor equal to zero.

$$x + 9 = 0 \quad \text{or} \quad x - 2 = 0$$

Final Answer: $x = -9$ or $x = 2$

23. $x(6 - x) = -40$

Solution: The first step will be to remove parentheses by the Distributive Property.

$$6x - x^2 = -40$$

Next, you must set the equation equal to zero. There are two methods! You decide which way you think is easier!!

METHOD I: In order to set the equation equal to zero with a POSITIVE coefficient of x^2, take everything to the RIGHT side of the equation!

$$\begin{array}{rcl} 6x - x^2 & = & -40 \\ -6x + x^2 & & -6x + x^2 \end{array}$$

$$0 = x^2 - 6x - 40$$

Now, factor the trinomial, using 10 and 4, with opposite signs.

$$(x-10)(x+4)=0$$

Set each factor equal to zero.

$$x-10=0 \quad \text{or} \quad x+4=0$$

Final Answer: $x=10$ or $x=-4$

METHOD II: $6x-x^2=-40$

In order to set the equation equal to zero you may choose to add $+40$ to each side of the equation

$$\begin{array}{rcl} 6x-x^2 & = & -40 \\ +40 & & +40 \end{array}$$

$$-x^2+6x+40=0$$

In order to factor this, you can multiply both sides of the equation by (-1).

$$(-1)\bullet\left(-x^2+6x+40\right)=(-1)\bullet 0$$

$$x^2-6x-40=0$$

Now, factor the trinomial, using 10 and 4, with opposite signs.

$$(x-10)(x+4)=0$$

Set each factor equal to zero.

$$x-10=0 \quad \text{or} \quad x+4=0$$

Final Answer: $x=10$ or $x=-4$

p. 188: 25. $2x^2 - 3x = 0$

Solution: Since this equation is already equal to zero, the first step will be to factor the left side. The first step in any factoring problem is to try to take out a common factor. In this case the common factor is x

$$x(2x - 3) = 0$$

Set each factor equal to zero:

$$x = 0 \quad \text{or} \quad 2x - 3 = 0$$
$$2x = 3$$
$$x = \frac{3}{2}$$

Final Answer: $x = 0$ or $x = \frac{3}{2}$

p. 188: 26. $5x^2 + 4x = 0$

Solution: Since this equation is already equal to zero, the first step will be to factor the left side. The first step in any factoring problem is to try to take out a common factor. In this case the common factor is x

$$x(5x + 4) = 0$$

Set each factor equal to zero:

$$x = 0 \quad \text{or} \quad 5x + 4 = 0$$
$$5x = -4$$
$$x = \frac{-4}{5}$$

Final Answer: $x = 0$ or $x = \frac{-4}{5} \; or \; -\frac{4}{5}$

p. 188: 27. $x(2x+7)=-5$

$$2x^2+7x+5=0$$

Notice that this is really just a TRINOMIAL, and as such, it can be factored into the product of two binomials. In this case, the FIRST times FIRST gives you $2x^2$ which is $2x \bullet x$.

$$(2x\_\_\_\_)(x\_\_\_\_)=0$$

Next, the LAST times LAST must give you $+5$, so try $5 \bullet 1$ or $1 \bullet 5$, where the numbers are the SAME sign. In order for the numbers to add and give you a $+7x$ you will need to put the 5 in the first binomial and the 1 in the second binomial. Then the OUTER times OUTER will be $+2x$, and the INNER times INNER will be $+5x$ for a total of $+7x$.

It looks like this:

$$(2x+5)(x+1)=0$$

Now, set each factor equal to zero, and solve for x. There are two solutions:

$$(2x+5)=0 \quad \text{or} \quad (x+1)=0$$
$$2x=-5 \qquad x=-1$$
$$x=-\frac{5}{2}$$

p. 188: 28. $x(3x+1)=2$

Solution: The first step is to multiply out the parentheses and set the equation equal to zero by subtracting 2 from each side of the equation.

$$3x^2+x=2$$

$$3x^2+x-2=0$$

The next step is to factor the resulting trinomial. There is no common factor, so it must begin like this:

$$(3x\quad)(x\quad)=0$$

$$(3x-2)(x+1)=0$$

$$3x=2 \text{ or } x=-1$$

$$x=\frac{2}{3}$$

Final Answer: $x=\frac{3}{2}$ or $x=-1$

p. 188: 30. $x(2x-9)=5$

Solution: The first step is to multiply out the parentheses and set the equation equal to zero by subtracting 5 from each side of the equation.

$$2x^2-9x=5$$

$$2x^2-9x-5=0$$

The next step is to factor the resulting trinomial. There is no common factor, so it must begin like this:

$$(2x\quad)(x\quad)=0$$

$$(2x+1)(x-5)=0$$

$$2x=-1 \text{ or } x=5$$

$$x=-\frac{1}{2}$$

Final Answer: $x=-\frac{1}{2}$ or $x=5$

p. 188: 34. Since the equation is already set equal to zero, the first step is to FACTOR the left side of the equation. There IS a common factor of 5x, so the first step in factoring must be to take out the common factor of 5x.

$$5x^3 + 20x^2 + 15x = 0$$

$$5x(x^2 + 4x + 3) = 0$$

The next step is to factor the resulting trinomial. There looks like this::

$$5x(\quad)(\quad) = 0$$

$$5x(x\quad)(x\quad) = 0$$

$$5x(x + 3)(x + 1) = 0$$

$$5x = 0 \text{ or } x + 3 = 0 \text{ or } x + 1 = 0$$

$$x = \frac{0}{5} \text{ or } x = -3 \text{ or } x = -1$$

Final Answer: $x = 0$ or $x = -3$ or $x = -1$

37. $4x^2 + 20x + 25 = 0$

Solution: Since this equation is already equal to zero, the first step will be to factor the left side. The first step is to notice the $4x^2$. Unfortunately, you can't take out a common factor, so it looks like an "advanced trinomial factoring" problem. However, it might not be too bad! Notice the perfect squares on each end of this trinomial. Maybe it's a perfect square trinomial! Let's try that first. Anyway, set up the two sets of parentheses, and prepare to factor the trinomial however it works out!

$$4x^2 + 20x + 25 = 0$$

$$(\_\_\_ + \_\_\_)(\_\_\_ + \_\_\_) = 0$$

First times First will be **2x times 2x.**

$$(2x + \_\_\_)(2x + \_\_\_) = 0$$

Last times Last will be **5 times 5.**

$$(2x + 5)(2x + 5) = 0 \qquad \text{OR} \qquad (2x + 5)^2 = 0$$

Set each factor equal to zero.

$$2x + 5 = 0 \quad \text{or} \quad 2x + 5 = 0$$
$$2x = -5 \qquad 2x = -5$$
$$x = -\frac{5}{2} \qquad x = -\frac{5}{2}$$

Final Answer: $x = -\frac{5}{2}$ or $x = -\frac{5}{2}$

$$2x + 5 = 0$$
$$2x = -5$$
$$x = -\frac{5}{2}$$

Final Answer: $x = -\frac{5}{2}$

38. $4x^2 + 9 = 12x$

Solution: First, you must set the equation equal to zero. Do this by adding $-12x$ to each side.

$$\begin{array}{rcl} 4x^2 + 9 & = & 12x \\ -12x & & -12x \\ \hline 4x^2 - 12x + 9 & = & 0 \end{array}$$

The first step will be to factor the left side. The first step is to notice the $4x^2$. Unfortunately, you can't take out a common factor, so it looks like an "advanced trinomial factoring" problem. However, it might not be too bad! Notice the perfect squares on each end of this trinomial. Maybe it's a perfect square trinomial! Let's try that first. Anyway, set up the two sets of parentheses, and prepare to factor the trinomial however it works out!

$$4x^2 - 12x + 9 = 0$$

$$(\_\_\_ - \_\_\_)(\_\_\_ - \_\_\_) = 0$$

First times First will be 2x times 2x.

$$(2x - \_\_\_)(2x - \_\_\_) = 0$$

Last times Last will be 3 times 3.

$$(2x - 3)(2x - 3) = 0 \qquad \text{OR} \qquad (2x - 3)^2 = 0$$

Set each factor equal to zero.

$$\begin{array}{rcl} 2x - 3 = 0 & \text{or} & 2x - 3 = 0 \\ 2x = 3 & & 2x = 3 \\ x = \frac{3}{2} & & x = \frac{3}{2} \end{array}$$

Final Answer: $x = \frac{3}{2}$ or $x = \frac{3}{2}$

$$\begin{array}{r} 2x - 3 = 0 \\ 2x = 3 \\ x = \frac{3}{2} \end{array}$$

Final Answer: $x = \frac{3}{2}$

42. $x^3 - 25x = 0$

Solution: Since this equation is already equal to zero, the first step will be to factor the left side. Remember FCFF? Factor the Common Factor First!! Factor out the common factor of x.

$$x(x^2 - 25) = 0$$

Next, notice that you have a difference of two squares!

$$x(x \_\_\_\_)(x \_\_\_\_) = 0$$

$$x(x - 5)(x + 5) = 0$$

Notice that there are 3 factors (in keeping with an x^3 equation). Set each factor equal to zero.

$$x = 0 \quad \text{or} \quad x - 5 = 0 \quad \text{or} \quad x + 5 = 0$$
$$x = 5 \qquad x = -5$$

Final Answer: $x = 0$ or $x = 5$ or $x = -5$

46. $x^3 - 9x^2 = 0$

Solution: Since this equation is already equal to zero, the first step will be to factor the left side. Remember FCFF? Factor the Common Factor First!! Factor out the common factor of x^2.

$$x^2(x-9)=0$$

$$x \bullet x \bullet (x-9)=0$$

Notice that there are 3 factors (in keeping with an x^3 equation). Set each factor equal to zero.

$$x=0 \quad \text{or} \quad x=0 \quad \text{or} \quad x-9=0$$
$$x=9$$

Final Answer: $x=0 \quad \text{or} \quad x=0 \quad \text{or} \quad x=9$

NOTE: The answer of $x=0$ is repeated to illustrate the 3 solutions for this x^3 equation.

p. 192: 55. $(x-4)^2 = 2x$

Solution: The first step in solving this equation is to square the binomial—i.e., remove the parentheses:

$$(x-4)(x-4) = 2x$$

$$x^2 - 8x + 16 = 2x$$

The next step is to set the equation equal to zero. Subtract $2x$ from each side:

$$\begin{array}{r} x^2 - 8x + 16 = 2x \\ -2x \quad\quad -2x \\ \hline \end{array}$$

$$x^2 - 10x + 16 = 0$$

Notice that this is a trinomial which factors:

$$(x - \_\_\_)(x - \_\_\_)$$

$$(x - 8)(x - 2) = 0$$

Therefore, $x = 8$, $x = 2$

p. 192: 56. $(x-4)^2 = 32 - 2x$

Solution: The first step in solving this equation is to square the binomial—i.e., remove the parentheses:

$$(x-4)(x-4) = 32 - 2x$$

$$x^2 - 8x + 16 = 32 - 2x$$

The next step is to set the equation equal to zero, by subtracting 32 and adding $+2x$ from each side:

$$\begin{array}{r} x^2 - 8x + 16 = 32 - 2x \\ +2x - 32 \quad -32 + 2x \\ \hline \end{array}$$

$$x^2 - 6x - 16 = 0$$

Notice that this is a trinomial which factors:

$$(x - 8)(x + 2) = 0$$

Therefore, $x = 8$, $x = -2$