2.06 Advanced Trinomial Factoring

Basic Algebra: One Step at a Time. Pages 165-172: 46, 47, 48, 49, 66

Extra Problem #9

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p. 171: 43-58. Notice that these exercises are actually a series of exercises in which the First times First must be $5x^2$, and the LAST times LAST must be 8. Obviously, there is only one way to get the First times First to be $5x^2$! It must be $5x$ times x. To make the LAST times LAST come out to 8, you can use either $1 \bullet 8$ or $2 \bullet 4$. These can be determined by trial and error as illustrated by the following exercises.

Shortcuts for Factoring Trinomials

1. **If the middle term is odd, then you can't use an "even-even" combination of numbers.**
2. **If you don't have a common factor in the problem, then you can't have a common factor in either factor of the answer.**

p. 171: 46. $5x^2 + 13x + 8$

SOLUTION: Notice that this is a **trinomial** since it has three terms. Since the coefficient of x^2 is NOT 1, and it can't be factored out as a common factor, this is what I call an "Advanced Trinomial Factoring" problem. Begin by remembering that this is actually "undoing" a **F OI L** (that is, **F L OI**).

In this case, the **First times First** is obviously ***5x*** times **x**. In the factored form the signs will BOTH be positive.

$5x^2 + 13x + 8$

$(5x \quad \_\_\_)(x \quad \_\_\_)$

The **Last times Last** must be two numbers whose product is 8. This would be either $1 \bullet 8$ or $2 \bullet 4$. It can't be $2 \bullet 4$, because the middle term is an odd number. The correct combination must be $1 \bullet 8$.

However, because of the ***5x*** times **x**, there are two ways to write the $1 \bullet 8$ combination:

$(5x + 1)(x + 8)$ $\qquad$ $(5x + 8)(x + 1)$

To decide which is the correct combination, you must do the **OUTER times OUTER** and the **INNER times INNER**, and see which one gives the sum of **13x**.

In the case of $(5x + 1)(x + 8)$, the **OUTER times OUTER** is **40x**, and the **INNER times INNER** is **1x**. This obviously does not add up to **13x**.

In the case of $(5x + 8)(x + 1)$, the **OUTER times OUTER** is **5x**, and the **INNER times INNER** is **8x**, which DOES add up to **13x**. This is the correct combination.

Final answer: $(5x + 8)(x + 1)$.

p. 171: 47. $5x^2 - 14x + 8$

SOLUTION: Notice that this is a **trinomial** since it has three terms. Since the coefficient of x^2 is NOT 1, and it can't be factored out as a common factor, this is what I call an "Advanced Trinomial Factoring" problem. Begin by remembering that this is actually "undoing" a **F OI L** (that is, **F L OI**).

In this case, the **First times First** is obviously ***5x*** times **x**. In the factored form, the signs will both be negative.

$$5x^2 - 14x + 8$$

$$(5x - \_\_\_)(x - \_\_\_)$$

The **Last times Last** must be two numbers whose product is 8. This would be either $1 \bullet 8$ or $2 \bullet 4$. Since the middle term is an even number, the correct combination might be (but not necessarily!) $2 \bullet 4$. It's a good place to start. However, because of the ***5x*** times **x**, there are two ways to write the $2 \bullet 4$ combination:

$(5x - 2)(x - 4)$ $\qquad$ $(5x - 4)(x - 2)$

To decide which is the correct combination, you must do the **OUTER times OUTER** and the **INNER times INNER**, and see which one gives the sum of **-14x**.

In the case of $(5x - 2)(x - 4)$, the **OUTER times OUTER** is **-20x**, and the **INNER times INNER** is **-2x**. This obviously does not add up to **-14x**.

In the case of $(5x - 4)(x - 2)$, the **OUTER times OUTER** is **-10x**, and the **INNER times INNER** is **-4x**, which DOES add up to **-14x**. This is the correct combination.

Final answer: $(5x - 4)(x - 2)$.

48. $5x^2 - 41x + 8$

SOLUTION: Notice that this is a trinomial since it has three terms. In this case, the First times First is obviously 5x times x. In the factored form, the signs will both be negative.

$$5x^2 - 41x + 8$$

$$(5x - \_\_\_)(x - \_\_\_)$$

The Last times Last must be two numbers whose product is 8. This would be either $1 \bullet 8$ or $2 \bullet 4$. It can't be $2 \bullet 4$, because the middle term is an odd number. The correct combination must be $1 \bullet 8$.

However, because of the 5x times x, there are two ways to write the $1 \bullet 8$ combination:

$(5x - 1)(x - 8)$ $\qquad$ $(5x - 8)(x - 1)$

To decide which is the correct combination, you must do the OUTER times OUTER and the INNER times INNER, and see which one gives the sum of 41x.

In the case of $(5x - 1)(x - 8)$, the OUTER times OUTER is -40x, and the INNER times INNER is -1x. This DOES add up to -41x., so this is the correct combination.

Final answer: $(5x - 1)(x - 8)$.

49. $5x^2 + 3x - 8$

SOLUTION: Notice that this is a **trinomial** since it has three terms. In this case, the **First times First** is obviously *5x* times *x*. In the factored form, the signs will be opposite, and the middle terms must be subtracted to get +3x.

$5x^2 + 3x - 8$

$(5x \quad \_\_\_)(x \quad \_\_\_)$

The **Last times Last** must be two numbers whose product is -8. This would be either $1 \bullet 8$ or $2 \bullet 4$. It can't be $2 \bullet 4$, because the middle term is an odd number. The correct combination must be $1 \bullet 8$.

However, because of the *5x* times *x*, there are two ways to write the $1 \bullet 8$ combination:

$(5x \quad 1)(x \quad 8) \qquad (5x \quad 8)(x \quad 1)$

To decide which is the correct combination, you must do the **OUTER times OUTER** and the **INNER times INNER**, and see which one gives the difference of +3x.

In the case of $(5x \quad 1)(x \quad 8)$, the **OUTER times OUTER** is 40x, and the **INNER times INNER** is 1x. This does not subtract to give you 3x., so this is not the correct combination.

In the case of $(5x \quad 8)(x \quad 1)$, the **OUTER times OUTER** is 5x, and the **INNER times INNER** is 8x. This DOES subtract to give you 3x., so this IS the correct combination.

In order to get the +3x, you need to have +8x and -5x. Therefore, the correct combination is $(5x + 8)(x - 1)$

Final answer: $(5x + 8)(x - 1)$.

66. $6x^2 + 19x + 8$

SOLUTION: Notice that this is a trinomial since it has three terms. In this case, the First times First is must either be 6x times x or 3x times 2x. It will take some "trial and error" to determine which, if either, combination works. Notice that in the factored form, the signs will be the SAME sign, both POSITIVE, and the middle term must add up to 19x.

$6x^2 + 19x + 8$

$(6x \quad \_\_\_)(x \quad \_\_\_)$ **OR** $(3x \quad \_\_\_)(2x \quad \_\_\_)$

The Last times Last must be two numbers whose product is $+8$. This would be either $1 \bullet 8$ or $2 \bullet 4$. It can't be $2 \bullet 4$, because the middle term is an odd number (see Shortcut #1 from the first page). The correct combination must be $1 \bullet 8$.

However, in the case of 6x times x, there are two ways to write the $1 \bullet 8$ combination:

$(6x \quad 1)(x \quad 8)$ $\qquad$ $(6x \quad 8)(x \quad 1)$

To decide if either of these is the correct combination, you must do the OUTER times OUTER and the INNER times INNER, to see which one gives the sum of +19x.

In the case of $(6x \quad 1)(x \quad 8)$, the OUTER times OUTER is 48x, and the INNER times INNER is 1x. This does not add to 19x, so this is not the correct combination.

In the case of $(6x \quad 8)(x \quad 1)$, there is a common factor of 2 in the first factor, so this is NOT a possibility (see Shortcut #2 from the first page of this section)!

Likewise, in the case of 3x times 2x, there are two ways to write the $1 \cdot 8$ combination:

$(3x \quad 1)(2x \quad 8) \qquad (3x \quad 8)(2x \quad 1)$

To decide if either of these is the correct combination, you must do the OUTER times OUTER and the INNER times INNER, to see which one gives the sum of +19x.

In the case of $(3x \quad 1)(2x \quad 8)$, there is a common factor of 2 in the second factor, so this is NOT a possibility (see Shortcut #2 from the first page of this section)!

In the case of $(3x \quad 8)(2x \quad 1)$, the OUTER times OUTER is 3x, and the INNER times INNER is 16x. This DOES add to 19x, so this IS the correct combination.

Therefore, the correct combination is $(3x + 8)(2x + 1)$

Final answer: $(3x + 8)(2x + 1)$.

Please note that the following exercise uses BOTH of the shortcuts mentioned on the first page of this section!!

#9. $2t^2 + 11t + 12$

SOLUTION: Since the coefficient of x^2 is NOT 1, and it can't be factored out as a common factor, this is what I call an "Advanced Trinomial Factoring" problem. Begin by remembering that this is actually "undoing" a F OI L (that is, F L OI).

In this case, the First times First is obviously 2x times x. In the factored form the signs will BOTH be positive.

$2t^2 + 11t + 12$

$(2x \quad \_\_\_)(x \quad \_\_\_)$

The Last times Last must be two numbers whose product is 12. This would be either $1 \bullet 12$ or $2 \bullet 6$ or $3 \bullet 4$. According to Shortcut #1 above, since the middle term is an ODD number, this CANNOT be an "even-even" combination, so it can't be $2 \bullet 6$. The correct combination must be either $1 \bullet 12$ or $3 \bullet 4$. It's probably $3 \bullet 4$!!

However, because of the 2x times x, there are two ways to write the $3 \bullet 4$ combination:

$(2x + 3)(x + 4)$ $\qquad$ $(2x + 4)(x + 3)$

Notice that in he second possibility $(2x + 4)(x + 3)$ there is a common factor of 2 in the first factor of the answer! This is NOT allowed, according to Shortcut #2.

Now try $(2x + 3)(x + 4)$.

To decide if this is the correct combination, you must do the OUTER times OUTER and the INNER times INNER, and see if you get a sum of 11x. In this case. the OUTER times OUTER is 8x, and the INNER times INNER is 3x, which adds up to 11x.

Final answer: $(2x + 3)(x + 4)$.