3.05 Fractional Equations

Basic Algebra: One Step at a Time. **Page 281 - 288: #12, 15, 16, 20, 24, 25, 29,30**

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P. 285. # 12. $\frac{3}{x-6}=\frac{5}{x-6}$

Since $\frac{a}{b}=\frac{c}{d}$ means that $a \bullet d = b \bullet c$, it follows that

$\frac{3}{x-6}=\frac{5}{x-6}$ means that $3 \bullet (x-6) = 5 \bullet (x-6)$. This eliminates ALL the fractions! However, remember that there are variables in the denominators! When you do this, you must be careful that the denominators must NEVER equal zero. Therefore, be careful that x cannot equal 6. That is, $x \neq 6$. You have to solve this very simple equation. Start by removing the parentheses.

$3 \bullet (x-6) = 5 \bullet (x-6)$

$3x-18=5x-30$

$-5x \quad -5x$ To get all the x terms on the left side, subtract $5x$ from each side.

$-2x-18=-30$

$+18 \quad +18$ Now, add $+18$ to each side to get all the number terms on the right side.

$-2x=-12$

$-2x=-12$ Divide both sides by -2.

$x=6$

However, $x \neq 6$, so this answer must be rejected. The final answer is NO SOLUTION, because $x=6$ makes the denominators equal to zero, which is NOT allowed !!

P. 285. # 15. $\frac{x-2}{x+4}=\frac{x}{x+2}$

Since $\frac{a}{b}=\frac{c}{d}$ means that $a\bullet d=b\bullet c$, it follows that

$\frac{x-2}{x+4}=\frac{x}{x+2}$ means that $(x-2)\bullet(x+2)=x\bullet(x+4)$. This eliminates ALL the fractions! However, remember that there are variables in the denominators! When you do this, you must be careful that the denominators must NEVER equal zero. Therefore, be careful that x cannot equal -4 or -2. That is, $x\neq -4,\quad x\neq -2$. As always, you have to solve this very simple equation. Start by removing the parentheses.

$(x-2)\bullet(x+2)=x\bullet(x+4)$

$x^2-4=x^2+4x$

$-x^2\quad -x^2$

$-4=\ 4x$

To get all the x terms on the left side, subtract x^2 from each side.

$-4=\ 4x$ Divide both sides by 4.

$x=-1$

The denominators are NOT zero, so this is an acceptable answer, and the check is optional. It's an easy check.

Check: $\frac{x-2}{x+4}=\frac{x}{x+2}$, where $x=-1$

$\frac{(-1)-2}{(-1)+4}=\frac{(-1)}{(-1)+2}$

$\frac{-3}{3}=\frac{-1}{1}$ It checks!!

P. 285. # 16. $\frac{x+6}{x}=\frac{x-2}{x-4}$

Since $\frac{a}{b}=\frac{c}{d}$ means that $a\bullet d=b\bullet c$, it follows that

$\frac{x+6}{x}=\frac{x-2}{x-4}$ means that $(x+6)\bullet(x-4)=x\bullet(x-2)$. This eliminates ALL the fractions! However, remember that there are variables in the denominators! When you do this, you must be careful that the denominators must NEVER equal zero. Therefore, be careful that x cannot equal 0 or 4. That is, $x\neq 0,\quad x\neq 4$. As always, you have to solve this very simple equation. Start by removing the parentheses.

$(x+6)\bullet(x-4)=x\bullet(x-2)$

$$\begin{array}{r} x^2+2x-24=x^2-2x \\ -x^2\qquad\quad -x^2 \\ \hline 2x-24=\quad -2x \end{array}$$

To get all the x terms on the left side, subtract x^2. from each side.

$$\begin{array}{r} 2x-24=\ -2x \\ -2x\qquad\quad -2x \\ \hline -24=\ -4x \end{array}$$

Add $-2x$ to each side of the equation.

$\frac{-24}{-4}=\frac{-4x}{-4}$ Divide both sides by sides by -4.

$6=x$

The denominators are NOT zero, so this is an acceptable answer, and the check is optional. It's an easy check.

Check: $\frac{x+6}{x}=\frac{x-2}{x-4}$, where $x=6$

$\frac{6+6}{6}=\frac{6-2}{6-4}$

$\frac{12}{6}=\frac{4}{2}$ It checks!!

P. 286. # 20. $\frac{4}{x-2}=\frac{x}{2}$

Since $\frac{a}{b}=\frac{c}{d}$ means that $a\bullet d=b\bullet c$, it follows that

$\frac{4}{x-2}=\frac{x}{2}$ means that $4\bullet 2=x\bullet(x-2)$ or $x\bullet(x-2)=4\bullet 2$ This eliminates ALL the fractions! However, remember that there are variables in the denominators! When you do this, you must be careful that the denominators must NEVER equal zero. Therefore, be careful that x cannot equal 2. That is, $x\neq 2$. Start by removing the parentheses.

$x\bullet(x-2)=4\bullet 2$

$x^2-2x = 8$

$-8 \quad -8$ This is a quadratic equation, so set it to zero by subtracting 8

$x^2-2x-8=0$ from each side.

$(x-4)(x+2)=0$ Factor the trinomial.

$x=4, \quad x=-2$

The denominators are NOT zero, so this is an acceptable answer, and the check is optional.

Check: $\frac{4}{x-2}=\frac{x}{2}$, where $x=4$

$\frac{4}{4-2}=\frac{4}{2}$

$\frac{4}{2}=\frac{4}{2}$ It checks!!

Check: $\frac{4}{x-2}=\frac{x}{2}$, where $x=-2$

$\frac{4}{(-2)-2}=\frac{-2}{2}$

$\frac{4}{(-4)}=\frac{-2}{2}$ It checks!!

P. 286. # 24. $\frac{x-2}{4}=\frac{3x-6}{10}$

Since $\frac{a}{b}=\frac{c}{d}$ means that $a \bullet d = b \bullet c$, it follows that

$\frac{x-2}{4}=\frac{3x-6}{10}$ means that $10 \bullet (x-2) = 4 \bullet (3x-6)$. This eliminates ALL the fractions!

Solve this very simple equation. Start by removing the parentheses.

$10x-20=12x-24$ To get all the x terms on the left side, subtract $12x$ from each side.

$10x-12x-20=12x-12x-24$

$-2x-20=-24$ Now, add $+20$ to each side to get all the number terms on the right side.

$-2x-20+20=-24+20$

$-2x=-4$ Divide both sides by -2.

$x=2$ Final Answer!! The check is easy! Substitute $x=2$ into the original problem.

$\frac{2-2}{4}=\frac{3 \bullet 2-6}{10}$

$\frac{0}{4}=\frac{0}{10}$ so $0=0$ which checks!

P. 287. # 25. $\frac{3(x-2)}{4}-\frac{x}{2}=2$

Notice that this is a fractional equation. The first step is to find the LCD, which is 4, and multiply both sides of the equation by the LCD. Since there are no variables in the denominators, the denominators will never be zero, so there will not be a possibility of rejected answers. Begin by multiplying both sides by 4.

$$4\bullet\frac{3(x-2)}{4}-4\bullet\frac{x}{2}=4\bullet 2$$

This looks pretty ugly, but when you reduce all the fractions, it really is not bad. In fact, ALL THE DENOMINATORS DIVIDE OUT!! When you reduce all the fractions, this is what is left—NO FRACTIONS!!

$$1\bullet 3(x-2)-2\bullet x=4\bullet 2$$

$$3x-6-2x=8$$

$$x-6=8$$

$x=14$ Final Answer!! (The check is optional, but simple!)

Check: $\frac{3(14-2)}{4}-\frac{14}{2}=2$

$$\frac{3(12)}{4}-\frac{14}{2}=2$$

$$\frac{36}{4}-\frac{14}{2}=2$$

$9-7=2$ It checks!!

P. 287. # 29. $\frac{x(x-8)}{6}+\frac{x-2}{2}=\frac{4}{3}$

Notice that this is a fractional equation. The first step is to find the LCD, which is 6, and multiply both sides of the equation by the LCD. Since there are no variables in the denominators, the denominators will never be zero, so there will not be a possibility of rejected answers. Begin by multiplying both sides by 6.

$$6\bullet\frac{x(x-8)}{6}+6\bullet\frac{x-2}{2}=6\bullet\frac{4}{3}$$

This looks pretty ugly, but when you reduce all the fractions, it really is not bad. In fact, ALL THE DENOMINATORS DIVIDE OUT!! When you reduce all the fractions, this is what is left—NO FRACTIONS!!

$$1\bullet x(x-8)+3\bullet(x-2)=2\bullet 4$$

$$x^2-8x+3x-6=8$$

$$x^2-8x+3x-6-8=0$$

$$x^2-5x-14=0$$

$$(x-7)(x+2)=0$$

$x=7,\quad x=-2$ Final Answer!! (The check is optional!)

P. 287. # 30. $\frac{x(x+1)}{6}-\frac{x}{3}=1$

Notice that this is a fractional equation. The first step is to find the LCD, which is 6, and multiply both sides of the equation by the LCD. Since there are no variables in the denominators, the denominators will never be zero, so there will not be a possibility of rejected answers. Begin by multiplying both sides by 6.

$$6\bullet\frac{x(x+1)}{6}-6\bullet\frac{x}{3}=6\bullet 1$$

ALL THE DENOMINATORS DIVIDE OUT!! When you reduce all the fractions, this is what is left—**NO FRACTIONS!!**

$$1 \bullet x(x+1) - 2 \bullet x = 6 \bullet 1$$

$$x^2 + x - 2x = 6$$

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$x = 3, \quad x = -2$ **Final Answer!! (The check is optional!)**