3.02 Multiplying and Dividing Fractions

Basic Algebra: One Step at a Time **Section 3.02** **Page 247-256: # 19, 24, 31, 32, 34, 36, 38, 42, 47**

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P. 251 # 19. $\dfrac{42x^7}{27y^3} \div \dfrac{14x^6}{9y^5}$

This is a division problem, so you must invert the second fraction and multiply. Several factors will divide out. Notice that these are FACTORS, not TERMS.

$$\frac{42x^7}{27y^3} \bullet \frac{9y^5}{14x^6}$$

First, 14 divides into 42 and it goes 3 times, and 9 divides into 27, and it goes 3 times.

$$\frac{3\,\cancel{42}x^7}{3\,\cancel{27}y^3} \bullet \frac{\cancel{9}y^5}{\cancel{14}x^6}$$

Next, the 3 factors divide out:

$$\frac{x^7}{y^3} \bullet \frac{y^5}{x^6}$$

Now, the x^6 factor divides out with the x^7 leaving an x factor in the numerator, and the y^3 factor divides out with the y^5 leaving a y^2 factor in the numerator. It looks like this:

$$\frac{x\,\cancel{x^7}}{\cancel{y^3}} \bullet \frac{\cancel{y^5}\,y^2}{\cancel{x^6}}$$

The final answer is $\dfrac{xy^2}{1}$ or xy^2

P. 252 # 24.

$$\frac{x^2 - x}{x^2 - x - 12} \bullet \frac{x^2 - 3x - 4}{x^2 - 1}$$

This is a multiplication problem, so you must factor, factor, factor, and factor! Factor everything, and see if any factors divide out.

$$\frac{x(x-1)}{(x-4)(x+3)} \bullet \frac{(x-4)(x+1)}{(x-1)(x+1)}$$

Notice that the (x-1) in the first numerator and the (x-1) in the second denominator divide out.

$$\frac{x\cancel{(x-1)}}{(x-4)(x+3)} \bullet \frac{(x-4)(x+1)}{\cancel{(x-1)}(x+1)}$$

$$\frac{x}{(x-4)(x+3)} \bullet \frac{(x-4)(x+1)}{(x+1)}$$

Also the (x-4) in the first denominator and the second numerator divide out.

$$\frac{x}{\cancel{(x-4)}(x+3)} \bullet \frac{\cancel{(x-4)}(x+1)}{(x+1)}$$

$$\frac{x}{(x+3)} \bullet \frac{(x+1)}{(x+1)}$$

And the (x+1) in the second numerator and denominator divide out.

$$\frac{x}{(x+3)} \bullet \frac{\cancel{(x+1)}}{\cancel{(x+1)}}$$

What is left is the x in the numerator and the (x+3) in the denominator.

$$\frac{x}{(x+3)}$$

P. 254 #31. $$\frac{x^2-49y^2}{x^2+12xy+35y^2} \bullet \frac{x^2-3xy-10y^2}{x^2-5xy-14y^2}$$

This is a multiplication problem, so you must factor, factor, factor, and factor! Factor everything, and see if any factors divide out.

$$\frac{(x-7y)(x+7y)}{(x+7y)(x+5y)} \bullet \frac{(x-5y)(x+2y)}{(x-7y)(x+2y)}$$

Notice that the (x+7y) in the first numerator and the (x+7y) in the first denominator divide out.

$$\frac{(x-7y)\cancel{(x+7y)}}{\cancel{(x+7y)}(x+5y)} \bullet \frac{(x-5y)(x+2y)}{(x-7y)(x+2y)}$$

$$\frac{(x-7y)}{(x+5y)} \bullet \frac{(x-5y)(x+2y)}{(x-7y)(x+2y)}$$

Also the (x+2y) in the second numerator and the second denominator divide out.

$$\frac{(x-7y)}{(x+5y)} \bullet \frac{(x-5y)\cancel{(x+2y)}}{(x-7y)\cancel{(x+2y)}}$$

$$\frac{(x-7y)}{(x+5y)} \bullet \frac{(x-5y)}{(x-7y)}$$

And the (x-7y) in the first numerator and second denominator divide out.

$$\frac{\cancel{(x-7y)}}{(x+5y)} \bullet \frac{(x-5y)}{\cancel{(x-7y)}}$$

What is left is the (x-5y) in the numerator and the (x+5y) in the denominator.

$$\frac{x-5y}{x+5y}$$

P. 254 #32. $\dfrac{x^2-8xy+16y^2}{x^2-3xy-10y^2} \bullet \dfrac{x^2-4y^2}{x^2-5xy+4y^2}$

This is a multiplication problem, so you must factor, factor, factor, and factor! Factor everything, and see if any factors divide out.

$$\frac{(x-4y)(x-4y)}{(x-5y)(x+2y)} \bullet \frac{(x-2y)(x+2y)}{(x-4y)(x-y)}$$

Notice that one of the (x-4y) factors in the first numerator and the (x-4y) in the second denominator divide out.

$$\frac{(x-4y)\cancel{(x-4y)}}{(x-5y)(x+2y)} \bullet \frac{(x-2y)(x+2y)}{\cancel{(x-4y)}(x-y)}$$

$$\frac{(x-4y)}{(x-5y)(x+2y)} \bullet \frac{(x-2y)(x+2y)}{(x-y)}$$

Also the (x+2y) in the second numerator and the (x+2y) in first denominator divide out.

$$\frac{(x-4y)}{(x-5y)\cancel{(x+2y)}} \bullet \frac{(x-2y)\cancel{(x+2y)}}{(x-y)}$$

$$\frac{(x-4y)}{(x-5y)} \bullet \frac{(x-2y)}{(x-y)}$$

The final answer is $\dfrac{(x-4y)(x-2y)}{(x-5y)(x-y)}$.

If you want to do so, you can multiply out the numerator and denominator, but this is not necessary. The factored form is actually preferred!

P. 254 # 34. $\dfrac{x^3y^2}{6xy+12x} \div \dfrac{y^3}{y^2-4}$

This is a division problem, so you must invert the second fraction and multiply. Meanwhile, you should try to factor everything that you can, in order to set up the next step of dividing out factors. NEVER DIVIDE OUT TERMS!! In the first denominator, there are common factors of 6 and x to take out. The second denominator is a difference of two squares which factors and becomes the second numerator.

$$\frac{x^3y^2}{6x(y+2)} \bullet \frac{(y-2)(y+2)}{y^3}$$

Notice that the (y+2) in the first denominator and the (y+2) in the second numerator divide out.

$$\frac{x^3y^2}{6x\,\cancel{(y+2)}} \bullet \frac{(y-2)\,\cancel{(y+2)}}{y^3}$$

$$\frac{x^3y^2}{6x} \bullet \frac{(y-2)}{y^3}$$

Also the y^2 in the first numerator divides out with the y^3 in the second denominator, leaving a y in the denominator.

$$\frac{x^3\cancel{y^2}}{6x} \bullet \frac{(y-2)}{\cancel{y^3}\,y}$$

$$\frac{x^3}{6x} \bullet \frac{(y-2)}{y}$$

Divide out the x factor, leaving

$$\frac{x^2}{6} \bullet \frac{(y-2)}{y}$$

The final answer is $\dfrac{x^2(y-2)}{6y}$

P. 254 # 36.

$$\frac{3x^2-x-2}{x^4-x^3} \div \frac{3x^2+5x+2}{x^3-2x^2}$$

This is a division problem, so you must invert the second fraction and multiply. Meanwhile, you should try to factor everything that you can, in order to set up the next step of dividing out factors. NEVER DIVIDE OUT TERMS!! In both numerators, you have trinomials to factor, while in the denominators, there are common factors to take out. The factoring, especially of these numerators, is the hard part of this problem!

$$\frac{(3x+2)(x-1)}{x^3(x-1)} \div \frac{(3x+2)(x+1)}{x^2(x-2)}$$

$$\frac{(3x+2)(x-1)}{x^3(x-1)} \bullet \frac{x^2(x-2)}{(3x+2)(x+1)}$$

Notice that the $(3x+2)$ in the first numerator and the $(3x+2)$ in the second denominator divide out, and the $(x-1)$ in the first numerator and denominator also divide out.

$$\frac{\cancel{(3x+2)}\ \cancel{(x-1)}}{x^3\ \cancel{(x-1)}} \bullet \frac{x^2(x-2)}{\cancel{(3x+2)}(x+1)}$$

$$\frac{1}{x^3} \bullet \frac{x^2(x-2)}{(x+1)}$$

Finally, you can divide out the x^2, leaving a factor of x in the denominator.

$$\frac{1}{\cancel{x^3}\,x} \bullet \frac{\cancel{x^2}(x-2)}{(x+1)}$$

The final answer is $\dfrac{x-2}{x(x+1)}$.

P. 255 # 38.

$$\frac{4x^2-9y^2}{4x^2-4xy-3y^3} \div \frac{4x^2+8xy+3y^2}{4x^2-y^2}$$

This is a division problem, so you must invert the second fraction and multiply. Meanwhile, you should try to factor everything that you can, in order to set up the next step of dividing out factors. NEVER DIVIDE OUT TERMS!! In the first numerator and second denominator, you have the difference of squares to factor. In the first denominator and the second numerator, you have trinomials. Factoring the difference of squares is pretty easy, but the factoring of these trinomials will probably be the most challenging part of this problem!

$$\frac{(2x-3y)(2x+3y)}{(2x-3y)(2x+y)} \div \frac{(2x+3y)(2x+y)}{(2x-y)(2x+y)}$$

Now, invert and multiply.

$$\frac{(2x-3y)(2x+3y)}{(2x-3y)(2x+y)} \cdot \frac{(2x-y)(2x+y)}{(2x+3y)(2x+y)}$$

Notice that the $(2x-3y)$ factors in the first fraction divide out, and the $(2x+y)$ in the second fraction divide out.

$$\frac{\cancel{(2x-3y)}(2x+3y)}{\cancel{(2x-3y)}(2x+y)} \cdot \frac{(2x-y)\cancel{(2x+y)}}{(2x+3y)\cancel{(2x+y)}}$$

$$\frac{(2x+3y)}{(2x+y)} \cdot \frac{(2x-y)}{(2x+3y)}$$

Finally, you can divide out the $(2x+3y)$ factors.

$$\frac{\cancel{(2x+3y)}}{(2x+y)} \cdot \frac{(2x-y)}{\cancel{(2x+3y)}}$$

The final answer is $\dfrac{2x-y}{2x+y}$.

P. 255 # 42. $\dfrac{25-x^2}{x^2-3x-10}$

This is just a fraction that needs to be reduced. Of course, the first step must be to factor the numerator and denominator Whatever you do, NEVER DIVIDE OUT TERMS!! In the numerator, you have the difference of squares to factor, and in the denominator, you have a trinomial to factor.

$$\frac{(5-x)(5+x)}{(x-5)(x+2)}$$

Notice that the $(5-x)$ in the numerator and the $(x-5)$ factor in the denominator are similar, but NOT exactly the same. They are in fact the negatives, one of the other. If you divide a number by its negative, the result is -1 **(see page 246 #45-52)**. Therefore, you can divide these factors out, and the result is -1

$$\frac{-1\cancel{(5-x)}(5+x)}{\cancel{(x-5)}(x+2)}$$

The final answer may be written as $\dfrac{-(x+5)}{(x+2)}$ or $-\dfrac{x+5}{x+2}$ or $\dfrac{-x-5}{x+2}$.

P. 256 # 47.

$$\frac{25-x^2}{4x^3y} \div \frac{x^2-10x+25}{12xy^3}$$

This is a division problem, so you must invert the second fraction and multiply. Factor whenever possible in order to reduce the fractions in the next step.

$$\frac{(5-x)(5+x)}{4x^3y} \bullet \frac{12xy^3}{(x-5)(x-5)}$$

Notice that the (5-x) in the first numerator is the negative of the (x-5) in the second denominator. A number divided by its negative is -1, so these factors divide out, leaving a -1 in the first numerator.

$$\frac{-1(5+x)}{4x^3y} \bullet \frac{12xy^3}{(x-5)}$$

Also, the 4 divides into the 12 leaving a factor of 3 in the numerator, and the x and the *y* divide out, leaving x^2 in the denominator and y^2 in the numerator

$$\frac{-1(5+x)}{x^2} \bullet \frac{3y^2}{(x-5)}$$

$$\frac{-3y^2(x+5)}{x^2(x-5)}$$

P. 289 # 8. $\dfrac{x^3y^2}{6xy+12x} \div \dfrac{y^3}{y^2-4}$

Solution: (See P. 254 # 34.)

Extra Problem #1 (from Arlete)

$$\left(\frac{x-3}{2}\right)^3 \bullet \left(\frac{1}{x-3}\right)^2$$

This is a multiplication problem with powers, so you should begin by raising each numerator and denominator to the respective power:

$$\frac{(x-3)^3}{2^3} \bullet \frac{1^2}{(x-3)^2}$$

This actually means:

$$\frac{(x-3)\,\cancel{(x-3)}\,\cancel{(x-3)}}{8} \bullet \frac{1}{\cancel{(x-3)}\,\cancel{(x-3)}}$$

The final answer is $\dfrac{x-3}{8}$.

Notice that the $(3x+2)$ in the first numerator and the $(3x+2)$ in the second denominator divide out, and the $(x-1)$ in the first numerator and denominator also divide out.

$$\frac{\cancel{(3x+2)}\,\cancel{(x-1)}}{x^3\,\cancel{(x-1)}} \bullet \frac{x^2(x-2)}{\cancel{(3x+2)}(x+1)}$$

$$\frac{1}{x^3} \bullet \frac{x^2(x-2)}{(x+1)}$$

Finally, you can divide out the x^2, leaving a factor of x in the denominator.

$$\frac{1}{\cancel{x^3}\,x} \bullet \frac{\cancel{x^2}\,(x-2)}{(x+1)}$$

The final answer is $\dfrac{x-2}{x(x+1)}$.

Extra Problem #2 (from Arlete)

$$\frac{x^2+5x+4}{(x+3)^2} \div \frac{x+1}{x^2-9}$$

This is a division problem, so you must invert the second fraction and multiply. Meanwhile, you should try to factor everything that you can, in order to set up the next step of dividing out factors. NEVER DIVIDE OUT TERMS!! In the first numerator, you have a trinomial to factor, while in the second denominator, there is a difference of squares to factor.

$$\frac{x^2+5x+4}{(x+3)^2} \div \frac{x+1}{x^2-9}$$

$$\frac{(x+4)(x+1)}{(x+3)(x+3)} \bullet \frac{(x-3)(x+3)}{x+1}$$

Notice that the $(x+1)$ in the first numerator and the $x+1$ in the second denominator divide out, and the $(x+3)$ in the first denominator and $(x+3)$ in the second numerator also divide out.

$$\frac{(x+4)\cancel{(x+1)}}{(x+3)\cancel{(x+3)}} \bullet \frac{(x-3)\cancel{(x+3)}}{\cancel{x+1}}$$

The final answer is $\dfrac{(x+4)(x-3)}{x+3}$.

Extra Problem # 17.

$$\frac{a^2+25}{a^2-4a+3} \bullet \frac{a-5}{a+5}$$

This is a multiplication problem, so the first step is to factor the numerators and denominators. However, the problem here is to realize that you CANNOT factor a^2+25, the sum of two squares!

$$\frac{a^2+25}{(a-3)(a-1)} \bullet \frac{a-5}{a+5}$$

Strangely enough, nothing divides out! This is NOT a typical problem! The answer is UGLY, and it does NOT simplify. Remember, you NEVER DIVIDE OUT TERMS!!

$$\frac{(a^2+25)(a-5)}{(a-3)(a-1)(a+5)}$$

Extra Problem # 25.

$$\frac{5a^2-180}{10a^2-10} \bullet \frac{20a+20}{2a-12}$$

As in nearly every multiplication problem, the first and most important step is to factor, factor, factor, factor!! However, in this problem, the critical step is to FACTOR COMPLETELY!! If you do NOT factor completely, you just won't get it!! Also, remember that the FIRST step in factoring is to FCFF: FACTOR THE COMMON FACTOR FIRST ! It will take TWO STEPS to factor completely in the first fraction!

$$\frac{5(a^2-36)}{10(a^2-1)} \bullet \frac{20(a+1)}{2(a-6)}$$

$$\frac{5(a-6)(a+6)}{10(a-1)(a+1)} \bullet \frac{20(a+1)}{2(a-6)}$$

Notice that two sets of factors divide out:

$$\frac{5\,\cancel{(a-6)}\,(a+6)}{10(a-1)\,\cancel{(a+1)}} \bullet \frac{20\,\cancel{(a+1)}}{2\,\cancel{(a-6)}}$$

Also, notice (either now or later!) that the 20 divides out with the 10 and 2.

$$\frac{5\,\cancel{(a-6)}\,(a+6)}{\cancel{10}\,(a-1)\,\cancel{(a+1)}} \bullet \frac{\cancel{20}\,\cancel{(a+1)}}{\cancel{2}\,\cancel{(a-6)}}$$

This leaves the final answer:

$$\frac{5(a+6)}{a-1}$$

Extra Problem #29.

29. $$\frac{5t^2+12t+4}{t^2+4t+4} \bullet \frac{t^2+8t+16}{5t^2+22t+8}$$

This is a multiplication problem, so you should try to factor everything that you can, in order to set up the next step of dividing out factors. NEVER DIVIDE OUT TERMS!! The hard part of this is probably going to be factoring the first numerator and the second denominator! If you have trouble factoring these two, then see my section on "Advanced Trinomial Factoring." Factoring the other trinomials should be pretty easy!

$$\frac{(5t+2)(t+2)}{(t+2)(t+2)} \bullet \frac{(t+4)(t+4)}{(5t+2)(t+4)}$$

Notice that the $(t+2)$ in the first fraction, and the $(t+4)$ in the second fraction divide out:

$$\frac{(5t+2)\cancel{(t+2)}}{(t+2)\cancel{(t+2)}} \bullet \frac{(t+4)\cancel{(t+4)}}{(5t+2)\cancel{(t+4)}}$$

Also, the $(5t+2)$ factors in the first numerator, and the $(5t+2)$ in the second denominator divide out.

$$\frac{\cancel{(5t+2)}\cancel{(t+2)}}{(t+2)\cancel{(t+2)}} \bullet \frac{(t+4)\cancel{(t+4)}}{\cancel{(5t+2)}\cancel{(t+4)}}$$

The final answer is $\dfrac{t+4}{t+2}$.

Extra Problem # 31.

$$\frac{2x^2-5x+3}{6x^2-5x-1} \bullet \frac{6x^2+13x+2}{2x^2+3x-9}$$

This is one TOUGH factoring problem!! You need to factor both numerators and both denominators, all of which are what I call "Advanced Trinomial Factoring." If you had trouble with this problem, perhaps it is more a problem with factoring than it is with simplifying fractions. See my section on "Advanced Trinomial Factoring." If you can factor all of this, then surely you can reduce the fraction!!!

$$\frac{(2x-3)(x-1)}{(6x+1)(x-1)} \bullet \frac{(6x+1)(x+2)}{(2x-3)(x+3)}$$

Notice all the factors that divide out:

$$\frac{\cancel{(2x-3)}\,\cancel{(x-1)}}{\cancel{(6x+1)}\,\cancel{(x-1)}} \bullet \frac{\cancel{(6x+1)}\,(x+2)}{\cancel{(2x-3)}\,(x+3)}$$

The final answer is $\frac{x+2}{x+3}$.

Extra Problem # 49.

$$\frac{a}{a-b} \div \frac{b}{b-a}$$

This is a division problem, so you must invert the second fraction and multiply.

$$\frac{a}{a-b} \bullet \frac{b-a}{b}$$

Notice that the $a-b$ in the first denominator is the negative of the $b-a$ in the second numerator. A number divided by its negative is -1, so these factors divide out, leaving a "-1" factor in the second numerator.

$$\frac{a}{\cancel{a-b}} \bullet \frac{-1\cancel{b-a}}{b}$$

$\frac{-1 \bullet a}{b}$ or $-\frac{a}{b}$.

Extra Problem # 65.

$$\frac{x-y}{x^2+2xy+y^2} \div \frac{x^2-y^2}{x^2-5xy+4y^2}$$

Since this is a DIVISION problem, don't forget, you must INVERT the second fraction and multiply!! As in nearly all multiplication or division problems, you need to factor the numerators and denominators. Just factor, factor, factor!!

$$\frac{x-y}{x^2+2xy+y^2} \bullet \frac{x^2-5xy+4y^2}{x^2-y^2}$$

$$\frac{x-y}{(x+y)(x+y)} \bullet \frac{(x-4y)(x-y)}{(x-y)(x+y)}$$

Notice that only one set of factors divides out:

$$\frac{x-y}{(x+y)(x+y)} \bullet \frac{(x-4y)\cancel{(x-y)}}{\cancel{(x-y)}(x+y)}$$

The final answer is $\frac{(x-y)(x-4y)}{(x+y)^3}$.