3.01 Reducing Fractions

Basic Algebra: One Step at a Time. **Pages 241 - 246: #17, (3 Extra Problems)**

Extra Problems: #15, 29, 33, 43, 45

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p. 244. # 17. $\frac{98x^2y^8}{14x^6y^6}$

Notice that this is actually three problems in one: $\frac{7\cancel{98}}{\cancel{14}} \cdot \frac{x^2}{x^6} \cdot \frac{y^8}{y^6}$

Now, reduce each fraction. In the first fraction, you can divide out the 14, leaving a 7 in the numerator.

In the second fraction, you can divide out the x^2, leaving x^4 in the denominator, since the highest power is in the denominator (it turns out that you must subtract exponents!).

In the third fraction, you can divide out the y^6, leaving y^2 in the numerator, since the highest power is in the numerator (again, you can subtract the exponents!).

$$\frac{7\cancel{98}}{\cancel{14}} \cdot \frac{\cancel{x^2}}{\cancel{x^6}\,x^4} \cdot \frac{y^2\,\cancel{y^8}}{\cancel{y^6}}$$

$$\frac{98}{14} \cdot \frac{x^2}{x^6} \cdot \frac{y^8}{y^6}$$

$$\frac{7}{1} \cdot \frac{1}{x^4} \cdot \frac{y^2}{1}$$

$$\frac{7y^2}{x^4}$$

Extra Problem #1 (from Arlete): $\frac{a^2-2ab+b^2}{a-b}$

The first and most important step is to FACTOR!! You NEVER divide out TERMS—only FACTORS: $\frac{(a-b)(a-b)}{a-b}$

Divide out the (a-b) factors: $\frac{\cancel{(a-b)}(a-b)}{\cancel{a-b}}$.

Final Answer: $\frac{(a-b)}{1}$ or $a-b$

Extra Problem #2 (from Arlete): $\frac{a^2+2ab+b^2}{a+b}$

The first and most important step is to FACTOR!! You NEVER divide out TERMS—only FACTORS: $\frac{(a+b)(a+b)}{a+b}$

Divide out the (a-b) factors: $\frac{\cancel{(a+b)}(a+b)}{\cancel{a+b}}$.

Final Answer: $\frac{(a+b)}{1}$ or $a+b$

Extra Problem #3 (from Arlete): $\frac{2x^2+x-6}{2x+4}$

Again, the first and most important step is to FACTOR!! You NEVER divide out TERMS—only FACTORS: $\frac{(2x-3)(x+2)}{2(x+2)}$

Divide out the (x+2) factors: $\frac{(2x-3)\cancel{(x+2)}}{2\cancel{(x+2)}}$

Final Answer: $\frac{(2x-3)}{2}$ or $\frac{2x-3}{2}$

Caution: Do NOT divide out the 2, since the 2x in the numerator is a TERM! NEVER DIVIDE OUT TERMS!!

Extra Problem:

#15: List all numbers such that the rational expression is undefined!

(**TRANSLATION into English:** Find all number for which the FRACTION is "Undefined"!)

SOLUTION: Now think! What is the ONE thing that is NOT allowed in fractions; the one thing that you have NEVER been allowed to do, and NO ONE will NEVER, EVER be allowed to do? [Answer: You are NEVER allowed to DIVIDE BY ZERO!! So you need to find ALL numbers that make the denominator ZERO!]

$$\frac{t^2 - 4}{2t^2 + 11t - 6}$$

Take the denominator and set it equal to ZERO! The NUMERATOR is completely irrelevant to the problem!

$$2t^2 + 11t - 6 = 0$$

This is a quadratic equation. Since the equation is already equal to zero, the next step is to FACTOR!! If you have trouble with this factoring, then see my explanation on the "Advanced Trinomial Factoring" page.

Since the First times First must be $2t^2$, it has to be in this form:

$$2t^2 + 11t - 6 = 0$$

$$(2t \_\_\_\_)(t \_\_\_\_) = 0$$

Since the Last times Last must be -6, you will need opposite signs, and a combination of $1 \text{ times } 6$ or $2 \text{ times } 3$. After much trial and error, the correct combination of numbers is $1 \text{ times } 6$.

$$(2t \quad 1)(t \quad 6) = 0$$

The Outer times Outer is $12t$, and the Inner times Inner is t. In order to get a total of $+11t$, you need $+12t$ and $-1t$, and it looks like this:

$$(2t - 1)(t + 6) = 0$$

Now, set each factor equal to zero and solve:

$$(2t - 1)(t + 6) = 0$$

$$2t - 1 = 0 \quad \text{or} \quad t + 6 = 0$$

$$2t = 1 \quad \text{or} \quad t = -6$$

$$t = \frac{1}{2}$$

Final Answer: The two numbers that cause this fraction to be undefined are $\frac{1}{2}$ and -6.

Extra Problem. #29: $\dfrac{6a^2 - 3a}{7a^2 - 7a}$

The first and most important step is to FACTOR!! You NEVER divide out TERMS—only FACTORS. Factor the common factor of 3a from the numerator and 7a from the denominator:

$$\frac{3a(2a-1)}{7a(a-1)}$$

Divide out the a factors: $\dfrac{3\cancel{a}(2a-1)}{7\cancel{a}(a-1)}$

Final Answer: $\dfrac{3(2a-1)}{7(a-1)}$

Extra Problem: #33: $$\frac{3a^2+9a-12}{6a^2-30a+24}$$

The first and most important step is to FACTOR!! You NEVER divide out TERMS—only FACTORS. Factor the common factor of 3 from the numerator and 6 from the denominator:

$$\frac{3(a^2+3a-4)}{6(a^2-5a+4)}$$

The numerator and denominator both have trinomials that factor!

$$\frac{3(a+4)(a-1)}{6(a-4)(a-1)}$$

Divide out the 3 and the (a-1) factors:

$$\frac{\cancel{3}(a+4)\cancel{(a-1)}}{\cancel{6}\,2(a-4)\cancel{(a-1)}}$$

Final Answer: $$\frac{a+4}{2(a-4)}$$

Extra Problem: #43: $$\frac{y^2+6y}{2y^2+13y+6}$$

Again, the first step is to FACTOR!! You NEVER divide out TERMS—only FACTORS. Factor the common factor of y from the numerator, and prepare to factor the trinomial denominator as you did in problem #15 on this same page of exercises. If you have trouble with this factoring, then see my explanation on the "Advanced Trinomial Factoring" page.

$$\frac{y^2+6y}{2y^2+13y+6}$$

$$\frac{y(y+6)}{(2y\_\_\_\_)(y\_\_\_\_)}$$

As in problem #15, the Last times Last must be 6, and the Outer times Outer and Inner times Inner must add up to $13y$. After trial and error, the combination given below using 1 times 6 works!

$$\frac{y(y+6)}{(2y\_\_1)(y\_\_6)}$$

The signs in the denominator are both positive!

$$\frac{y(y+6)}{(2y+1)(y+6)}$$

Divide out the (y+6) factors:

$$\frac{y\,\cancel{(y+6)}}{(2y+1)\,\cancel{(y+6)}}$$

Final Answer: $$\frac{y}{2y+1}$$

Extra Problem: #45: $$\frac{4x^2-12x+9}{10x^2-11x-6}$$

As always, the first step is to FACTOR!! You NEVER divide out TERMS—only FACTORS. The numerator is a perfect square trinomial, and you will have to factor the denominator in the same way that you did in problems #15 and #43 on this same page of exercises. If you have trouble with this factoring, then see my explanation on the "Advanced Trinomial Factoring" page.

$$\frac{4x^2-12x+9}{10x^2-11x-6}$$

$$\frac{(2x\_\_\_\_)(2x\_\_\_\_)}{(5x\_\_\_\_)(2x\_\_\_\_)}$$

As in problem #15, the Last times Last must be 6, and the Outer times Outer and Inner times Inner must add up to $-11x$. After trial and error, the combination given below using 2 times 3 works!

$$\frac{(2x-3)(2x-3)}{(5x\_\_2)(2x\_\_3)}$$

In the denominator, the signs are opposite. The Outer times Outer is $15x$, and the Inner times Inner is $4x$. In order to obtain a middle term of $-11x$, you need a $-15x$ and a $+4x$. It should look like this:

$$\frac{(2x-3)(2x-3)}{(5x+2)(2x-3)}$$

Divide out the (2x-3) factors:

$$\frac{(2x-3)\cancel{(2x-3)}}{(5x+2)\cancel{(2x-3)}}$$

Final Answer: $$\frac{2x-3}{5x+2}$$