

3.04 Adding and Subtracting Fractions

Basic Algebra: One Step at a Time.

Page 265 - 280: #19, 20, 22, 24, 33, 34, 36, 43, 51, 54, 55, 57, 58, 59, 66, 70, 74, 4 Extra Problems
Extra Problem: #25

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Part I: Fractions with a Common Denominator

P. 267: 19.
$$\frac{x^2 - 12x}{x - 6} + \frac{x^2 - 5x + 30}{x - 6}$$

Solution: Notice that you already have a common denominator. This means that the LCD (which is $x - 6$) becomes the denominator of the fraction, and you ADD the numerators together:

$$\frac{x^2 - 12x + x^2 - 5x + 30}{x - 6}$$

Now, combine like terms in the numerator, remembering that $x^2 + x^2 = 2x^2$

$$\frac{2x^2 - 17x + 30}{x - 6}$$

The most difficult part of this problem is probably factoring the numerator. It might make it easier if you realize that the denominator has a factor of $x - 6$, which means that the only reason you would need to factor the numerator would be if it too had a factor of $x - 6$. If it does, it will have to start off like this, with both signs negative:

$$\frac{(2x - \underline{\quad})(x - \underline{\quad})}{x - 6}$$

Next, notice that to get the last times last to be 30, with one of the numbers -6, the other number must be -5. It must be this way:

$$\frac{(2x - 5)(x - 6)}{x - 6}$$

Notice that the middle term in this trinomial was $-12x - 5x$, which is $-17x$ as it should be. Now, divide out the $x - 6$ factors, and the

Final answer is $2x - 5$

p. 267. # 20. $\frac{y^2+4y}{3y+5} + \frac{2y^2+4y+5}{3y+5}$

Solution: First notice that this is the addition of two fractions. The first priority must be to have a common denominator, which in this case is $3y+5$! The LCD becomes THE denominator of the entire problem, and it looks like this:

$$\frac{\quad}{3y+5}$$

Next, ADD the numerators.

$$\frac{(y^2+4y)+(2y^2+4y+5)}{3y+5}$$

Combine like terms: $\frac{3y^2+8y+5}{3y+5}$

Factor the numerator: $\frac{(3y+5)(y+1)}{3y+5}$

Reduce the fraction: $\frac{\cancel{(3y+5)}(y+1)}{\cancel{3y+5}}$

Final answer: $\frac{(y+1)}{1}$ or $y+1$.

p. 268. # 22.

$$\frac{5x^2}{x-2} - \frac{4x^2+3x-2}{x-2}$$

Solution: First notice that this is a “Fraction Subtraction”!! The first priority must be to have a common denominator, which in this case is $x-2$! The LCD becomes THE denominator of the entire problem, and it looks like this:

$$\frac{\quad}{x-2}$$

Next, subtract the numerators. Be careful to distribute the negative through the second numerator!

$$\frac{5x^2 - (4x^2 + 3x - 2)}{x-2}$$

$$\frac{5x^2 - 4x^2 - 3x + 2}{x-2}$$

Combine like terms:

$$\frac{x^2 - 3x + 2}{x-2}$$

Factor:

$$\frac{(x-2)(x-1)}{x-2}$$

Reduce the fraction:

$$\frac{\cancel{(x-2)}(x-1)}{\cancel{x-2}}$$

Final answer:

$$\frac{(x-1)}{1} \text{ or } x-1.$$

p. 268. # 24.

$$\frac{2x^3 - 6x^2 - 3}{x^3 + x^2 - 6x} - \frac{x^3 - 8x - 3}{x^3 + x^2 - 6x}$$

Solution: First notice that this is a “Fraction Subtraction”, very similar to the other exercises on this page, but this one is probably “uglier”!! The first priority must be to have a common denominator, which in this case is $x^3 + x^2 - 6x$! There is NO need to factor the numerators at all. The LCD is the priority here. As before, the LCD becomes THE denominator of the entire problem, and it looks like this:

$$\frac{-}{x^3 + x^2 - 6x}$$

Next, subtract the numerators. Be careful to distribute the negative through the second numerator!

$$\frac{(2x^3 - 6x^2 - 3) - (x^3 - 8x - 3)}{x^3 + x^2 - 6x}$$

$$\frac{2x^3 - 6x^2 - 3 - x^3 + 8x + 3}{x^3 + x^2 - 6x}$$

Combine like terms:

$$\frac{x^3 - 6x^2 + 8x}{x^3 + x^2 - 6x}$$

Factor the common factor of x from the numerator and the denominator:

$$\frac{x(x^2 - 6x + 8)}{x(x^2 + x - 6)}$$

The x divides out:

$$\frac{(x^2 - 6x + 8)}{(x^2 + x - 6)}$$

Factor the trinomials in the numerator and denominator.

$$\frac{(x - 4)(x - 2)}{(x + 3)(x - 2)}$$

Reduce the fraction:

$$\frac{(x - 4)\cancel{(x - 2)}}{(x + 3)\cancel{(x - 2)}}$$

Final answer:

$$\frac{x - 4}{x + 3}$$

Part II: Fractions without a Common Denominator

ADDITION AND SUBTRACTION OF FRACTIONS Summary

I. FIND THE LEAST COMMON DENOMINATOR (LCD).

- A. Factor each denominator to determine what factors are needed for the common denominator.
- B. For each of the denominator factors, determine the highest power of each factor. The **LCD** is the product of each factor raised to its highest power.
- C. The LCD becomes the denominator of the fraction.

II. PLAY "WHAT'S MISSING?"

- A. Compare each denominator to the **LCD**, and determine the missing factors for each denominator.
- B. Multiply each numerator and denominator by "What's missing!"

III. ADD OR SUBTRACT NUMERATORS.

- A. Add (or subtract) numerators, and place over the common denominator.
- B. Combine like terms and reduce the resulting fraction, if possible.

p. 273. # 33.

$$\frac{7}{8x} + \frac{3}{5x}$$

Solution:

Step I: (Find the LCD): The LCD is $40x$

Step II: (What's Missing?): 1st denominator has $8x$, missing 5 .

2nd denominator has $5x$, missing 8 .

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{7}{8x} + \frac{3}{5x}$$
$$\frac{7}{8x} \cdot \frac{5}{5} + \frac{3}{5x} \cdot \frac{8}{8}$$

Step III: (Add or Subtract): Add the numerators and place over the LCD.

$$\begin{array}{r} + \\ \hline 40x \\ \\ 35 + 24 \\ \hline 40x \end{array}$$

Final answer:

$$\frac{59}{40x}$$

p. 273. # 34.

$$\frac{7}{8x} - \frac{5}{6y}$$

Solution:

Step I: (Find the LCD): The LCD is $24xy$

Step II: (What's Missing?): 1st denominator has $8x$, missing $3y$.

2nd denominator has $6y$, missing $4x$.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{7}{8x} - \frac{5}{6y}$$
$$\frac{7 \cdot 3y}{8x \cdot 3y} - \frac{5 \cdot 4x}{6y \cdot 4x}$$

Step III: (Add or Subtract): Subtract the numerators and place over the LCD.

$$\frac{\quad}{24xy}$$

Final answer:

$$\frac{21y - 20x}{24xy}$$

This is the final answer. It does NOT simplify! Be careful! Do NOT combine unlike terms, and do NOT "cancel" terms!!

p. 273. # 36. $\frac{3}{8xy^2} + \frac{5}{6x^2}$

Solution:

Step I: (Find the LCD): The LCD is $24x^2y^2$

Step II: (What's Missing?): 1st denominator has $8xy^2$, missing $3x$.

2nd denominator has $6x^2$, missing $4y^2$.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{3}{8xy^2} + \frac{5}{6x^2}$$
$$\frac{3}{8xy^2} \cdot \frac{3x}{3x} + \frac{5}{6x^2} \cdot \frac{4y^2}{4y^2}$$

Step III: (Add or Subtract): Add the numerators and place over the LCD.

$$\frac{\quad}{24x^2y^2}$$

Final answer: $\frac{9x + 20y^2}{24x^2y^2}$

Since the numerator does not have like terms, and it does not factor, this is the final answer!

P. 275 # 43. $\frac{4x}{x^2 - 5x + 6} + \frac{3}{x^2 - 3x + 2}$

Solution:

Step I: (Find the LCD): Factor the denominators

$$\frac{4x}{(x-3)(x-2)} + \frac{3}{(x-2)(x-1)} \quad \text{The LCD} = (x-3)(x-1)(x-2)$$

Step II: (What's Missing?):

1st denominator has $(x-3)(x-2)$, missing $(x-1)$.

2nd denominator has $(x-2)(x-1)$, missing $(x-3)$.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{4x}{(x-3)(x-2)} \cdot \frac{(x-1)}{(x-1)} + \frac{3}{(x-2)(x-1)} \cdot \frac{(x-3)}{(x-3)}$$

Step III: (Add or Subtract): Add the numerators and place over the LCD.

$$\frac{4x^2 - 4x + 3x - 9}{(x-3)(x-2)(x-1)}$$

Final answer: $\frac{4x^2 - x - 9}{(x-3)(x-2)(x-1)}$

No Extra Charge! Here is another very similar problem.

$$\frac{4x}{x^2 - 6x + 5} + \frac{3}{x^2 - 3x + 2}$$

Solution:

Step I: (Find the LCD): Factor the denominators

$$\frac{4x}{(x-5)(x-1)} + \frac{3}{(x-2)(x-1)} \quad \text{The LCD} = (x-5)(x-1)(x-2)$$

Step II: (What's Missing?):

1st denominator has $(x-5)(x-1)$, missing $(x-2)$.

2nd denominator has $(x-2)(x-1)$, missing $(x-5)$.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{4x}{(x-5)(x-1)} \cdot \frac{(x-2)}{(x-2)} + \frac{3}{(x-2)(x-1)} \cdot \frac{(x-5)}{(x-5)}$$

Step III: (Add or Subtract): Add the numerators and place over the LCD.

$$\frac{4x^2 - 8x + 3x - 15}{(x-5)(x-1)(x-2)}$$

Final answer:
$$\frac{4x^2 - 5x - 15}{(x-5)(x-1)(x-2)}$$

P. 277 # 51.

$$\frac{5}{x^2 - 10x + 25} - \frac{3}{x^2 - 5x}$$

Solution:

Step I: (Find the LCD): $\frac{5}{(x-5)(x-5)} - \frac{3}{x(x-5)}$

$$\frac{5}{(x-5)^2} - \frac{3}{x(x-5)} \quad \text{The LCD} = x(x-5)^2$$

Step II: (What's Missing?): 1st denominator has $(x-5)^2$, missing x .

2nd denominator has $x(x-5)$, missing $(x-5)$.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{5}{(x-5)^2} \cdot \frac{x}{x} - \frac{3}{x(x-5)} \cdot \frac{(x-5)}{(x-5)}$$

Step III: (Add or Subtract): Subtract the numerators and place over the LCD.

$$\frac{5 \cdot (x) - 3 \cdot (x-5)}{x(x-5)^2}$$

$$\frac{5x - 3x + 15}{x(x-5)^2}$$

$$\frac{2x + 15}{x(x-5)^2}$$

P. 277 Alternate Problem Similar to #51.

Solution:

Step I: (Find the LCD): Factor the first denominators!

$$\frac{5}{x^2 - 10x + 25} + \frac{3}{x^2 - 5x}$$

$$\frac{5}{(x-5)(x-5)} + \frac{3}{x(x-5)}$$

$$\frac{5}{(x-5)^2} + \frac{3}{x(x-5)} \quad \text{The LCD} = x(x-5)^2$$

Step II: (What's Missing?): 1st denominator has $(x-5)^2$, missing x .

3rd denominator has $x(x-5)$, missing $(x-5)$.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{5}{(x-5)^2} \cdot \frac{x}{x} + \frac{3}{x(x-5)} \cdot \frac{(x-5)}{(x-5)}$$

Step III: (Add or Subtract): Subtract the numerators and place over the LCD.

$$\frac{5 \cdot (x) + 3 \cdot (x-5)}{x(x-5)^2}$$

$$\frac{5x + 3x - 15}{x(x-5)^2}$$

$$\frac{8x - 15}{x(x-5)^2}$$

P. 277 # 54.

$$\frac{3x^2-4}{2x^2-4x} - \frac{x}{x-2} - \frac{3}{2x}$$

Solution:

Step I: (Find the LCD): Factor the first denominator!

$$\frac{3x^2-4}{2x(x-2)} - \frac{x}{x-2} - \frac{3}{2x} \quad \text{The LCD} = 2x(x-2)$$

Step II: (What's Missing?): 1st denominator has $2x(x-2)$, nothing missing.

2nd denominator has $x-2$, missing $2x$.

3rd denominator has $2x$, missing $(x-2)$.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{3x^2-4}{2x(x-2)} - \frac{x \cdot 2x}{x-2 \cdot 2x} - \frac{3 \cdot (x-2)}{2x \cdot (x-2)}$$

Step III: (Add or Subtract): Subtract the numerators and place over the LCD.

$$\frac{(3x^2-4) - x \cdot 2x - 3 \cdot (x-2)}{2x(x-2)}$$
$$\frac{3x^2-4 - 2x^2 - 3x+6}{2x(x-2)}$$
$$\frac{x^2-3x+2}{2x(x-2)}$$

Factor the numerator:

$$\frac{(x-2)(x-1)}{2x(x-2)}$$

Reduce the fraction:

$$\frac{\cancel{(x-2)}(x-1)}{2x \cancel{(x-2)}}$$

$$\frac{x-1}{2x}$$

P. 277: 55.

$$\frac{2a^2 - 7ab - 12b^2}{2a(3a - 4b)} + \frac{2a + 4b}{3a - 4b}$$

Since this is an addition problem, the first step is to find the LCD, which is $2a(3a - 4b)$. The next step is to play “What’s Missing?” by observing that the first denominator isn’t missing any factors, but the second fraction is missing a $2a$. Therefore, you must multiply numerator and denominator of the second fraction by $2a$.

$$\frac{2a^2 - 7ab - 12b^2}{2a(3a - 4b)} + \frac{2a + 4b}{3a - 4b}$$

$$\frac{2a^2 - 7ab - 12b^2}{2a(3a - 4b)} + \frac{2a \cdot (2a + 4b)}{2a \cdot (3a - 4b)}$$

$$\frac{2a^2 - 7ab - 12b^2 + 2a \cdot (2a + 4b)}{2a(3a - 4b)}$$

$$\frac{2a^2 - 7ab - 12b^2 + 4a^2 + 8ab}{2a(3a - 4b)}$$

$$\frac{6a^2 + ab - 12b^2}{2a(3a - 4b)}$$

Now comes the hard part! This numerator might factor, then again, it may not! The only reason it would be necessary to factor this would be if there were a factor of $(3a - 4b)$. This helps narrow down some of the choices for factoring the numerator, into

$$\frac{(3a - 4b)(2a + 3b)}{2a(3a - 4b)}$$

Check it out! It really works. Now divide out the factors of $(3a - 4b)$, and you have:

$$\frac{2a + 3b}{2a}, \text{ which is the final answer!}$$

By the way, DO NOT divide out terms, like $2a$!

P. 278: 57. $\frac{1}{a^2-7a+12} + \frac{2}{a^2-4a+3} - \frac{3}{a^2-5a+4}$

This problem came from a book called *New School Algebra* by G.A. Wentworth, published in 1898 by Ginn and Company. Since this is an addition/subtraction problem, the first step is to factor each denominator in order to find the LCD.

$$\frac{1}{(a-3)(a-4)} + \frac{2}{(a-3)(a-1)} - \frac{3}{(a-4)(a-1)}$$

The LCD consists of three binomial factors $(a-3)(a-4)(a-1)$. The next step is to play "What's Missing?" by observing which factors are missing from each of the three fractions. Notice that the first denominator is missing the $(a-1)$ factor, the second fraction is missing the $(a-4)$ factor, and the third fraction is missing the $(a-3)$. So, you must multiply numerators and denominators of each fraction by the respective missing factor:

$$\begin{aligned} & \frac{1}{(a-3)(a-4)} + \frac{2}{(a-3)(a-1)} - \frac{3}{(a-4)(a-1)} \\ & \frac{1}{(a-3)(a-4)} \cdot \frac{(a-1)}{(a-1)} + \frac{2}{(a-3)(a-1)} \cdot \frac{(a-4)}{(a-4)} - \frac{3}{(a-4)(a-1)} \cdot \frac{(a-3)}{(a-3)} \\ & \frac{a-1}{(a-3)(a-4)(a-1)} + \frac{2a-8}{(a-3)(a-4)(a-1)} - \frac{3a+9}{(a-3)(a-4)(a-1)} \end{aligned}$$

When you combine like terms in the numerator, everything subtracts out, leaving

$$\frac{0}{(a-3)(a-4)(a-1)} \text{ which is just } 0. \quad \text{Final Answer!}$$

P. 278: 58.
$$\frac{1}{x-2} + \frac{1}{x^2-3x+2} - \frac{2}{x^2-4x+3}$$

This problem came from a book called *New School Algebra* by G.A. Wentworth, published in 1898 by Ginn and Company.

Solution: The first step is to factor each denominator in order to find the LCD for the problem.

$$\frac{1}{x-2} + \frac{1}{(x-2)(x-1)} - \frac{2}{(x-3)(x-1)}$$

The LCD is $(x-2)(x-1)(x-3)$, which becomes the denominator of the fraction. The first fraction is missing the factors $(x-1)(x-3)$, the second fraction is missing the $(x-3)$, and the third fraction is missing the $(x-2)$. You must multiply numerator and denominator of each fraction by the "missing factors." It will look like this:

$$\frac{1}{x-2} \cdot \frac{(x-1)(x-3)}{(x-1)(x-3)} + \frac{1}{(x-2)(x-1)} \cdot \frac{(x-3)}{(x-3)} - \frac{2}{(x-3)(x-1)} \cdot \frac{(x-2)}{(x-2)}$$

Put down the LCD:

$$(x-1)(x-2)(x-3)$$

and you ADD or SUBTRACT the numerators:

$$\frac{1 \cdot (x-1)(x-3) + 1 \cdot (x-3) - 2 \cdot (x-2)}{(x-1)(x-2)(x-3)}$$

Now, multiply out the parentheses in the numerator, and combine like terms

$$\frac{x^2 - 4x + 3 + x - 3 - 2x + 4}{(x-1)(x-2)(x-3)}$$

$$\frac{x^2 - 5x + 4}{(x-1)(x-2)(x-3)}$$

The numerator factors, so this fraction MIGHT reduce. You will have to factor it and try to reduce the fraction.

$$\frac{(x-4)(x-1)}{(x-1)(x-2)(x-3)}$$

Divide out the factor of $(x-1)$

$$\frac{(x-4)\cancel{(x-1)}}{\cancel{(x-1)}(x-2)(x-3)}$$

$$\frac{x-4}{(x-2)(x-3)}$$

Final Answer:

P. 278: 59.

$$\frac{3}{10a^2 + a - 3} - \frac{4}{2a^2 + 7a - 4}$$

This problem came from a book called *New School Algebra* by G.A. Wentworth, published in 1898 by Ginn and Company.

Solution:

This is a “**fraction subtraction**”, so the first step is to find the **Least Common Denominator** for the two denominators. The first step of the first step is to factor the two denominators. This step may well be the most difficult part of the entire problem! If you have trouble with this factoring, please click on this link for a more detailed explanation of “[Advanced Trinomial Factoring](#)”:

$$\frac{3}{(5a + 3)(2a - 1)} - \frac{4}{(2a - 1)(a + 4)}$$

From this you can see that the LCD is $(5a + 3)(2a - 1)(a + 4)$, and the first fraction is “missing” a factor of $(a + 4)$, while the second fraction is “missing” a factor of $(5a + 3)$. Now, multiply the numerator and denominator of each fraction by the respective missing factor:

$$\frac{3 \cdot (a + 4)}{(5a + 3)(2a - 1) \cdot (a + 4)} - \frac{4 \cdot (5a + 3)}{(2a - 1)(a + 4) \cdot (5a + 3)}$$

The LCD will be THE denominator of the entire fraction, and then multiply out the numerators being careful of the signs in the second part.

$$\frac{3a + 12 - 20a - 12}{(5a + 3)(2a - 1)(a + 4)}$$

Combine like terms: $\frac{-17a}{(5a + 3)(2a - 1)(a + 4)}$ **Final Answer!**

P. 280 #66. $\frac{5}{x-5} + \frac{5}{5-x}$

Solution:

In this case, notice that the second denominator is NOT the same as the first denominator, but it IS similar. In fact, the $x-5$ and the $5-x$ are negatives of one another. It might be helpful to multiply the numerator and denominator of the second fraction by -1 .

$$\frac{5}{x-5} + \frac{-1 \cdot 5}{-1 \cdot (5-x)}$$

$$\frac{5}{x-5} + \frac{-5}{-5+x} \quad \text{or} \quad \frac{5}{x-5} + \frac{-5}{x-5}$$

Now, you have a common denominator of $x-5$, so you can add the numerators together:

$$\frac{5-5}{x-5} \text{ which is } \frac{0}{x-5} \text{ or } 0$$

p. 280 # 70. $\frac{x^2}{x-2} + \frac{4}{2-x}$

Solution:

Notice that the second denominator $2-x$ is very similar to the $x-2$ factor in the first denominator. It will be very helpful to multiply the numerator and denominator of the second fraction by -1 .

$$\frac{x^2}{x-2} + \frac{-1 \cdot 4}{-1 \cdot 2-x}$$

$$\frac{x^2}{x-2} + \frac{-4}{x-2}$$

Put down the LCD as the denominator of the problem, and add (or subtract) numerators:

$$\frac{x^2-4}{x-2}$$

Next, factor the difference of squares:

$$\frac{(x-2)(x+2)}{x-2}$$

Reduce the fraction by dividing out the $(x-2)$. **The final answer is:**

$$\frac{(x+2)}{1} \quad \text{or} \quad x+2$$

Extra Problem. $\frac{x^3}{x-2} + \frac{4x}{2-x}$

Solution:

Notice that the second denominator $2-x$ is very similar to the $x-2$ factor in the first denominator. It will be very helpful to multiply the numerator and denominator of the second fraction by -1 .

$$\frac{x^3}{x-2} + \frac{-1 \cdot 4x}{-1 \cdot 2-x}$$

$$\frac{x^3}{x-2} + \frac{-4x}{x-2}$$

Put down the LCD as the denominator of the problem, and add (or subtract) numerators:

$$\frac{x^3 - 4x}{x-2}$$

Factor the common factor of x from the numerator:

$$\frac{x(x^2 - 4)}{x-2}$$

Next, factor the difference of squares:

$$\frac{x(x-2)(x+2)}{x-2}$$

Reduce the fraction by dividing out the $(x-2)$.

The final answer is: $\frac{x(x+2)}{1}$ or $x(x+2)$.

P. 280 # 74. $\frac{x}{x^2 - 25} - \frac{5}{5 - x}$

Solution:

Find the LCD by factoring the first denominator:

$$\frac{x}{(x-5)(x+5)} - \frac{5}{5-x}$$

Notice that the second denominator $5-x$ is very similar to the $x-5$ factor in the first denominator. It will be very helpful to multiply the numerator and denominator by -1 .

$$\frac{x}{(x-5)(x+5)} - \frac{-1 \bullet 5}{-1 \bullet (5-x)}$$

$$\frac{x}{(x-5)(x+5)} - \frac{-5}{x-5}$$

$$\frac{x}{(x-5)(x+5)} + \frac{5}{x-5}$$

Now, it should be clear that the LCD = $(x-5)(x+5)$. The first fraction has the common denominator, but the second fraction needs a factor of $(x+5)$, so multiply numerator and denominator of the second fraction by $(x+5)$.

$$\frac{x}{(x-5)(x+5)} + \frac{5 \bullet (x+5)}{(x-5) \bullet (x+5)}$$

Put down the LCD as the denominator of the problem, and add numerators:

$$\frac{x + 5x + 25}{(x-5)(x+5)}$$

Combine like terms in the numerator:

Final answer: $\frac{6x + 25}{(x-5)(x+5)}$

Extra Problem #1 (from Arlete) $\frac{b}{b-1} + \frac{2b}{b^2-1}$

Step I: (Find the LCD):

You must factor the denominator to find the LCD.

$$\frac{b}{b-1} + \frac{2b}{(b-1)(b+1)}$$

The LCD is $(b-1)(b+1)$

Step II: (What's Missing?): 1st denominator has $(b-1)$, missing $(b+1)$.

2nd denominator is not missing anything.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{b}{b-1} + \frac{2b}{(b-1)(b+1)}$$
$$\frac{b \cdot (b+1)}{(b-1) \cdot (b+1)} + \frac{2b}{(b-1)(b+1)}$$

Step III: (Add or Subtract):

Now that you have a common denominator, just add the numerators and place over the LCD.

$$\frac{\quad +}{(b-1)(b+1)}$$
$$\frac{b^2 + b + 2b}{(b-1)(b+1)}$$

Combine like terms and factor the numerator if possible:

$$\frac{b^2 + 3b}{(b-1)(b+1)}$$
$$\frac{b(b+3)}{(b-1)(b+1)}$$

Extra Problem #2 (from Arlete) $\frac{a-1}{a+1} + \frac{a+1}{a-1}$

Step I: (Find the LCD). The LCD is $(a+1)(a-1)$

Step II: (What's Missing?). 1st denominator has $(a+1)$, missing $(a-1)$.

2nd denominator has $(a-1)$, missing $(a+1)$.

Multiply numerator and denominator of each fraction by "What's Missing":

$$\frac{a-1}{a+1} + \frac{a+1}{a-1}$$

$$\frac{(a-1) \bullet (a-1)}{(a+1) \bullet (a-1)} + \frac{(a+1) \bullet (a+1)}{(a-1) \bullet (a+1)}$$

Step III: (Add or Subtract the Numerators).

Now that you have a common denominator, just place the numerators over the LCD.

$$\frac{\quad + \quad}{(a+1)(a-1)}$$

$$\frac{(a-1) \bullet (a-1) + (a+1) \bullet (a+1)}{(a+1)(a-1)}$$

CAUTION: Do NOT divide out the $(a+1)$ or the $(a-1)$ since there are TERMS in the numerator! Instead. Multiply out the numerator and combine like terms.

$$\frac{a^2 - \cancel{2a} + 1 + a^2 + \cancel{2a} + 1}{(a+1)(a-1)}$$

$$\frac{2a^2 + 2}{(a+1)(a-1)}$$

This can be factored, but it does not reduce. Final answer:

$$\frac{2(a^2 + 1)}{(a+1)(a-1)} \text{ or } \frac{2a^2 + 2}{(a+1)(a-1)}$$

Extra Problem: #25.

$$\frac{x-6}{x^2+5x+6} + \frac{9}{x^2+5x+6}$$

Solution: First notice that this is the addition of two fractions. The first priority must be to have a common denominator, which in this case is x^2+5x+6 ! The LCD becomes THE denominator of the entire problem, and it looks like this:

$$\frac{\quad}{x^2+5x+6} + \frac{\quad}{x^2+5x+6}$$

Next, ADD the numerators.

$$\frac{(x-6) + (9)}{x^2+5x+6}$$

Combine like terms: $\frac{x+3}{x^2+5x+6}$

Factor the denominator: $\frac{x+3}{(x+3)(x+2)}$

Reduce the fraction: $\frac{\cancel{x+3} 1}{\cancel{(x+3)}(x+2)}$

Final answer: $\frac{1}{x+2}$

NOTE: DON'T FORGET THE "1" in the numerator! Without the 1, it's WRONG!!