3.07 Literal Equations

Basic Algebra: One Step at a Time, Page 293-296: #9, 10, plus 2 extra problems,12, 22, 25, 27, 28, 31, 32, 33.

Extra Problems: 17, 21, 31, 37, 53.

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See Section 3.07, with explanations, examples, exercises, and answers, coming soon!

9. Solve for x: $a(x+b)=c(x+d)$.

Solution: First, remove parentheses by the distributive property.

$$ax+ab=cx+cd$$

Next, get all the x terms on the left side by subtracting cx from each side. At the same time, subtract ab to each side to get all the non- x terms on the right side of the equation

$$\begin{array}{rcl} ax \cancel{+ab} & = & \cancel{cx}+cd \\ -cx \cancel{-ab} & & \cancel{-cx}-ab \\ \hline ax-cx & = & cd-ab \end{array}$$

Now, factor the common factor of x:

$$x(a-c) = cd-ab$$

Finally, since the x has been multiplied by $(a-c)$, you must divide both sides of the equation by $(a-c)$.

$$\frac{x\bullet \cancel{(a-c)}}{\cancel{(a-c)}} = \frac{cd-ab}{(a-c)}$$

$$x = \frac{cd-ab}{a-c}$$

NOTE: Don't be tempted to divide out the a or the c! These are "terms"! Never divide out TERMS--only FACTORS!!

10. Solve for x: $a(x-b)=c(d-x)$.

Solution: First, remove parentheses by the distributive property.

$$ax - ab = cd - cx$$

Next, get all the x terms on the left side by adding cx from each side. At the same time, add ab to each side to get all the non- x terms on the right side of the equation

$$\begin{array}{rcl} ax - ab &=& cd - cx \\ +cx \cancel{+ab} && +ab \cancel{+cx} \\ \hline ax + cx &=& cd + ab \end{array}$$

Now, factor the common factor of x:

$$x(a+c) \;=\; cd + ab$$

Finally, since the x has been multiplied by $(a+c)$, you must divide both sides of the equation by $(a+c)$.

$$\frac{x \bullet \cancel{(a+c)}}{\cancel{(a+c)}} = \frac{ab+cd}{(a+c)}$$

$$x = \frac{ab+cd}{a+c}$$

NOTE: Don't be tempted to divide out the a or the c! These are "terms"! Never divide out TERMS--only FACTORS!!

Extra Problem (from Chris).

Solve for x: $a(x-b) = cx + ab$.

Solution: First, remove parentheses by the distributive property.

$$ax - ab = cx + ab$$

Next, get all the x terms on the left side by subtracting cx from each side. At the same time, add $+ab$ to each side to get all the non-x terms on the right side of the equation

$$\begin{array}{rcr} ax - ab & = & cx + ab \\ -cx + ab & & -cx + ab \\ \hline ax - cx & = & 2ab \end{array}$$

Now, factor the common factor of x:

$$x(a-c) = 2ab$$

Finally, since the x has been multiplied by $(a-c)$, you must divide both sides of the equation by $(a-c)$.

$$\frac{x \bullet \cancel{(a-c)}}{\cancel{(a-c)}} = \frac{2ab}{(a-c)}$$

$$x = \frac{2ab}{a-c}$$

Extra Problem #2

Solve for x: $1 - 3xy = 7(5xz + y)$.

Solution: **First, remove parentheses by the distributive property.**

$$1 - 3xy = 35xz + 7y$$

Next, get all the x terms on the right side by adding $3xy$ from each side. At the same time, subtract $7y$ from each side to get all the non-x terms on the left side of the equation

$$\begin{array}{rcl} 1 \quad \cancel{-3xy} & = & 35xz \cancel{+7y} \\ -7y \quad \cancel{+3xy} & & +3xy \quad \cancel{-7y} \\ \hline 1 - 7y & = & 35xz + 3xy \end{array}$$

Now, factor the common factor of x:

$$1 - 7y = 35xz + 3xy$$

$$1 - 7y = x(35z + 3y)$$

Finally, since the x has been multiplied by $(35z + 3y)$, you must divide both sides of the equation by $(35z + 3y)$.

$$\frac{1 - 7y}{(35z + 3y)} = \frac{x\cancel{(35z + 3y)}}{\cancel{(35z + 3y)}}$$

$$x = \frac{1 - 7y}{35z + 3y}$$

12. **Solve for x:** $Y - a = m(x - b)$.

Solution: First, remove parentheses by the distributive property.

$$Y - a = mx - mb$$

Next, notice that there is only one x term, which is on the right side of the equation. Therefore, you must get the non- x terms all on the left side by adding mb from each side.

$$\begin{array}{rcl} Y - a & = & mx - mb \\ +mb & & \cancel{+mb} \\ \hline Y - a + mb & = & mx \end{array}$$

Finally, in order to solve for x,

$$Y - a + mb = mx$$

you must divide both sides of the equation by m.

$$\frac{Y - a + mb}{m} = \frac{\cancel{m}x}{\cancel{m}}$$

$$x = \frac{Y - a + mb}{m}$$

NOTE: Don't be tempted to divide out the m! The m in the numerator is a "term"! Never divide out TERMS--only FACTORS!!

22. $C = 2\pi r$, solve for r.

Solution: Since you are solving for r, and the r has been multiplied by 2π, you must "undo" the multiplication, by dividing both sides by 2π:

$$\frac{C}{2\pi} = \frac{2\pi r}{2\pi}$$

$$\frac{C}{2\pi} = \frac{\cancel{2\pi} r}{\cancel{2\pi}}$$

$$r = \frac{C}{2\pi}$$

25. $A = \frac{1}{2}bh$, solve for h.

Solution: Since there is a denominator of 2, multiply both sides by 2 to clear the fraction!

$$2 \bullet A = \cancel{2} \bullet \frac{1}{\cancel{2}}bh$$

$$2A = bh$$

Next, remember that you are solving for h, and the h has been multiplied by b. In order to "undo" the multiplication, you must divide both sides by b:

$$\frac{2A}{b} = \frac{\cancel{b}h}{\cancel{b}}$$

$$h = \frac{2A}{b}$$

P. 296: 27. $V = \frac{1}{3}\pi r^2 h$, solve for h.

Solution: Since there is a denominator of 3, multiply both sides by 3 to clear the fraction!

$$3 \bullet V = 3 \bullet \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

Next, remember that you are solving for h, and the h has been multiplied by π and r^2. In order to "undo" the multiplication, you must divide both sides by π and r^2:

$$\frac{3V}{\pi r^2} = \frac{\cancel{\pi}\,\cancel{r^2}\,h}{\cancel{\pi}\,\cancel{r^2}}$$

$$\frac{3V}{\pi r^2} = h$$

$$h = \frac{3V}{\pi r^2}$$

P. 296: 28. $V = \frac{1}{3}\pi r^2 h$, solve for r.

Solution: Since there is a denominator of 3, multiply both sides by 3 to clear the fraction!

$$3 \bullet V = 3 \bullet \frac{1}{3}\pi r^2 h$$

$$3V = \pi r^2 h$$

Next, remember that you are solving for r, and the r^2 has been multiplied by π and h. In order to "undo" the multiplication, you must divide both sides by π and h:

$$\frac{3V}{\pi h} = \frac{\cancel{\pi}\,r^2\,\cancel{h}}{\cancel{\pi}\,\cancel{h}}$$

$$\frac{3V}{\pi h} = r^2$$

$$r^2 = \frac{3V}{\pi h}$$

P. 296: 31. $F = \frac{9}{5}C + 32$, solve for C.

There are at least two ways to solve for C in this problem. Both are equally correct, but one is much easier than the other. The easy way to solve this is to notice that the C has had two operations performed on it.

METHOD I:

First, C is multiplied by the fraction $\frac{9}{5}$, and then 32 was added. To solve for C, you must UNDO these two operations in reverse order. So, first undo the $+32$, by subtracting 32 from each side:

$$F = \frac{9}{5}C + 32$$

$$F - 32 = \frac{9}{5}C + 32 - 32$$

$$F - 32 = \frac{9}{5}C$$

Now, undo the multiplication by $\frac{9}{5}$ by multiplying both sides of the equation by the reciprocal of $\frac{9}{5}$ which is $\frac{5}{9}$:

$$F - 32 = \frac{9}{5}C$$

$$\frac{5}{9} \bullet (F - 32) = \frac{5}{9} \bullet \left(\frac{9}{5}C\right)$$

$$\frac{5}{9} \bullet (F - 32) = C$$

METHOD II:

The other method involves multiplying both sides of the equation by the denominator which is 5:

$$F = \frac{9}{5}C + 32$$

$$5 \bullet F = 5 \bullet \frac{9}{5}C + 5 \bullet 32$$

$$5F = 9C + 160$$

Subtract 160 from each side:

$$5F - 160 = 9C + 160 - 160$$

$$5F - 160 = 9C$$

To solve for C just divide both sides by 9:

$$\frac{5F - 160}{9} = \frac{9C}{9}$$

$$C = \frac{5F - 160}{9}$$

This is a slightly different form of the answer obtained in the first method, if you factor out the 5, the answer will be the same as above:

$$C = \frac{5(F - 32)}{9}.$$

P. 296. # 32. $C = \frac{5}{9}(F - 32)$, solve for F.

Notice that this problem is the same as the answer to #31, so the problem will be solved in reverse!

$$C = \frac{5}{9}(F - 32)$$

Begin by "undoing" the fraction $\frac{5}{9}$ by multiplying both sides by $\frac{9}{5}$.

$$\frac{9}{5}C = \frac{9}{5} \bullet \frac{5}{9}(F - 32)$$

$$\frac{9}{5}C = F - 32$$

Next, "undo" the -32 by adding a $+32$ to each side of the equation.

$$\frac{9}{5}C = F - 32$$

$$\frac{9}{5}C + 32 = F - 32 + 32$$

$$\frac{9}{5}C + 32 = F$$

$$F = \frac{9}{5}C + 32$$

In conclusion, notice that the problem for #11 is the answer for #12, and vice-versa.

33. $\frac{1}{F}=\frac{1}{S}+\frac{1}{U}$

The first step is to find the LCD, which is FSU (to all the Florida Gator and Miami Hurricane fans out there, GO FLORIDA STATE!!)

$$FSU \bullet \frac{1}{F} = FSU \bullet \frac{1}{S} + FSU \bullet \frac{1}{U}$$

In the first position, the F divides out, leaving SU.

In the second position the S divides out, leaving UF!

In the third position, the U divides out, leaving FS.

$$\cancel{F}\,SU \bullet \frac{1}{\cancel{F}} = F\,\cancel{S}U \bullet \frac{1}{\cancel{S}} + FS\,\cancel{U} \bullet \frac{1}{\cancel{U}}$$

$$SU \quad = \quad UF \quad + \quad FS$$

Now, in order to solve for S, you have to get all the S terms on one side of the equation. You can do that by subtracting FS from each side of the equation.

$$\begin{array}{rcl} SU & = & UF + FS \\ -FS & & -FS \\ \hline SU - FS = UF & & \end{array}$$

Now, to solve for S, you have to factor out the S on the left side of the equation:

$$SU - FS = UF$$

$$S(U - F) = UF$$

and divide both sides by $(U-F)$: $\frac{S\,\cancel{(U-F)}}{\cancel{(U-F)}} = \frac{UF}{(U-F)}$

$$S = \frac{UF}{U-F}$$

IMPORTANT NOTE: This problem is very much like my own career, in that after I started (and graduated!) at FSU and I ended up (and graduated also!) at UF— except that I did NOT change colors!!

Extra Problems: #17, 21, 31, 37, 53

17. $P = 2L + 2W$, solve for L.

Solution: Since you are solving for L, you must isolate the L term. Begin by eliminating the $2W$ by subtracting $2W$ from each side:

$$P - 2W = 2L + 2W - 2W$$

$$P - 2W = 2L$$

This equation can be written

$$2L = P - 2W$$

Next, divide both sides by 2

$$\frac{2L}{2} = \frac{P - 2W}{2}$$

You can divide out the 2 factors on the left side, but NOT on the right side, since the $2W$ is a "TERM." **NEVER DIVIDE OUT TERMS!!!**

$$\frac{\cancel{2}L}{\cancel{2}} = \frac{P - 2W}{2}$$

$$L = \frac{P - 2W}{2}$$ Final Answer!! (Other forms are also acceptable!!)

21. $A=\frac{1}{2}bh$, solve for h.

Solution: Since there is a denominator of 2, multiply both sides by 2 to clear the fraction!

$$2 \bullet A = \cancel{2} \bullet \frac{1}{\cancel{2}} bh$$

$$2A = bh$$

Next, remember that you are solving for h, and the h has been multiplied by b. In order to "undo" the multiplication, you must divide both sides by b:

$$\frac{2A}{b} = \frac{\cancel{b}h}{\cancel{b}}$$

$$h = \frac{2A}{b}$$

31. $F = \frac{9}{5}C + 32$, solve for C.

There are at least two ways to solve for C in this problem. Both are equally correct, but one is much easier than the other. The easy way to solve this is to notice that the C has had two operations performed on it.

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First, C is multiplied by the fraction $\frac{9}{5}$, and then 32 was added. To solve for C, you must UNDO these two operations in reverse order. So, first undo the $+32$, by subtracting 32 from each side:

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$$F - 32 = \frac{9}{5}C$$

Now, undo the multiplication by $\frac{9}{5}$ by multiplying both sides of the equation by the reciprocal of $\frac{9}{5}$ which is $\frac{5}{9}$:

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$$\frac{5}{9} \bullet (F - 32) = \frac{5}{9} \bullet \left(\frac{9}{5}C\right)$$

$$\frac{5}{9} \bullet (F - 32) = C$$

METHOD II:

The other method involves multiplying both sides of the equation by the denominator which is 5:

$$F = \frac{9}{5}C + 32$$

$$5 \bullet F = 5 \bullet \frac{9}{5}C + 5 \bullet 32$$

$$5F = 9C + 160$$

Subtract 160 from each side:

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$$5F - 160 = 9C$$

To solve for C just divide both sides by 9:

$$\frac{5F - 160}{9} = \frac{9C}{9}$$

$$C = \frac{5F - 160}{9}$$

This is a slightly different form of the answer obtained in the first method, if you factor out the 5, the answer will be the same as above:

$$C = \frac{5(F - 32)}{9}.$$

37. $$R = r + \frac{400(W-L)}{N}$$, solve for L.

Solution: Since you are solving for L, you must isolate the L term. First, let's eliminate the fraction from the problem by multiplying both sides of the equation by the denominator N.

$$N \bullet R = N \bullet r + N \bullet \frac{400(W-L)}{N}$$

As you can see, the denominator divides out:

$$N \bullet R = N \bullet r + \cancel{N} \bullet \frac{400(W-L)}{\cancel{N}}$$

$$N \bullet R = N \bullet r + 400(W-L)$$

Next, remove the parentheses by the Distributive Property:

$$NR = Nr + 400W - 400L$$

In order to solve for L, it might help to get the L term on the left side of the equation (to make the coefficient positive!). Add $400L$ to each side:

$$NR + 400L = Nr + 400W - 400L + 400L$$

$$NR + 400L = Nr + 400W$$

Next, in order to isolate the $400L$, subtract NR from each side:

$$NR - NR + 400L = Nr - NR + 400W$$

$$400L = Nr - NR + 400W$$

Finally, divide both sides by 400:

$$\frac{400L}{400} = \frac{Nr - NR + 400W}{400}$$

37. Continued

The 400 on the left side divides out, but NOT on the right side. Remember, on the right side the $400W$ is a TERM, and you NEVER divide out terms!!!

$$\frac{\cancel{400}L}{\cancel{400}} = \frac{Nr - NR + 400W}{400}$$

The final answer is $$L = \frac{Nr - NR + 400W}{400}$$

By the way, several other forms of the answer are equally acceptable. For example, the three terms in the numerator can be arranged in ANY order!

53. Solve for c: $ac = bc + d$.

Solution: First, get all the c terms on the left side by subtracting bc from each side.

$$\begin{array}{r} ac = bc + d \\ \underline{-bc \quad -bc} \\ ac - bc = d \end{array}$$

Next, you have to get the c in one place, so use the distributive property (factor out the common factor of c!) to write

$$c(a - b) = d$$

Finally, since the c has been multiplied by $(a-b)$, you must divide both sides of the equation by $(a-b)$.

$$\frac{c \bullet \cancel{(a-b)}}{\cancel{(a-b)}} = \frac{d}{(a-b)}$$

$$c = \frac{d}{a-b}$$