

4.06 Equation of a Line

Basic Algebra: One Step at a Time

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P. 346: 24. Find the equation of the line through $(3, -2)$ and $(-4, 4)$.

Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!). In this case, you are given two points. The first step is to find the slope between the two points, using the well-known formula for the slope between two points:

$$m = \frac{\text{RISE}}{\text{RUN}} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{\text{RISE}}{\text{RUN}} = \frac{4 - (-2)}{(-4) - 3}$$

$$m = \frac{\text{RISE}}{\text{RUN}} = \frac{6}{-7} \text{ or } -\frac{6}{7}$$

Now, write down the formula: $y = mx + b$, where $m = -\frac{6}{7}$. You can use either point, let's say $x = 3$, $y = -2$.

$$-2 = -\frac{6}{7} \cdot 3 + b$$

P. 346: 24. Continued

To clear the fraction, multiply by the denominator which is 7.

$$7 \cdot (-2) = 7 \cdot -\frac{6}{7} \cdot 3 + 7 \cdot b$$

Divide out the 7:

$$7 \cdot (-2) = \cancel{7} \cdot -\frac{6}{\cancel{7}} \cdot 3 + 7 \cdot b$$

Solve for b:

$$-14 = -18 + 7 \cdot b$$

$$4 = 7b$$

$$b = \frac{4}{7}$$

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = -\frac{6}{7}x + \frac{4}{7}$$

Check your answer, be substituting $x = 3$ to see if $y = -2$.

$$y = -\frac{6}{7}x + \frac{4}{7}$$

$$y = -\frac{6}{7}3 + \frac{4}{7}$$

$$y = -\frac{18}{7} + \frac{4}{7}$$

$$y = -\frac{14}{7} = -2 \quad \text{It checks!!}$$

Final answer:

$$y = -\frac{18}{7} + \frac{4}{7}$$

P. 348. # 31a)

Find the equation of the line through $(4, 2)$ and parallel to $4x + 3y = 12$.

Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!). In this case, you are given a point, but instead of being given the slope of the line, you are given the equation of a given line. Your line must be parallel to this given line, which means that the given line has the same slope as the line you need to find.

Find the slope of the given line by solving for y in terms of x .

$$4x + 3y = 12$$

Add $-4x$ to each side of the equation:

$$3y = -4x + 12$$

Divide both sides by 3:

$$\frac{3y}{3} = \frac{-4x}{3} + \frac{12}{3}$$

$$y = -\frac{4}{3}x + 4$$

The slope of the given line is $m = -\frac{4}{3}$. The slope of a line parallel is also $m = -\frac{4}{3}$.

Now find the equation of a line with $m = -\frac{4}{3}$ passing through $(4, 2)$,

Start with the formula: $y = mx + b$, where $m = -\frac{4}{3}$, $x = 4$, $y = 2$.

$$2 = -\frac{4}{3} \cdot 4 + b$$

To clear the fraction, multiply by the denominator which is 3.

$$3 \cdot 2 = 3 \cdot -\frac{4}{3} \cdot 4 + 3 \cdot b$$

Divide out the 3: $6 = -16 + 3b$

Solve for b : $22 = 3b$

$$b = \frac{22}{3}$$

P. 348: 31a) Continued

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = -\frac{4}{3}x + \frac{22}{3}$$

Check your answer, be substituting $x = 4$ to see if $y = 2$

$$y = -\frac{4}{3}x + \frac{22}{3}$$

$$y = -\frac{4}{3} \cdot 4 + \frac{22}{3}$$

$$y = -\frac{16}{3} + \frac{22}{3}$$

$$y = \frac{6}{3} = 2 \text{ It checks!!}$$

Final answer:

$$y = -\frac{4}{3}x + \frac{22}{3}$$

P. 348. # 31b)

Find the equation of the line through $(4, 2)$ and perpendicular to $4x + 3y = 12$.

Solution:

In the previous part of this problem, you found the slope of the given line to be

$m = -\frac{4}{3}$, so the slope of a line perpendicular to this line is the negative

reciprocal of this slope, which is $m = \frac{3}{4}$.

Now find the equation of a line with $m = \frac{3}{4}$ passing through $(4, 2)$,

Start with the formula: $y = mx + b$, where $m = \frac{3}{4}$, $x = 4$, $y = 2$.

$$2 = \frac{3}{4} \cdot 4 + b$$

In this case, the fraction clears itself, so you can just divide out the 4. However, if you prefer to work it like most of the other problems like this, you can clear the fraction as before. The result will be the same. HOWEVER, let's do it the easy way!!

$$2 = \frac{3}{4} \cdot 4 + b$$

Divide out the 4: $2 = 3 + b$

Solve for b: $b = -1$

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = \frac{3}{4}x - 1$$

Check your answer, be substituting $x = 4$ to see if $y = 2$

$$y = \frac{3}{4}x - 1$$

$$y = \frac{3}{4} \cdot 4 - 1$$

Divide out the 4:

$$y = 3 - 1$$

$$y = 2 \text{ It checks!!}$$

Final answer:

$$y = \frac{3}{4}x - 1$$

Extra Problem #1

Find the equation of the line through $(4, 2)$ and parallel to $4x - 3y = 12$.

Solution:

In order to find the equation of any line, you must have a point (from which to start!) and a slope (a direction in which to go!). In this case, you are given a point, but instead of being given the slope of the line, you are given the equation of a given line. Your line must be parallel to this given line, which means that the given line has the same slope as the line you need to find.

Find the slope of the given line by solving for y in terms of x .

$$4x - 3y = 12$$

Add $-4x$ to each side of the equation:

$$-3y = -4x + 12$$

Divide both sides by -3 :

$$\frac{-3y}{-3} = \frac{-4x}{-3} + \frac{12}{-3}$$

$$y = \frac{4}{3}x - 4$$

The slope of the given line is $m = \frac{4}{3}$, so the slope of a line parallel is also $m = \frac{4}{3}$.

Now find the equation of a line with $m = \frac{4}{3}$ passing through $(4, 2)$,

Start with the formula: $y = mx + b$, where $m = \frac{4}{3}$, $x = 4$, $y = 2$.

$$2 = \frac{4}{3} \cdot 4 + b$$

To clear the fraction, multiply by the denominator which is 3 .

$$3 \cdot 2 = \cancel{3} \cdot \frac{4}{\cancel{3}} \cdot 4 + 3 \cdot b$$

Divide out the 3: $6 = 16 + 3b$

Solve for b : $-10 = 3b$

$$b = -\frac{10}{3}$$

Extra Problem #1 Continued

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = \frac{4}{3}x - \frac{10}{3}$$

Check your answer, be substituting $x = 4$ to see if $y = 2$

$$y = \frac{4}{3}x - \frac{10}{3}$$

$$y = \frac{4}{3} \cdot 4 - \frac{10}{3}$$

$$y = \frac{16}{3} - \frac{10}{3}$$

$$y = \frac{6}{3} = 2 \text{ It checks!!}$$

Final answer:

$$y = \frac{4}{3}x - \frac{10}{3}$$

Extra Problem #2

Find the equation of the line through $(4, 2)$ and perpendicular to $4x - 3y = 12$.

Solution:

In the previous part of this problem, you found the slope of the given line to be $m = \frac{4}{3}$, so the slope of a line perpendicular to this line is the negative reciprocal

of this slope, which is $m = -\frac{3}{4}$.

Now find the equation of a line with $m = -\frac{3}{4}$ passing through $(4, 2)$,

Start with the formula: $y = mx + b$, where $m = -\frac{3}{4}$, $x = 4$, $y = 2$.

$$2 = -\frac{3}{4} \cdot 4 + b$$

In this case, the fraction clears itself, so you can just divide out the 4. However, if you prefer to work it like most of the other problems like this, you can clear the fraction as before. The result will be the same. HOWEVER, let's do it the easy way!!

$$2 = -\frac{3}{4} \cdot 4 + b$$

Divide out the 4 :

$$2 = -3 + b$$

Solve for b:

$$b = 5$$

Be sure to answer the question! Find the equation of the line $y = mx + b$

$$y = -\frac{3}{4}x + 5$$

Check your answer, by substituting $x = 4$ to see if $y = 2$

$$y = -\frac{3}{4}x + 5$$

$$y = -\frac{3}{4} \cdot 4 + 5$$

Divide out the 4:

$$y = -3 + 5$$

$$y = 2 \text{ It checks!!}$$

Final answer:

$$y = -\frac{3}{4}x + 5.$$