4.07 Systems of Equations (2x2)

Basic Algebra: One Step at a Time, **Page 349-368: #23, 30, 33, 34, 39, 49, 52, 61**

Dr. Robert J. Rapalje, Retired
Central Florida, USA

23. Solve by the elimination method (i.e, eliminate one of the variables!)

$$-x+3y=-5$$
$$2x+3y=-17$$

Solution:

Notice that you have a +3y in both equations. If you were to multiply both sides of one of the equations by -1, you would have a +3y and a -3y, which would subtract out. Either equation will do, so multiply both sides of the second equation by -1.

$$\begin{aligned} -x+3y &= -5 \\ -1(2x+3y) &= -1(-17) \\ \hline -x+3y &= -5 \\ -2x-3y &= 17 \end{aligned}$$

When you add the two equations together, the y terms subtract out:

$$-3x = 12$$

Divide both sides by -3:

$$\frac{-3x}{-3}=\frac{12}{-3}$$
$$x=-4$$

Substitute $x=-4$ into either equation (the first equation will do), and find y.

$$\begin{aligned} -x+3y &= -5 \\ -(-4)+3y &= -5 \\ 4+3y &= -5 \\ 3y &= -9 \\ y &= -3 \end{aligned}$$

Final answer: $(-4,-3)$

Check in the OTHER equation: $2x+3y=-17$

$$2(-4)+3(-3)=-17$$
$$-8-9=-17$$ It checks!!

30. Solve by the elimination method (i.e, eliminate one of the variables!)

$$3x+2y=38$$
$$x+5y=\ \ 4$$

Solution:

You can eliminate either the x terms or the *y* terms, depending upon which looks the easier method to do. To eliminate the x terms, with coefficients of 3 and 1, the common multiple is 3, whereas to eliminate the *y* terms, with coefficients 2 and 5, the common multiple is 10. It's probably easier to eliminate the x terms. If you have a 3x in the first equation, you need a -3x in the second equation to subtract it out. To get a -3x, you need to multiply both sides of the second equation by -3.

$$3x+2y=38$$
$$\underline{-3(\ x+5y)=-3(\ 4)}$$

$$3x+2y=38$$
$$-3x-15y=-12$$

When you add the two equations together, the x terms subtract out:

$$-13y\quad=\ 26$$

Divide both sides by -13:

$$\frac{-13y}{-13}=\frac{26}{-13}$$
$$y=-2$$

Substitute $y=-2$ into either equation (the first equation will do), and find x.

$$3x+2y=38$$
$$3x\ \ +2(-2)=38$$
$$3x-4=38$$
$$3x=42$$
$$x=14$$

Final answer: $(14,-2)$

Check in the OTHER equation: $x+5y=4$

$$(14)+5(-2)=4$$
$$14\ \ -\ \ 10\ \ =4$$

It checks!!

33. **Solve by the elimination method (i.e, eliminate one of the variables!)**

$$3x+5y=\ \ 2$$
$$2x+3y=-4$$

Solution:

You can eliminate either the x terms or the y terms, depending upon which looks the easier method to do. To eliminate the x terms, with coefficients of 3 and 2, the common multiple is 6, whereas to eliminate the y terms, with coefficients 5 and 3, the common multiple is 15. It's probably easier to eliminate the x terms. You will need to get a 6x and a -6x to subtract out the x terms. To get a 6x, in the first equation, you need to multiply both sides of the first equation by 2, and to get a -6x in the second equation, you need to multiply both sides of the second equation by -3.

$$2(3x+5y)=\ \ 2(\ 2)$$
$$-3(\ 2x+3y)=-3(-4)$$

$$6x+10y=\ \ 4$$
$$-6x-\ \ 9y=12$$

When you add the two equations together, the x terms subtract out:

$$y\ =\ 16$$

Substitute $y\ =\ 16$ into either equation (the first equation will do), and find x.

$$3x+5y=2$$
$$3x\ \ +5(16)=2$$
$$3x+80=2$$
$$3x=-78$$
$$x=-26$$

Final answer: $(-26,16)$

Check in the OTHER equation: $2x+3y=-4$

$$2(-26)+3(16)=-4$$
$$-52\ +\ 48\ \ =-4$$ It checks!!

34. Solve by the elimination method (i.e, eliminate one of the variables!)

$$5x+2y=-12$$
$$3x-5y=-\ 1$$

Solution:

You can eliminate either the x terms or the y terms, depending upon which looks the easier method to do. To eliminate the x terms, with coefficients of 5 and 3, the common multiple is 15, whereas to eliminate the y terms, with coefficients 2 and -5, the common multiple is 10. To eliminate the y terms, you will need to get a 10y and a -10y to subtract them out. To get a 10y, in the first equation, you need to multiply both sides of the first equation by 5, and to get a -10y in the second equation, you need to multiply both sides of the second equation by 2.

$$5(5x+2y)=5(-12)$$
$$2(\ 3x-5y)=2(-1)$$
$$25x+10y=-60$$
$$6x-10y=\ \ -2$$

When you add the two equations together, the x terms subtract out:

$$31x\ =-62$$
$$x\ =-2$$

Substitute $x\ =-2$ into either equation (the first equation will do), and find *y*.

$$5x+2y=-12$$
$$5(-2)\ +2y=-12$$
$$-10+2y=-12$$
$$2y=-2$$
$$y=-1$$

Final answer: $(-2,-1)$

Check in the OTHER equation: $3x-5y=-1$

$$3(-2)-5(-1)=-1$$
$$-6\ +\ 5\ \ =-1$$ It checks!!

39. **Solve by the elimination method (i.e, eliminate one of the variables!)**

$$4x - 7y = -28$$
$$4x + 7y = -28$$

Solution:

This one is actually easy! To eliminate the *y* terms, just add the two equations together.

$$8x \qquad = -56$$
$$x = -7$$

Substitute $x = -7$ **into either equation (the first equation will do), and find *y*.**

$$4x - 7y = -28$$
$$4(-7) - 7y = -28$$
$$-28 - 7y = -28$$
$$-7y = 0$$
$$y = 0$$

Final answer: $(-7, 0)$

Check in the OTHER equation:

$$4x + 7y = -28$$
$$4(-7) - 7(0) = -28$$
$$-28 \quad - \quad 0 \quad = -28$$

It checks!!

49. Solve by the substitution method.

$$x = 5y + 24$$
$$y = 3x - 2$$

Solution:

Since this set of equations has an equation in the form of $x =$ and also one in the form of $y =$, there are TWO ways to solve this by substitution. You can either substitute the $x =$ into the second equation, or substitute the $y =$ into the first equation. Let's substitute the $x =$ from the first equation into the second equation. It looks like this

$$x = \quad 5y + 24$$
$$y = 3(\qquad\quad) - 2$$
$$y = 3(5y + 24) - 2$$
$$y = 15y + 72 - 2$$
$$y = 15y + 70$$

$$-14y = 70$$

Divide both sides by -14:

$$y = -5$$

Now, substitute this into the first equation:

$$x = 5y + 24$$
$$x = 5(-5) + 24$$

$$x = -25 + 24$$
$$x = -1$$

Final answer: $(-1, -5)$

Check in the OTHER equation:

$$y = 3x - 2$$

$$-5 = 3(-1) - 2$$ It checks!!

52. **Solve by the substitution method.**

$$7x - 4y = 40$$
$$y - x = 2$$

Solution:

In order to solve this system of equations by the substitution method, you need to solve for one of the variables in one of the equations in terms of the other variable. Perhaps it would be easiest to solve for y in the second equation by adding x to each side of the equation.

$$y = x + 2$$

Now, substitute this into the first equation:

$$7x - 4(\quad) = 40$$
$$7x - 4(x + 2) = 40$$
$$7x - 4x - 8 = 40$$
$$3x - 8 = 40$$
$$3x = 48$$
$$x = 16$$

Substitute $x = 16$ back into either equation (the "$y =$" equation will be easiest!), and find y.

$$y = x + 2$$
$$y = 16 + 2$$
$$y = 18$$

Final answer: $(16, 18)$

Check in the OTHER equation:

$$7x - 4y = 40$$
$$7(16) - 4(18) = 40$$
$$102 - 72 = 40$$ **It checks!!**

61. Solve by any algebraic method

$$17x + 8y = 4$$
$$32x + 18y = -16$$

Solution:

The best algebra method is elimination! You do NOT want to get into a process of solving for either x or y in either of these equations. It gets really ugly fast!!

Next is the question of how to eliminate the x or the y variables. You can go to a lot of trouble to find the least common multiple of 8 and 18, or just use the product of 18 times 8. That is, you multiply the first equation by the coefficient in the second equation, which is 18, and you multiply the second equation by the coefficient in the first equation, which is 8. And then also, multiply one of the equations by a negative, in order to make the variable subtract out. In color, it looks like this:

$$18(17x + 8y) = 18(4)$$
$$-8(32x + 18y) = -8(-16)$$
$$306x + 144y = 72$$
$$-256x - 144y = 128$$

When you add the two equations together, the y terms subtract out leaving:

$$50x = 200$$

Divide both sides by 50:

$$\frac{50x}{50} = \frac{200}{50}$$
$$x = 4$$

Substitute $x = 4$ into either equation (the first equation will do), and find y.

$$17x + 8y = 4$$
$$17(4) + 8y = 4$$
$$68 + 8y = 4$$
$$8y = -64$$
$$y = -8$$

Final answer: $(4, -8)$

Check in the OTHER equation:

$$32x + 18y = -16$$
$$32(4) + 18(-8) = -16$$
$$128 - 144 = -16$$

It checks!!