Math in Living C O L O R !!

To see Section 5.02 with explanations, examples, and exercises, click here!

5.02 Cube Roots and More

Basic Algebra: One Step at a Time. Page 403 - 412: #54, 58, 93, 94.

Dr. Robert J. Rapalje, Retired
Central Florida, USA

The expression $\sqrt[3]{x}$ is called the cube root of x, and it asks the question, "What cubed would equal x?" Likewise, $\sqrt[4]{x}$ means the fourth root of x, $\sqrt[5]{x}$ means the fifth root of x, etc. In general, $\sqrt[n]{x}$ means the nth root of x, where the radicand is x, and the index of the radical is n.

The operations of square root, cube root, fourth root, etc. are actually inverse operations for the operations of squaring, cubing, raising to the fourth power, etc. When taking square roots in the last section, it was essential to be familiar with the perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, and 169. Also, remember that the even powers (x^2, x^4, x^6, x^8, x^{10}, etc.) were and are perfect squares. Now, when taking a cube root, it is essential to be familiar with (i.e., memorize them!!) the perfect cubes, and other powers, especially the numbers 1, 8, 27, 64, and 125.

$2^3 = 8$	$2^4 = 16$	$2^5 = 32$
$3^3 = 27$	$3^4 = 81$	
$4^3 = 64$		
$5^3 = 125$		

And again, the list goes on. However, these are the main numbers that we use, and with which you need to be familiar. You really need to have the numbers 8, 27, 64, and 125 in your head before you continue this lesson!!

Taking a cube root of a number is actually the inverse operation of cubing. Suppose you cubed the number 5. The answer of course is 125. Now, what would you have to do to the 125 to get back to the 5? You would take the cube root of 125, written $\sqrt[3]{125}$, which is 5.

NOTE: Frequently the terminology "square root" and "radical" are used interchangeably. They are NOT the same. The term "radical" may be used generally to refer to a square root, cube root, etc. The term "square root" does not include cube roots, fourth roots, etc.

P. 409: 54. $\sqrt[3]{54}$

Solution: Before you begin cube root problem, you must remember the perfect cube numbers, i.e., $2^3 = 8,\ \ 3^3 = 27,\ 4^3 = 64,\ \ 5^3 = 125$, and find the perfect cube that divides evenly into 54.

The only perfect cube that divides evenly into 54 is 27. Break down the 54 into $27 \bullet 2$.

$$\sqrt[3]{} \bullet \sqrt[3]{}$$

$$\sqrt[3]{27} \bullet \sqrt[3]{2}$$

Final answer: $3 \bullet \sqrt[3]{2}$

Since this is a numerical problem you can check the answer by calculating the decimal value of the problem and then calculating the value of the answer to see if these values are the same.

$\sqrt[3]{54}$ = 3.77976315

$3 \bullet \sqrt[3]{2}$ = 3.77976315 -- It checks!!

P. 409: 58. $\sqrt[3]{80}$

Solution: Before you begin cube root problem, you must remember the perfect cube numbers, i.e., $2^3 = 8,\ 3^3 = 27,\ 4^3 = 64,\ 5^3 = 125$, and find the perfect cube that divides evenly into 80.

The only perfect cube that divides evenly into 80 is 8. Break down the 80 into $8 \bullet 10$.

$$\sqrt[3]{\quad} \bullet \sqrt[3]{\quad}$$

$$\sqrt[3]{8} \bullet \sqrt[3]{10}$$

$$2 \bullet \sqrt[3]{10}$$

Since this is a numerical problem you can check the answer by calculating the decimal value of the problem and then calculating the value of the answer to see if these values are the same.

$\sqrt[3]{80}$ = 4.30886938

$2 \bullet \sqrt[3]{10}$ = 4.30886938 -- It checks!!

P. 412: 93. $\sqrt[5]{320x^9y^{32}}$

Solution: Make two separate fifth roots:

$$\sqrt[5]{\quad} \bullet \sqrt[5]{\quad}$$

Find the perfect 5th powers in the above, place it in the (first) RED radical.

Remember that the $2^5 = 32$ is the main perfect 5th power.

Variables raised to powers must be multiples of 5, like x^5 or y^{30}

$$\sqrt[5]{32x^5y^{30}} \bullet \sqrt[5]{\quad}$$

Place the other factors that are "left-over" in the (second) BLUE radical.

$$\sqrt[5]{32x^5y^{30}} \bullet \sqrt[5]{10x^4y^2}$$

Take the 5th root of the perfect power. Leave the second root alone!!

$$2xy^6 \bullet \sqrt[5]{10x^4y^2}$$

P. 412: 94. $\sqrt[4]{160x^{40}y^{90}}$

Make two separate fourth roots:

$$\sqrt[4]{\quad} \bullet \sqrt[4]{\quad}$$

Find the perfect 4th powers in the above, place it in the (first) RED radical.

Remember that the $2^4 = 16$ and $3^4 = 81$ are the main perfect 5th powers.

Variables raised to powers must be multiples of 4, like x^{40} or y^{88}

$$\sqrt[4]{16x^{40}y^{88}} \bullet \sqrt[4]{\quad}$$

Place the other factors that are "left-over" in the (second) BLUE radical.

$$\sqrt[4]{16x^{40}y^{88}} \bullet \sqrt[4]{10y^2}$$

Take the 4th root of the perfect power. Leave the second root alone!!

$$2x^{10}y^{22} \bullet \sqrt[4]{10y^2}$$