1.03 *Calculators--In Your Head or In Your Hand*

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

At the time of this writing (1997), there are a variety of different calculators available at a variety of prices. If you are planning to take higher mathematics courses, or if you wish to use a state-of the art calculator, you will probably want a "graphics" calculator. A graphics calculator will certainly be useful in this course, but not necessary. An "ordinary" scientific calculator will be sufficient. For this course, several features will be necessary. **First**, you will need a **"power" function.** Look for the **"X^Y"** or the **"Y^x" function** on the calculator (on the TI-85, look for the "^" key). The opposite of **raising X or Y to a power** is **taking a root of X or Y.** The is the **root function**, which is indicated on the calculator by the keys $X^{\frac{1}{Y}}$ (*or* $\sqrt[Y]{X}$) or $Y^{\frac{1}{X}}$ (*or* $\sqrt[X]{Y}$). For convenience, your calculator probably has the **square (X^2)** and **square root ($\sqrt{X}$) functions.**

Second, you will need a **"scientific notation" function.** This will probably be an **"EXP"** or an **"EE"** key on the calculator. **Third**, you will want a calculator with **parentheses "("** and **")"** keys. It will also be helpful to have **memory function(s), reciprocal function (1/X),** and the function $a\ b/c$. Most scientific calculators have all of these functions. **However, the functions to look for if you are buying a calculator (not all have these!) are conversion from decimals to fractions and conversion from standard decimal form to scientific notation and back.** These are very handy features. In this section, you will find some explanation, examples, and exercises which will supplement your calculator instructions and help you practice with your calculator. Remember

that each brand and model of calculator is different, and it is important that you learn to work with your calculator, and that you practice with it!!!

Now that you have a calculator . . . What kind of calculator do you have? First, is it a "graphics" calculator, or "non-graphics"? Most calculators today have what are called "direct" as well as "indirect" functions(operations). The "direct" function is the one that is right on the key. This is accessed "directly" by just pressing the key. An "indirect" function is one that is directly above the key, usually printed in a different color. Sometimes there are two indirect functions, usually in two different colors. To access the "indirect" functions, you first press the key in the upper left corner of the calculator, (usually called **"2nd"**, sometimes **"shift"** or **"inv"**,) then you press the key below the color-coded function you want to access. Sometimes there is a **"3rd"** function in another color, with additional functions color-coded to it. With graphics calculators, there are also special "Function" keys and also "Menu" keys.

PRINCIPLE

You must practice with and become familiar with your own calculator. During an examination is not a good time to do this!

PRINCIPLE

When learning a new skill always begin with examples for which you already know the answer.

POWERS AND ROOTS

To begin using the calculator, try raising a number to a power. You may have an X^2 key. If so, this is the easiest way to square a number. When raising to other powers, you must use Y^X *or* X^Y (for TI-85, look for the "^" on the right side of the calculator.)

EXAMPLE: 2^3 **: Press the keys "2", " Y^X ", "3", "=" or "ENTER". The answer, of course, is 8.**

EXAMPLE: 14^5 **: Press the keys "14", " Y^X ", "5", "=" or "ENTER". The answer (of course?) is 537824.**

The opposite (or inverse) operation of **squaring** is the operation of **square root.** The opposite (inverse) of **cubing** is **cube root.** The opposite(inverse) of **raising a number to a power** is **taking a root of a number.** As examples, consider:

$7^2 = 49$. **The opposite operation is** $\sqrt{49} = 7$.

$2^3 = 8$. **The opposite operation is** $\sqrt[3]{8} = 2$.

$14^5 = 537824$. **The opposite operation is** $\sqrt[5]{537824} = 14$.

At this point, a very important question should be answered about your calculator relating to the order in which you must enter the numbers. The question is this: when you calculate the $\sqrt{9}$, **do you enter "9" first and then "$\sqrt{\ }$"? Or do you enter "$\sqrt{\ }$" first and then the "9"? This question is important, and knowing which way your calculator operates will save you much time and trouble in later calculations! Use your calculator to do this both ways to**

see which way gives you the correct answer of "3". You need to make note of whether you enter the base number or the operation first. To help you remember and for notation purposes, throughout this book, we define "TYPE I" and "TYPE II" calculators:

TYPE I CALCULATOR: "$\sqrt{9}$" = "9", "$\sqrt{\ }$" ANSWER IS 3

TYPE II CALCULATOR: "$\sqrt{9}$" = "$\sqrt{\ }$", "9" ANSWER IS 3.

Now that you have determined the type of calculator you have, try to calculate $\sqrt[3]{125}$ (you already know the answer is 5, right?) First, some calculators have a special cube root key (probably it says " $\sqrt[3]{X}$ "). If so, then

TYPE I CALCULATOR: " $\sqrt[3]{125}$ " = "125", " $\sqrt[3]{X}$ " ANSWER IS 5

TYPE II CALCULATOR: " $\sqrt[3]{125}$ " = " $\sqrt[3]{X}$ ", "125" ANSWER IS 5.

If you do not have a special cube root key, then you need to use the $\sqrt[X]{Y}$, $\sqrt[Y]{X}$, $X^{\frac{1}{Y}}$, **or** $Y^{\frac{1}{X}}$ function, whichever your calculator has. This key will be necessary even if you have the special cube root key, since you need to to find fourth roots, fifth roots, etc. Notice that this is probably a **"second function,"** so begin at the upper left key on the calculator:

TYPE I CALC: " $\sqrt[3]{125}$ " = "125", "2nd", " $\sqrt[X]{Y}$ ", "3", "=" ANSWER IS 5

TYPE II CALC: " $\sqrt[3]{125}$ " = "3", "2nd", " $\sqrt[X]{Y}$ ", "125", "=" ANSWER IS 5.

TI-85: "3","2nd","MATH","F5(MISC)","MORE","F4 ($\sqrt[X]{\ }$), "125", "ENTER".

If you have a TI-85, there are easier ways to set this up. Ask for some help and do not be intimidated--it is much easier than this.

Use your calculator to compute the following exercises (in real life, not everything comes out even--round to nearest hundredth):

1. 5^3
2. 13^6
3. 12^3
4. 3^{12}
5. $\sqrt{49}$
6. $\sqrt{169}$
7. $\sqrt{1600}$
8. $\sqrt{1024}$
9. $\sqrt{203401}$
10. $\sqrt{915849}$
11. $\sqrt{750}$
12. $\sqrt{1352}$
13. $\sqrt[3]{64}$
14. $\sqrt[3]{216}$
15. $\sqrt[3]{1728}$
16. $\sqrt[3]{1{,}000{,}000}$
17. $\sqrt[3]{4913}$
18. $\sqrt[3]{300763}$
19. $\sqrt[4]{81}$
20. $\sqrt[5]{32}$
21. $\sqrt[5]{1024}$
22. $\sqrt[12]{531441}$
23. $\sqrt[3]{1024}$
24. $\sqrt[5]{123456}$

COMBINED OPERATIONS

Frequently, a calculation involves more than one operation. Consider the following $\frac{12 \cdot 5}{3 \cdot 2}$

(You can quickly see that the answer is 60/6 which is 10.)

If you didn't know any better (which perhaps you do not!), you might use the following calculator steps:

"12", "X", "5", "÷", "3", "X", "2", "=" (ANSWER -- 40! Wrong!)

What happened, and what can be done to correct the problem? Follow through the steps in the above calculation: You first multiplied 12 times 5 to obtain 60. Then you divided by 3 to obtain 20. However, next you **multiplied by 2**, which gives 40.

There are at least three ways to correct this problem. Try these, and let me know which is the easiest:

1. **PARENTHESES--Put parentheses around the numerator and denominator.** **NUMER: "(", "12", "X", "5", ")", "÷",**
   DENOM: "(", "3", "X", "2", ")", "="
   ANSWER -- 10.

2. **NUMERATOR MULTIPLY, DENOMINATOR DIVIDE.** ***Realize that <u>all</u> numerator factors represent multiplication, and <u>all</u> denominator factors represent division.***

   "12", "X", "5", "÷", "3", "÷", "2", "=" ANSWER -- 10.

3. **ZIG-ZAG METHOD--** $\frac{12 \nearrow 5}{3 \cdot 2 \searrow}$

   "12", "÷", "3", "X", "5", "÷", "2", "=" ANSWER -- 10.

EXERCISES:

1. $\frac{12 \cdot 5}{3 \cdot 2}$ 2. $\frac{32 \cdot 12}{24 \cdot 8}$ 3. $\frac{36 \cdot 9}{27 \cdot 2}$ 4. $\frac{42 \cdot 12}{8 \cdot 7}$

5. $\frac{12 \cdot 40}{3 \cdot 2 \cdot 4}$ 6. $\frac{4 \cdot 12}{8 \cdot 3 \cdot 5}$ 7. $\frac{3.6 \cdot 98}{4.9 \cdot 20}$ 8. $\frac{425 \cdot 32}{9 \cdot 0.04 \cdot 83.6}$

SCIENTIFIC NOTATION

When working with very large (astronomical!) numbers or very small (microscopic!) numbers, it is often convenient to write the numbers in **scientific notation.** In **scientific notation,** the number must be expressed as a **number between 1 and 9.99... times a power of 10.** Numbers with magnitude **10 or greater** will be expressed with a **positive power of 10,** while numbers with magnitude **smaller than 1** must be expressed with a **negative power of 10.**

Before using the calculator to compute with scientific notation, first it may be helpful to review the concept and make sure you understand how to convert from **standard decimal form** to **scientific notation** and back, using the calculator in your head! Your calculator may or may not convert from one to the other (this is something you can easily do in your head!), but it will

certainly perform scientific notation calculations and frequently give you answers in scientific notation. **First, the "non-calculator" explanation!**

EXAMPLES:

STANDARD DECIMAL FORM		SCIENTIFIC NOTATION
1. 300	= 3.0 X 100	= 3.0 X 10^2
2. 3000	= 3.0 X 1000	= 3.0 X 10^3
3. 3250	= 3.25 X 1000	= 3.25 X 10^3
4. 32,500,000	move decimal 7 places left	= 3.25 X 10^7
5. 0.03	= 3.0 X 1/100	= 3.0 X 10^{-2}
6. 0.0003	= 3.0 X 1/10000	= 3.0 X 10^{-4}
7. 0.0000000325	move 8 places right	= 3.25 X 10^{-8}
8. 0.00000000032	move 10 places right	= 3.2 X 10^{-10}

EXERCISES: Express in scientific notation.

1. 450 = _____ 2. 7500 = _____

3. 12,000,000 = _____ 4. 720,000,000,000 = _____

5. 0.00325 = _____ 6. 0.000000246 = _____

7. 0.00000000325 = _____ 8.0.00000000436 = _____

9. 480,000,000 = _____ 10. 93,200,000,000 = _____

11. 0.0000876 = _____ 12. 0.0000122 = _____

Your calculator may or may not be able to convert from standard decimal form to scientific notation and vice versa. For some calculators (certain Casios and the TI-85), you use the **"MODE"**

button, and follow the instructions in your manual. For other calculators (certain Texas Instrument models) there are separate functions called **"SCI"** and **"FLO"**. (**"FLO"** means "floating point", which means a decimal format.) If you have this function on your calculator, it might be a good idea to go back and use your calculator on some of the previous exercises.

EXERAMPLE (This is a cross between an exercise and an example!!)
Use your calculator to compute the following exercises.

13a) 6,000 x 7,000 = _____

b) 60,000 x 70,000 = _____

c) 60,000,000 x 70,000,000 = _____

At some point in these calculations, depending upon your calculator, the answer began with a reasonable **42,000,000** for **#13a**, and then "degenerated" to what on your calculator probably looks like **4.2 15** or **4.2 E 15** or 4.2^{15} as the answer to **#13c**. What happened? Is the calculator correct? Actually, the problem is that the calculator can only display so many digits--some calculators can only display eight digits, others can display ten digits--on the screen. It is therefore necessary for **the calculator to convert the answer to *scientific notation*!!** The answer to **#13c** is actually 4.2×10^{15}.

Most calculators have an **EE (or EXP) function**, which makes **scientific notation** very easy to enter. If you have an **EE or EXP key** on your calculator, to enter 7.2×10^{12}, simply enter **7.2, EE (or EXP), 12, enter.** It doesn't get any easier than that!

CAUTION: DO NOT ENTER "7.2", "X", "EE", ETC.

There is one more function or skill you will need for scientific notation. What about **negative exponents**, such as 7.2×10^{-12}? This is done using what is called the **"+/-" key.** Whatever you did to

enter **7.2 x 10^{12}**, try it now by touching the **"+/-" key** just before (or on some calculators <u>after</u>) you enter **12.** Try it to be sure: **7.2, EE (EXP), "+/-", 12, enter:** _____(Your answer)
(One more for practice--enter into your calculator: 8.6 x 10^{39}.)

Frequently, there will be **combined operations**, as the following example: $\dfrac{7.2\ X\ 10^{12}\ \cdot\ 6.3\ X\ 10^{-8}}{3.5\ X\ 10^{-4}\ \cdot\ 8.1\ X\ 10^{14}}$

It will be helpful to work up to this example "one step at a time!"

14. $7.2\ X\ 10^{12}\ \cdot\ 6.3\ X\ 10^{-8}\ =$

"7.2", "EE", "12", "X", "6.3", "EE", "+/-", "8", "="

15. $3.5\ X\ 10^{-4}\ \cdot\ 8.1\ X\ 10^{14}\ =$

16. $8.42\ X\ 10^{13}\ \cdot\ 5.8\ X\ 10^{8}\ =$

17. $8.42\ X\ 10^{-13}\ \cdot\ 5.8\ X\ 10^{-8}\ =$

18. $\dfrac{7.2\ X\ 10^{12}}{3.5\ X\ 10^{-4}}\ =$

19. $\dfrac{6.3\ X\ 10^{-8}}{8.1\ X\ 10^{14}}\ =$

20. $\dfrac{7.2\ X\ 10^{12}}{8.1\ X\ 10^{-14}}\ =$

21. $\dfrac{7.2\ X\ 10^{-12}}{8.1\ X\ 10^{-14}}\ =$

22. $\dfrac{10^{12}}{8\ X\ 10^{-4}}$ [HINT: 10^{12} must be entered as 1 X 10^{12}]

23. $\dfrac{10^{-12}}{4\ X\ 10^{-4}}$

24. $\dfrac{10^{-6}}{6\ X\ 10^{-10}}$

25. $\dfrac{7.2\ X\ 10^{12}\ \cdot\ 6.3\ X\ 10^{-8}}{3.5\ X\ 10^{-4}\ \cdot\ 8.1\ X\ 10^{14}}$ =

26. $\dfrac{6.3\ X\ 10^{-23}\ \cdot\ 9.5\ X\ 10^{4}}{7.5\ X\ 10^{12}\ \cdot\ 4.3\ X\ 10^{-3}}$ =

27. $\dfrac{6.3\ X\ 10^{23}\ \cdot\ 9.5\ X\ 10^{-4}}{7.5\ X\ 10^{-12}\ \cdot\ 4.3\ X\ 10^{3}}$ =

28. $\dfrac{6.3\ X\ 10^{-23}\ \cdot\ 9.5\ X\ 10^{-4}}{7.5\ X\ 10^{-12}\ \cdot\ 4.3\ X\ 10^{-3}}$ =

Combined operations:

29. $\dfrac{72{,}000^2 \cdot 0.0063}{0.035 \cdot 81^2} =$ _____.

30. $\dfrac{0.72 \cdot 6{,}300{,}000^2}{350^2 \cdot 810} =$ _____.

31. $\dfrac{72{,}000^2}{\sqrt{0.035}} =$

32. $\dfrac{\sqrt[3]{0.72} \cdot 6{,}300{,}000}{\sqrt{350}} =$

33. $\dfrac{1{,}940^4}{8{,}560{,}000 \cdot 0.067^3} =$

34. $\dfrac{0.0096^2 \cdot \sqrt{0.00042}}{8350^3} =$

35. $(7.2 \cdot 10^{12})^2 \cdot (6.3 \cdot 10^5)^3 =$

36. $(9.4 \cdot 10^{-12})^3 \cdot (8.3 \cdot 10^5)^2 =$

37. $\dfrac{(6.3\ X\ 10^{-8})^3}{(8.1\ X\ 10^{14})^2} =$

38. $\dfrac{(7.2\ X\ 10^{12})^3}{(8.1\ X\ 10^{-14})^2} =$

ANSWERS 1.03

p. 31: **1.** 125; **2.** 4826809; **3.** 1728; **4.** 531441; **5.** 7; **6.** 13; **7.** 40; **8.** 32; **9.** 451; **10.** 957; **11.** 27.39; **12.** 36.77; **13.** 4; **14.** 6; **15.** 12; **16.** 100; **17.** 17; **18.** 67; **19.** 3; **20.** 2; **21.** 4; **22.** 3; **23.** 10.08; **24.** 10.43.

p. 33: **1.** 10; **2.** 2; **3.** 6; **4.** 9; **5.** 20; **6.** 0.4; **7.** 3.6; **8.** 451.89.

p. 34-38: **1.** 4.5×10^{2}; **2.** 7.5×10^{3}; **3.** 1.2×10^{7}; **4.** 7.2×10^{11}; **5.** 3.25×10^{-3}; **6.** 2.46×10^{-7}; **7.** 3.25×10^{-9}; **8.** 4.36×10^{-9}; **9.** 4.8×10^{9}; **10.** 9.32×10^{10}; **11.** 8.76×10^{-5}; **12.** 1.22×10^{-5}; **13a)** 42,000,000 or 4.2×10^{7}; **b)** 4,200,000,000 or 4.2×10^{9}; **c)** 4.2×10^{15}; **14.** 4.54×10^{5}; **15.** 2.84×10^{11}; **16.** 4.88×10^{22}; **17.** 4.88×10^{-20}; **18.** 2.06×10^{16}; **19.** 7.78×10^{-23}; **20.** 8.89×10^{25}; **21.** 8.89×10^{1} or 88.89; **22.** 1.25×10^{15}; **23.** 2.5×10^{-9}; **24.** 1.67×10^{3} or 1666.67; **25.** 1.6×10^{-6} **26.** 1.86×10^{-28}; **27.** 1.86×10^{28}; **28.** 1.86×10^{-12} **29.** 1.42×10^{5}; **30.** 2.88×10^{5}; **31.** 2.77×10^{10}; **32.** 3.02×10^{5}; **33.** 5.50×10^{9}; **34.** 3.24×10^{-18}; **35.** 1.30×10^{43}; **36.** 5.72×10^{-22}; **37.** 3.81×10^{-52}; **38.** 5.69×10^{64}.