1.02 *Order of Operations, Signed Numbers, Absolute Value*

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In the first section, different number systems were defined, and properties relating to addition (subtraction) and multiplication (division) of numbers within those systems were identified. In order to execute these basic operations it is necessary to establish a few traditions (agreements!) as to how these operations will be carried out. The "signs" of the numbers must also be addressed. In this section it is preferred that you use the calculator "in your head." In the next section, you will be encouraged to use the calculator "in your hand." If you do not already have one, be looking for a good sale on inexpensive calculators!

ORDER OF OPERATIONS

In your head, perform the following calculations:

1. $4 + 6 \cdot 2 =$ _______ **2.** $16 \div 8 \cdot 2 =$ _______

3. $5 \cdot 2^2 =$ _______ **4.** $(3 + 4)^2 =$ _______

Solutions:

1. If addition is performed first, the answer is 20.
If multiplication is performed first, the answer is 16.

2. If division is performed first, the answer is 4.
If multiplication is performed first, the answer is 1.

3. If multiplication is performed first, the answer is 100.
If the power is performed first, the answer is 20.

4. If you add first, the answer is 49. If you square the 3 and then square the 4 (which the problem does <u>not</u> say to do!), then the answer would be 25.

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A mathematics system must have unique solutions to basic operations such as these. In order to perform these operations and always arrive at the same conclusion, we must agree to certain traditions which are called the **order of operations agreement.** This agreement is generally accepted across the world and in most arenas of life. If you obey these traditions, you will obtain consistent results in agreement with everyone else.

ORDER OF OPERATIONS

LEVEL 1: **Simplify within symbols of grouping: parentheses ( ), brackets [ ], or set braces { }.**

LEVEL 2: **Raise to the power (i.e. exponents).**

LEVEL 3: **Multiply or divide in order from left to right.**

LEVEL 4: **Add or subtract in order from left to right.**

Now, we can say of the examples on the previous page, the "correct answers" are: **1.** 16; **2.** 4; **3.** 20; **4.** 49

EXERCISES: **Simplify each of the following according to the order of operations agreement.**

1. $2 + 8 \cdot 4$

= ________

= ________

2. $20 \div 4 \cdot 5$

= ________

= ________

3. $2 \cdot 5^2$

= ________

= ________

4. $32 \div 4 \cdot 8$

= ________

= ________

5. $100 \div 5^2$

= ________

= ________

6. $(2 \cdot 5)^2$

= ________

= ________

7. $12 + 8 \div 2$

= ____________

= ____________

8. $12 \div (3 + 3)$

= ____________

= ____________

9. $5 + 9 \cdot 2$

= ____________

= ____________

10. $(100 \div 5)^2$

= ____________

= ____________

11. $12 \div 3 + 3$

= ____________

= ____________

12. $(5 + 9) \cdot 2$

= ____________

= ____________

13. $20 - 5 \cdot 0$

= ____________

= ____________

14. $12 + 3 \div 3$

= ____________

= ____________

15. $12 \div (3 \div 3)$

= ____________

= ____________

16. $5 + 5^2$

= ____________

= ____________

17. $(12 + 3) \div 3$

= ____________

= ____________

18. $18 + 2 \cdot 0$

= ____________

= ____________

19. $8 + 3 \cdot 2^2$

= ________________

= ________________

= ________________

20. $5 \cdot 3^2 + 11 - 2^2$

= ________________

= ________________

= ________________

21. $24 - 3 \cdot 4 + 6 \div 2$

= ____________________

= ____________________

= ____________________

22. $12 + 18 \div 3^2 + 6$

= ____________________

= ____________________

= ____________________

23. $12 + (18 \div 3)^2 + 6$

= ____________________

= ____________________

= ____________________

24. $6 + 6^2 \div 3 \cdot 2$

= ____________________

= ____________________

= ____________________

= ____________________

25. $(16 + 2^2) \div 2 \cdot 2$

= ____________________

= ____________________

= ____________________

= ____________________

26. $8 + 2 \cdot 3^2 - 5$

= ____________________

= ____________________

= ____________________

= ____________________

27. $5^2 + 4^2 \div 2^2 + 3^2$

= ____________________

= ____________________

= ____________________

= ____________________

28. $(16 + 2)^2 \div (2 + 2)$

= ____________________

= ____________________

= ____________________

= ____________________

29. $3 \cdot 4^2 + 7 - 2 \cdot 3^2 \div 6 + 3 \cdot 5^2$

= ______________________

= ______________________

= ______________________

= ______________________

= ______________________

30. $24 - 12 \div 2 \cdot 3 + 6 \cdot 2^3$

= ______________________

= ______________________

= ______________________

= ______________________

= ______________________

31. $(5 \cdot 2)^2 - 20 \div 2 \cdot (8 - 3) + 10 - 3 + 7$

32. $35 - 20 \div 5 + 7^2 \cdot 2 - 6 \cdot 3 + 9 + 10 \div 2$

33. $\dfrac{(11 - 5)^2 + 3 \cdot 2^2}{(5 - 3)^2 + 4(7 - 2)}$

34. $\dfrac{(11 - 5)^2 - 3 \cdot 2^2}{(5 - 3)^2 + 4(7 - 2)}$

35. $\dfrac{3(5 + 3) + 3(9 - 5)}{6 \cdot 2^2 - 2 \cdot 3^2} - \dfrac{2 \cdot 3^3 - 2 \cdot 5^2}{4^2 - (7 + 5)}$

36. $\dfrac{(6 \div 2 \cdot 3)^2 + 2 \cdot 3^2}{(5 + 2)^2 - 4 \cdot 2^2} + \dfrac{(20 + 5) \cdot 2^2}{(20 - 5 \cdot 2)^2}$

SIGNED NUMBERS

Frequently in algebra you are required to simplify expressions with negative as well as positive numbers. When adding numbers it is best to think in terms of MONEY **$$$$$**! A positive number is like money coming <u>in</u> to your possession, or income; a negative number is like money going <u>out</u>, like expenditures or debts. When writing a negative number, it is helpful to write the negative number in parentheses.

For example, 8 + (-5) means you have 8 dollars in your possession and you spend 5 dollars. The result is +3 or 3, which means you have 3 dollars.

What if the larger "amount" is negative? For example, 5+(-12) means you have 5 dollars and spend 12 dollars. The result is (-7) or a debt of 7 dollars.

What if both "amounts" are negative? If you have two debts, like (-7) + (-12), the result is (-19) or a total debt of 19 dollars.

In summary, it is obvious that when you add positive numbers, you get a positive number. When you add positive and negative numbers, you subtract the numbers and the sign of the answer is the same as the sign of the larger "magnitude." When you add negative numbers, you always add the numbers and the sign is always negative.

ADDITION RULES

RULE	SIGN OF ANSWER	WHAT TO DO	EXAMPLES
(+)+(+)	+	Add the numbers	(+8)+(+4) = +12
(-)+(-)	-	Add the numbers	(-8)+(-4) = -12
(+)+(-)	Sign of larger "amount"	Subtract the nos	(+12)+(-8) = +4
(-)+(+)	Sign of larger "amount"	Subtract the nos	(-12)+(+8) = -4

EXERCISES:

1. $(-12) + 20 =$ ____ **2.** $(-20) + 12 =$ ____ **3.** $(-12) + (-8) =$ ____

4. $16 + (-4) =$ ____ **5.** $(-34) + (-16) =$ ____ **6.** $(-14) + (-28) =$ ____

7. $(-125) + 28 =$ ____ **8.** $(-52) + 120 =$ ____ **9.** $(-12) + (-87) =$ ____

10. $(-12) + 34 + (-26) =$ ______ **11.** $(-28) + (-125) + 95 =$ ______

12. $200 + (-120) + (-85) =$ ______ **13.** $(-200) + (-135) + 75 =$ ______

When subtracting negative numbers, remember that the negative of a negative is the positive of the number. For examples, $-(-8)$ is a $(+8)$ or just 8; $-(-10)$ is $+10$ or 10; $-(-x)$ is x; etc. Of course, the negative of a positive is a negative. As examples, $-(+8)$ is (-8); $-(+10)$ is -10; etc.

SUBTRACTION RULES

$$-(-X) = +X$$

$$-(+X) = -X$$

EXERCISES:

14. $-(-12) =$ _____ **15.** $-(-18) =$ _____ **16.** $-(-120) =$ _______

17. $8 - (-12) = 8 +$ ___
$=$ ________

18. $-8 - (-12) = -8 +$ ____
$=$ ________

19. $12 - (-8) =$ ________
$=$ ________

20. $-12 - (-8) =$ ________
$=$ ________

21. $(-38) - (-12) =$ ________
$=$ ________

22. $38 - (-12) =$ ________
$=$ ________

When multiplying or dividing positive and negative numbers, remember that multiplying is actually a short way to add. For example, 4 x 3 means 4 threes. In this way 4 x (-3) means 4 three dollar debts or (-12). Likewise, (-3) x 4 is also (-12). In the same way that a **negative of a negative** is a positive, a **negative times a negative** is a positive. The following are rules for multiplication and division of signed numbers. Remember also, that the word **"of"** means **"times"**.

RULES FOR MULTIPLICATION AND DIVISION

RULE	SIGN OF ANSWER	RULE	SIGN OF ANSWER
(+)X(+)	+	(+) ÷ (+)	+
(+)X(-)	−	(+) ÷ (-)	−
(-)X(+)	−	(-) ÷ (+)	−
(-)X(-)	+	(-) ÷ (-)	+

EXERCISES:

23. $7(-8) =$ ______ 24. $(-6)(-9) =$ ______ 25. $(-23)\cdot 4 =$ ______

26. $(-17)(-3) =$ ______ 27. $(-16)(9) =$ ______ 28. $(-5)\cdot(-24) =$ ______

29. $7(-8)(-2) =$ ______ 30. $5(-9)(2) =$ ______ 31. $(-2)(-4)(-25) =$ ______

32. $(-7)(-8)(-2) =$ ______ 33. $5(-9)(-2) =$ ______ 34. $(-14)(-4)(-25) =$ ______

When raising to a power, remember that a positive number raised to any power is automatically positive. A negative number raised to an even power is always positive, and a negative number to an odd power is negative.

POWER RULES

$(\text{POSITIVE})^{\text{ANY POWER}}$ = POSITIVE

$(\text{NEGATIVE})^{\text{EVEN POWER}}$ = POSITIVE

$(\text{NEGATIVE})^{\text{ODD POWER}}$ = NEGATIVE

EXERCISES:

35. $(-2)^2 =$ _____ 36. $(-5)^2 =$ _____ 37. $(-4)^2 =$ _____

38. $(-5)^3 =$ _____ 39. $(-3)^3 =$ _____ 40. $(-4)^3 =$ _____

41. $(-1)^{12} =$ _____ 42. $(-1)^9 =$ _____ 43. $(-1)^{15} =$ _____

44. $(-1)^{24} =$ _____ 45. $(-10)^3 =$ _____ 46. $(-3)^4 =$ _____

47. $(-2)^3(-1) =$ _____ 48. $(-3)^4\ (-1)^8 =$ _____ 49. $(-2)^4\ (-1)^3 =$ _____

50. $(-2)^5\ (-1)^2 =$ _____ 51. $(-5)^2\ (-2)^3 =$ _____ 52. $(-3)^2\ (-2)^3 =$ _____

53. $(-2)^2\ (-3)^3 =$ _____ 54. $(-2)^3\ (-3)^3 =$ _____ 55. $(-3)^2\ (-2)^2 =$ _____

However, be careful when the negative number does not have parentheses. Is there a difference between $(-2)^2$ and -2^2 ? The quantity $(-2)^2$ means (-2) times (-2), which is +4. However, -2^2 means the negative of (two to the second power). By order of operations agreement, this means to raise two to the second power, and then take the negative. This result is -4. The important question to ask is this: "What is it that you are raising to the power?" In the case of $(-2)^2$, you are raising (-2) to the second power. However, with -2^2 only the 2 is squared, not the "-". Therefore,

$$(-2)^2 = 4, \text{ but } -2^2 = -4$$

Also notice that in (-2^2), only the 2 (<u>not</u> the negative) is squared. Therefore $(-2^2) = -4$. Notice that $(-2)^2 \neq (-2^2)$.

Complete the following exercises:

56. $-2^4 =$ ____ 57. $(-2)^4 =$ ____ 58. $(-5)^2 =$ ____

59. $-5^2 =$ ____ 60. $(-2)^3 =$ ____ 61. $-2^3 =$ ____

62. $-3^4 =$ ____ 63. $(-3)^4 =$ ____ 64. $(-5)^3 =$ ____

65. $-5^3 =$ ____ 66. $-1^{10} =$ ____ 67. $-1^{13} =$ ____

68. $-2^2\ (-3)^3 =$ ____ 69. $-2^3\ (-3)^2 =$ ____ 70. $(-3)^3\ (-2^2) =$ ____

71. $(-2^2)(-3^3) =$ ____ 72. $(-2^3)(-3^2) =$ ____ 73. $(-3^3)(-2^2) =$ ____

74. $(-1)^2\ (-2^3) =$ ____ 75. $(-2^3)(-1^2) =$ ____ 76. $(-1^3)(-2^2) =$ ____

ABSOLUTE VALUE

The **absolute value** of a number, denoted by vertical bars on each side of a number or a quantity, represents the **size** or **magnitude** of that number. Another way to think of absolute value of a number is the distance on the number line of that number from zero. As examples, $|-3|$ is 3; $|-7|$ is 7; $|10|$ is 10; $|0|$ is 0. Notice that the absolute value definition does not apply to what is outside the absolute value bars. For example, $-|3|$ is -3; $-|-3|$ is -3; $-|7|$ is -7; $-|-7|$ is -7.

EXERCISES:

77. $|-4| + 3|-3|$

78. $|-5| - 3|-3|$

79. $-|-8| + 3|-9|$

80. $-3|-5| - 5|-6|$

81. $-4|4-6| - 8|-8+3|$

82. $9|-7+1| - 3|5-9|$

83. $|-11| - |-5|^2$

84. $|-5 - 3|^2 - 4|7 - 2|$

85. $|-8 - 5|^2 + |3 - 12|^2$

86. $|-8^2 - 5| + |3^2 - 12|$

87. $|-8^2 + 5| - |3^2 - 12|$

88. $-|-5^2 - 3^2|$

89. $\dfrac{|-8^2 - 5| + |3^2 - 12|}{|8 - (-4)^2|}$

90. $\dfrac{|(-8)^2 + 5| - |3^2 - 12|}{-(-4^2 + 7^2)}$

91. $\dfrac{|-8^2 - 5| + |-6^2 - 5^2|}{-|(-4)^2 + 7^2|}$

92. $\dfrac{|-8^2 + 4| - |2^5 - 12|}{|-4 - (-4)^2|}$

ANSWERS 1.02

p.15-19: **1.** 34; **2.** 25; **3.** 50; **4.** 64; **5.** 4; **6.** 100; **7.** 16; **8.** 2; **9.** 23; **10.** 400; **11.** 7; **12.** 28; **13.** 20; **14.** 13; **15.** 12; **16.** 30; **17.** 5; **18.** 18; **19.** 20; **20.** 52; **21.** 15; **22.** 20; **23.** 54; **24.** 30; **25.** 20; **26.** 21; **27.** 38; **28.** 81; **29.** 127; **30.** 54; **31.** 64; **32.** 125; **33.** 2; **34.** 1; **35.** 5; **36.** 4.

p.21-26: **1.** 8; **2.** -8; **3.** -20; **4.** 12; **5.** -50; **6.** -42; **7.** -97; **8.** 68; **9.** -99; **10.** -4; **11.** -58; **12.** -5; **13.** -260; **14.** 12; **15.** 18; **16.** 120; **17.** 20; **18.** 4; **19.** 20; **20.** -4; **21.** -26; **22.** 50; **23.** -56; **24.** 54; **25.** -92; **26.** 51; **27.** -144; **28.** 120; **29.** 112; **30.** -90; **31.** -200; **32.** -112; **33.** 90; **34.** -1400; **35.** 4; **36.** 25; **37.** 16; **38.** -125; **39.** -27; **40.** -64; **41.** 1; **42.** -1; **43.** -1; **44.** 1; **45.** -1000; **46.** 81; **47.** 8; **48.** 81; **49.** -16; **50.** -32; **51.** -200; **52.** -72; **53.** -108; **54.** 216; **55.** 36; **56.** -16; **57.** 16; **58.** 25; **59.** -25; **60.** -8; **61.** -8; **62.** -81; **63.** 81; **64.** -125; **65.** -125; **66.** -1; **67.** -1; **68.** 108; **69.** -72; **70.** 108; **71.** 108; **72.** 72; **73.** 108; **74.** -8; **75.** 8; **76.** 4; **77.** 13; **78.** -4; **79.** 19; **80.** -45 **81.** -48: **82.** 42; **83.** -14; **84.** 44; **85.** 250; **86.** 72; **87.** 56; **88.** -34; **89.** 9; **90.** -2; **91.** -2; **92.** 2.