1.06 *Linear Inequalities and Properties of Inequalities*

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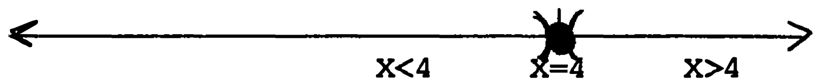
In describing equations and inequalities, the **Trichotomy Axiom** and **interval notation** will be helpful. Consider the variable X and any number, for example, 4. As the word "tri" means "three," according to the Trichotomy Axiom, there are three ways to compare the variable X to the number 4: **X=4**, **X<4**, **or** **X>4**.

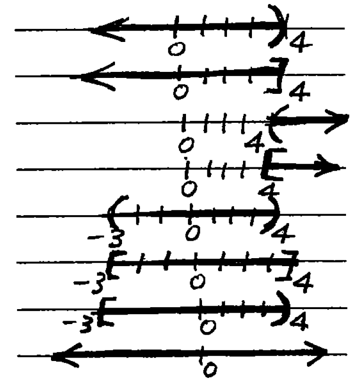


The first category **X=4** consists of just one point. The second category **X<4** represents an entire interval of numbers on the numberline to the left of the number 4. Likewise, the third category **X>4** represents an entire interval of numbers on the numberline to the right of the number 4. Notice in each of these cases, the endpoint 4 is not included. If the endpoint 4 is to be included, then you write **X≤4** or **X≥4** respectively. **Remember that X < 4 is equivalent to 4 > X**, and that **X≤4 is equivalent to 4≥X.**

Suppose the variable X represents all values between -3 and 4. In this case, X must be greater than -3 and at the same time less than 4. Another way to write this is -3 is less than X and at the same time X is less than 4. In math symbols, this is **-3<X and X<4.** This can be written **-3<X<4**, which in English just means that **"X is between -3 and 4, not including the endpoints."**

Since in each of these **intervals** there are infinitely many numbers, it is helpful to describe the interval with **interval notation** (illustrated on the next page!). Interval notation is always given from **left to right**, with brackets **"["** or **"]"** indicating **included endpoints**, and parentheses **"("** or **")"** indicating that **endpoints are not included.** If the interval extends all the way to the right (infinity) or to the left (negative infinity), the symbols "∞" or "-∞" are used. The tradition has always been to consider -∞ and ∞ non-inclusive, since infinity is not something that can be "contained" or "included."

Variable Notation	Interval Notation	Graph on Numberline
$X < 4$	$(-\infty, 4)$	
$X \le 4$	$(-\infty, 4]$	
$X > 4$	$(4, \infty)$	
$X \ge 4$	$[4, \infty)$	
$-3 < X < 4$	$(-3, 4)$	
$-3 \le X \le 4$	$[-3, 4]$	
$-3 \le X < 4$	$[-3, 4)$	
All Reals	$(-\infty, \infty)$	



Realize that inequalities, like equations, must often be solved. Remember that when you solved equations, according to the **addition rule for equations,** you were allowed to add (or subtract) the same number from both sides of an equation. Also, according to the **multiplication rule for equations,** you were allowed to multiply (or divide) both sides of an equation by the same number. Wouldn't it be nice if the same rules applied to inequalities as well as equations? Well, it is <u>almost</u> that simple, but not quite. It will be necessary to modify the multiplication rule slightly.

Take the inequality $-2 < 4$, and add $+2$ to each side

$$\underline{+2 \quad +2}$$

$$0 < 6 \quad \text{Still true!}$$

Use the same inequality $-2 < 4$, and add -2 to each side

$$\underline{-2 \quad -2}$$

$$-4 < 2 \quad \text{Still true!}$$

Use the same inequality and multiply (or divide) both sides by $+2$.

$$-2 < 4$$

$$+2(-2) < +2(4)$$

$$-4 < 8 \quad \text{Still true!}$$

Now try multiplying (or dividing) both sides by a negative, say -2.

$$-2 < 4$$

$$-2(-2) < -2(4)$$

$$\mathbf{+4 > -8} \quad \textbf{INEQUALITY MUST BE REVERSED!}$$

These examples verify (but do **not prove!**) the following rules:

RULES FOR INEQUALITIES

1. If any number is added or subtracted from both sides of an inequality, then the inequality sign remains the same.

2. If both sides of an inequality are multiplied or divided by a positive number, then the inequality sign remains the same.

3. If both sides of an inequality are multiplied or divided by a negative number, then the inequality sign must be reversed.

In summary, remember that these are the same rules as the ones used to solve equations, except that **when you multiply or divide both sides of an inequality by a negative number, you must change the direction of the inequality sign.** Remember also that $X>4$ means the same as $4<X$. The expression $-2<X<6$ means that X represents any number between -2 and 6, and the expression $6>X>-2$ means the same as $-2<X<6$. However, the notation $-2<X<6$, from smallest to largest (left to right), is preferred.

In the following examples, solve the inequalities. Give answers in interval notation:

EXAMPLE 1

$$\begin{array}{rcr} 7X + 12 & \geq & 5X + 4 \\ \underline{-5X \qquad} & & \underline{-5X \qquad} \\ 2X + 12 & \geq & + 4 \\ \underline{\quad - 12} & & \underline{-12} \\ 2X & \geq & -8 \end{array}$$

$X \geq -4$ **(DO NOT REVERSE!)**

[-4, ∞)

EXAMPLE 2

$$\begin{array}{rcr} 5X - 12 & > & 7X + 4 \\ \underline{-7X \qquad} & & \underline{-7X \qquad} \\ -2X - 12 & > & + 4 \\ \underline{\quad + 12} & & \underline{+12} \\ -2X & > & 16 \end{array}$$

$X < -8$ **(REVERSE SIGN!)**

(-∞, -8)

EXERCISES: Solve for X. Give answers in interval notation.

1. $5 + 3X \leq X - 3$

2. $- 3X + 4 > 4X - 3$

3. $5 - 3X < X - 3$

4. $- 3X + 15 \geq 3X - 3$

Also, you should be aware that inequalties, like equations, can be **conditional, identities, or contradictions.**

EXAMPLE 3

$$\begin{array}{rcl} 5X + 12 & \geq & 5X + 20 \\ \underline{-5X} & & \underline{-5X} \\ 12 & \geq & 20 \end{array}$$

Never true
NO SOLUTION
or **EMPTY SET**

EXAMPLE 4

$$\begin{array}{rcl} 5X - 12 & < & 5X + 20 \\ \underline{-5X} & & \underline{-5X} \\ - 12 & < & + 20 \end{array}$$

True for all X
$(-\infty, \infty)$

EXERCISES: Solve for X. Give answers in interval notation.

5. $5 - 3(X - 4) \leq 2(X - 4)$

6. $- 3(X + 4) > 4(2X + 3) - 5X$

7. $2X - 8(7 - X) > 5(2X - 4)$

8. $-8 - 3(X - 4) < 3(6 - X)$

9. $2X - 8 < 2(2 - 5X) + 12X$

10. $-2(2 + 3X) \geq 3(5 - X) + 8$

EXAMPLE 5

$$\begin{array}{rcrcr} -5 & \leq & 2X + 3 & \leq & 7 \\ \underline{-3} & & \underline{-3} & & \underline{-3} \\ -8 & \leq & 2X & \leq & 4 \\ -4 & \leq & X & \leq & 2 \end{array}$$

[-4, 2]

EXAMPLE 6

$$\begin{array}{rcrcr} -1 & < & 3 - 2X & \leq & 5 \\ \underline{-3} & & \underline{-3} & & \underline{-3} \\ -4 & < & -2X & \leq & 2 \end{array}$$

$$2 > X \geq -1$$

[-1, 2)

EXAMPLE 7

$$-5 \leq \frac{2X + 3}{3} \leq 7$$

$$\begin{array}{rcrcr} -15 & \leq & 2X + 3 & \leq & 21 \\ -18 & \leq & 2X & \leq & 18 \\ -9 & \leq & X & \leq & 9 \end{array}$$

[-9, 9]

EXAMPLE 8

$$-1 < \frac{3 - 2X}{3} \leq 5$$

$$\begin{array}{rcrcr} -3 & < & 3 - 2X & \leq & 15 \\ -6 & < & -2X & \leq & 12 \end{array}$$

$$3 > X \geq -6$$

[-6, 3)

11. $-7 \leq 3 - 2X < 5$

12. $-5 < 2X + 3 < -1$

13. $-7 \leq \dfrac{3 - 2X}{3} < 5$

14. $-5 < \dfrac{2X + 3}{3} < -1$

15. $-3 \leq \dfrac{-3X + 6}{2} \leq 6$

16. $-9 < \dfrac{3 - 3X}{2} \leq 6$

FORMAL EXPLANATION

As an introduction to a more formal explanation of inequalities, consider again the properties of equations from section 1.04. Which, if any, of these properties of equations apply also to inequalities?

ADDITION PROPERTY FOR EQUATIONS

If $a=b$, then $a + c = b + c$
If $a=b$, then $a - c = b - c$.

The same number may be added (or subtracted) from both sides of an equation.

I. ADDITION PROPERTY FOR INEQUALITIES

If $a < b$, then $a + c < b + c$
If $a < b$, then $a - c < b - c$
If $a > b$, then $a + c > b + c$
If $a > b$, then $a - c > b - c$.

The addition property for inequalities is basically the same as it is for equations--the same number may be added (or subtracted) from both sides of an inequality, and the inequality remains the same. Of course, this property is also valid for "$\leq$" and "$\geq$."

MULTIPLICATION PROPERTY FOR EQUATIONS If $a=b$, then $ac = bc$
If $a=b$ and $c\neq 0$, then $a/c = b/c$.

Both sides of an equation may be multiplied or divided by the same non-zero number.

II. MULTIPLICATION PROPERTY FOR INEQUALITIES must be given in two distinct parts:

A. **If $a < b$ and $c > 0$, then $ac < bc$.**
If $a > b$ and $c > 0$, then $ac > bc$.
If $a \leq b$ and $c > 0$, then $ac \leq bc$.
If $a \geq b$ and $c > 0$, then $ac \geq bc$.

This means that if both sides of an inequality are multiplied (or divided) by a <u>positive</u> number, then the inequality sign remains the same.

B. **If $a < b$ and $c < 0$, then $ac > bc$.**
If $a > b$ and $c < 0$, then $ac < bc$.
If $a \leq b$ and $c < 0$, then $ac \geq bc$.
If $a \geq b$ and $c < 0$, then $ac \leq bc$.

This means that if both sides of an inequality are multiplied (or divided) by a <u>negative</u> number, then the inequality sign must be reversed.

REFLEXIVE PROPERTY FOR EQUATIONS: $a = a$. Any number is equal to itself. **Clearly, a number is <u>not</u> less than or greater than itself. Therefore, there is NO reflexive property for inequalities.**

SYMMETRIC PROPERTY FOR EQUATIONS: If $a = b$, then $b = a$. The order in which the equality is given does not matter. For example, you can say "X=4" or "4=X", the meaning is the same--the value of X is 4. **Clearly, if $a < b$, then it is <u>not</u> true that $b < a$. There is NO symmetric property for inequalities.**

TRANSITIVE PROPERTY FOR EQUATIONS: If $a = b$ and $b = c$, then $a = c$. The word "trans" means "across." If you can get from point "a" to "b", and then from "b" to "c", then you can get from "a" <u>across</u> "b" to "c."

III. TRANSITIVE PROPERTY FOR INEQUALITIES.

If $a < b$ and $b < c$, then $a < c$.
If $a > b$ and $b > c$, then $a > c$.

The transitive property is also valid for "≤" and "≥."

EXERCISES: In each of the following exercises, fill in the blank with the name of the property.

1. **If $X + 3 = 5$, then $X = 2$** ______________________
   BECAUSE YOU ADDED -3 TO BOTH SIDES OF THE EQUATION.

2. **If $X - 3 = 5$, then $X = 8$** ______________________
   BECAUSE YOU ADDED 3 TO BOTH SIDES OF THE EQUATION.

3. **If $X + 3 < 5$, then $X < 2$** ______________________
   BECAUSE YOU ADDED -3 TO BOTH SIDES OF THE INEQUALITY.

4. **If $X + 3 > 5$, then $X > 2$** ______________________
   BECAUSE YOU ADDED -3 TO BOTH SIDES OF THE INEQUALITY.

5. **If $7X = 21$, then $X = 3$** ______________________
   BECAUSE YOU MULTIPLIED BOTH SIDES OF THE EQUATION BY 1/7.

6. **If $7X > 21$, then $X > 3$** ______________________
   BECAUSE YOU MULTIPLIED BOTH SIDES OF THE INEQUALITY BY 1/7.

7. If $7X < 21$, then $X < 3$ ______________________
BECAUSE YOU MULTIPLIED BOTH SIDES OF THE INEQUALITTY BY 1/7.

8. If $-7X > 21$, then $X <-3$ ______________________
BECAUSE YOU MULTIPLIED BOTH SIDES OF THE INEQUALITY BY -1/7.

9. If $-7X = 21$, then $X =-3$ ______________________
BECAUSE YOU MULTIPLIED BOTH SIDES OF THE EQUATION BY -1/7.

10. If $-7X < 21$, then $X >-3$ ______________________
BECAUSE YOU MULTIPLIED BOTH SIDES OF THE INEQUALITY BY -1/7.

11. If $X = 3$ and $3 = Y$, then $X = Y$ __________ PROPERTY OF__________.

12. If $5 > 3$ and $3 > 1$, then $5 > 1$ __________ PROPERTY OF__________.

13. If $-3< 3$ and $3 < 6$, then $-3< 6$ __________ PROPERTY OF__________.

14. If $X < 3$ and $3 < Y$, then $X < Y$ __________ PROPERTY OF__________.

15. If $4X > 20$, then $X > 5$ ______________________

16. If $4X > 20$ and $20 > 16$, then $4X>16$ ______________________

17. If $X + 4 < 12$, then $X < 8$ ______________________

18. If $X + 4 = 12$, then $X = 8$ ______________________

19. If $4X = 20$, then $X = 5$ ______________________

20. If $4X = 20$ and $20 = 2Y$, then $4X=2Y$ ______________________

21. If $-4X> 20$, then $X < -5$ ______________________

22. If $X > -5$, then $4X > -20$ ______________________

23. If $X + 4 = 12$, then $2X + 8 = 24$ ______________________

24. If $X = 12$, then $X + 8 = 20$ ______________________

25. If $X = 12$, then $12 = X$ ______________________

26. $6 = 6$ ______________________

27. If $12 < X$ and $X < 20$, then $12<20$ ______________________

28. If $-X < 12$, then $X > -12$ ______________________

29. If $3X + 12 > 27$, then $3X > 15$ ______________________

30. If $3X > 15$, then $X > 5$ ______________________

COMPOUND INEQUALITIES

Before introducing the idea of compound inequalities (two inequalities connected with the by the words **"and"** or **"or"**) it will be helpful to make use of **set notation**, and define the words **"and"** and **"or"**.

$A \cap B$ **(A "intersection" B) is the set containing all members that are common to <u>both</u> set A <u>and</u> set B. This consists of the <u>overlap</u> of the sets A and B. It is like the "intersection" of two streets or highways--where the two streets come together and overlap.**

$A \cup B$ **(A "union" B) is the set containing all members that are in either set A <u>or</u> set B <u>or</u> both. The union consists of everything included in either set. In terms of streets or highways, the union includes both entire streets.**

Suppose you were given two sets of numbers as follows:

$A = \{2,4,6,8\}$ and $B = \{1,4,6,8\}$

$A \cap B = \{4,6,8\}$ (Include only the overlap of A <u>and</u> B!)

$A \cup B = \{1,2,4,6,8\}$ (Include <u>everything</u> that is in A <u>or</u> B <u>or</u> in both!)

Frequently two simple statements (equations or inequalities) are connected with the words **"and"** or **"or"** to form a **compound statement.** When the word **"and"** is used, the statement is called a **conjunction**, and the solution is the **intersection** of the two solution sets. When the word **"or"** is used, the statement is called a **disjunction**, and the solution is the **union** of the two solution sets.

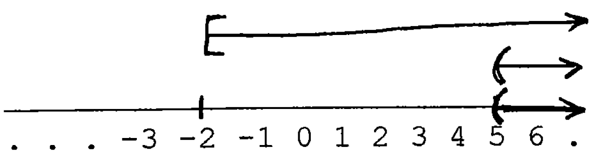
SUMMARY

A and B = $A \cap B$	Intersection Overlap Common to both
A or B = $A \cup B$	A or B or both Includes everything Either/Or

In each of the following examples and exercises, you will be asked to solve **compound inequalities** with the key words **"and"** or **"or"**. The answers of course will be graphed on a numberline and given in **interval notation.** The secret to success in this (learned after more than 20 or 25 years of experience!) is this: Draw a numberline, but then do the problem <u>ABOVE</u> THE NUMBERLINE (NOT <u>ON</u> THE NUMBERLINE--SAVE THE NUMBERLINE FOR YOUR FINAL ANSWER! The following examples illustrate this. Remember, do the problem in the space provided <u>above</u> the numberline, then bring the **intersection** or **union** down <u>on</u> the numberline, and finally give the answer in **interval notation.**

EXAMPLE 1

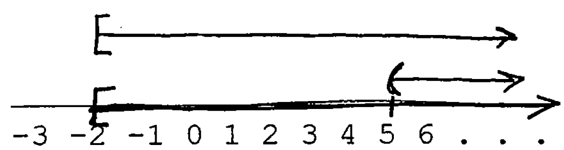
$X > 5$ and $X \geq -2$



ANSWER: $(5, \infty)$

EXAMPLE 2

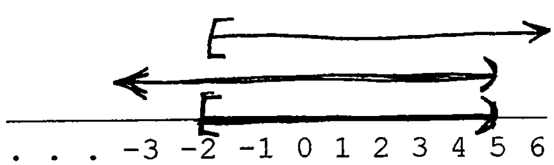
$X > 5$ or $X \geq -2$



ANSWER: $[-2, \infty)$

EXAMPLE 3

$X < 5$ and $X \geq -2$

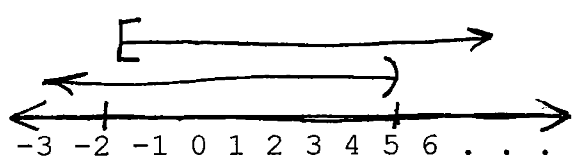


ANSWER: $[-2, 5)$

This is called **"BETWEENNESS."**

EXAMPLE 4

$X < 5$ or $X \geq -2$

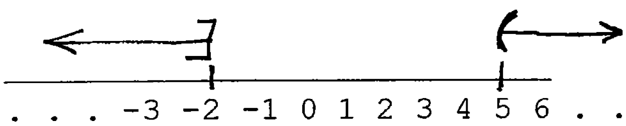


ANSWER: $(-\infty, \infty)$

This may also be written **"All Real Numbers."**

EXAMPLE 5

X > 5 and X ≤ -2

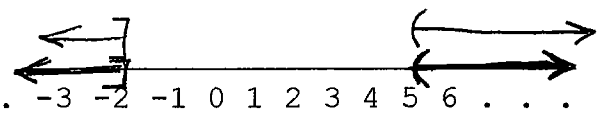


ANSWER: No Solution

Empty Set { }

EXAMPLE 6

X > 5 or X ≤ -2



ANSWER: Two separate intervals

$(-\infty, -2] \cup (5, \infty)$

This is called "**EXTREMES.**"

EXERCISES: Solve the compound inequalities and give interval notation.

1. X < 4 and X ≤ -2

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

2. X < 4 or X ≤ -2

. -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

3. X > 4 and X ≤ -2

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

4. X > 4 or X ≤ -2

. -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

5. $X < 4$ or $X \geq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

6. $X < 4$ and $X \geq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

7. $X \geq 4$ or $X \geq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

8. $X \geq 4$ and $X \geq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

9. $X < -4$ or $X \geq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

10. $X < -4$ and $X \geq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

11. $X > -5$ and $X \leq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

12. $X > -5$ or $X \leq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

13. $-4X < 4$ or $X - 7 \leq -2$

14. $-4X < 4$ and $X - 7 \leq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

-3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

15. $-4X \geq 4$ or $-X \geq -2$

16. $-4X \geq 4$ and $-X \leq -2$

. . . -3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

-3 -2 -1 0 1 2 3 4 5 6 . . .

ANSWER:

17. $-2X-6 < -4$ or $3X-6 \geq -6$

18. $-2X < -4$ and $3X-6 \geq -6$

19. $X-3 > -5$ and $-3X \geq -6$

20. $X-3 < -5$ or $-3X-6 \leq -6$

1.07 *Laws of Exponents*

We will begin with a summary of the laws of exponents. You are probably familiar with these laws from your previous algebra background.

	LAWS OF EXPONENTS	GENERALIZATION
1.	When you multiply (with the same base number), you add exponents.	$X^m \cdot X^n = X^{(m+n)}$
2.	When you divide (with the same base number), you subtract exponents.	$\frac{X^m}{X^n} = X^{(m-n)}$
3.	When you raise a power to a power, you multiply exponents.	$(X^m)^n = X^{mn}$
4.	When a product or a quotient is raised to a power, you raise each factor to the power.	$(XY)^m = X^m \cdot Y^m$ $\left(\frac{X}{Y}\right)^m = \frac{X^m}{Y^m}$
5.	Any non-zero number raised to the zero power is 1.	$X^0 = 1$
6.	Any number raised to a negative power is 1 divided by that number raised to the positive power.	$X^{-n} = \frac{1}{X^n}$
7.	One (1) divided by any number raised to a negative power is that number raised to the positive power.	$\frac{1}{X^{-n}} = X^n$
8.	A fraction raised to a negative power is the reciprocal of the fraction raised to the positive power.	$\left(\frac{X}{Y}\right)^{-n} = \left(\frac{Y}{X}\right)^n$

"QUICKIES"

Simplify each of the following. Express without negative exponents.

1. $X^4 \cdot X^7 =$ ______ 2. $\dfrac{X^8}{X^2} =$ ______ 3. $(X^4)^7 =$ ______

4. $\left(X^3\right)^0 =$ ______ 5. $\dfrac{X^{10}}{X^5} =$ ______ 6. $X^4 \cdot X^0 =$ ______

7. $2^4 \cdot 2^6 =$ ______ 8. $\left(2^3\right)^6 =$ ______ 9. $\dfrac{2^{10}}{2^5} =$ ______

10. $\dfrac{X^3}{X^{-2}} =$ ______ 11. $\dfrac{1}{X^{-3}} =$ ______ 12. $\left(\dfrac{X^2}{Y^3}\right)^4 =$ ______

13. $\left(X^3\ Y^4\right)^3 =$ ______ 14. $\left(\dfrac{X^4\ Y^2}{Z^5}\right)^2 =$ ______ 15. $\left(\dfrac{X^2}{Y^3}\right)^0 =$ ______

16. $X^{-3} =$ ______ 17. $Y^{-5} =$ ______ 18. $2^{-3} =$ ______

19. $3^{-2} =$ ______ 20. $3X^0 =$ ______ 21. $(3X)^0 =$ ______

22. $(3X)^{-1} =$ ______ 23. $3X^{-1} =$ ______ 24. $(3X)^{-2} =$ ______

25. $(3X)^{-4} =$ ______ 26. $3X^{-3} =$ ______ 27. $(3X^{-1})^{-2} =$ ______

"TWO-STEP"

28. $(X^4 \cdot X^3)^3 =$ ______

$=$ ______

29. $(X^6 \cdot X^{-2})^3 =$ ______

$=$ ______

30. $(X^5 \cdot X^{-2})^6 =$ ______

$=$ ______

31. $\left(\dfrac{X^8}{X^2}\right)^5 =$ ______

$=$ ______

32. $\left(\dfrac{X^4}{X^{-2}}\right)^7 =$ ______

$=$ ______

33. $\dfrac{X^4 \cdot X^{10}}{X^{-6}} =$ ______

$=$ ______

34. $\dfrac{X^{-4} \cdot X^{10}}{X^{-6}} =$ ______

$=$ ______

35. $\dfrac{X^4 \cdot X^{-10}}{X^{-6}} =$ ______

$=$ ______

$=$ ______

36. $\dfrac{X^{-4} \cdot X^{-10}}{X^{-6}} =$ ______

$=$ ______

$=$ ______

"WATCH YOUR STEP!"

37. $(2X^3)^4 \cdot (X^4Y^{-3})^2$

$=$ ______

$=$ ______

$=$ ______

$=$ ______

38. $(3X^3Y^{-2})^2 \cdot (2X^{-4}Y^5)^2$

$=$ ______

$=$ ______

$=$ ______

$=$ ______

39. $(3^{-1}X^{3}Y^{-2})^{-2} \cdot (2X^{-4}Y^{5})^{-2}$

= ______________________

= ______________________

= ______________________

= ______________________

40. $(3X^{-3}Y^{2})^{2} \cdot (2^{-1}X^{4}Y^{-5})^{-2}$

= ______________________

= ______________________

= ______________________

= ______________________

41. $\dfrac{X^{-11}\,(X^{-2})^{4}}{(X^{2})^{-6}}$

= ______________________

= ______________________

= ______________________

= ______________________

42. $\dfrac{2^{-7} \cdot X^{4}}{2^{-9} \cdot X^{-4}}$

= ______________________

= ______________________

= ______________________

= ______________________

43. $\dfrac{(3X^{-3}Y^{2})^{-2}}{(3^{-1}X^{4}Y^{-5})^{-2}}$

= ______________________

= ______________________

= ______________________

= ______________________

44. $\dfrac{(3^{-1}X^{4}Y^{-5})^{-2}}{(3X^{-3}Y^{2})^{-2}}$

= ______________________

= ______________________

= ______________________

= ______________________

In the next exercises remember, the rules are exactly the same--but the level of abstraction has increased. Now there are variables in the exponents, which give the exercises a slightly different (more complicated) appearance. Just know and obey the laws of exponents, and combine like terms when possible (in the exponents).

EXAMPLE 1: $\frac{X^{3a}\,X^{2b}}{X^{3c}}$

$= \frac{X^{3a\,+\,2b}}{X^{3c}}$ ADD EXPONENTS!

$= X^{3a\,+\,2b\,-\,3c}$ SUBTRACT EXPONENTS!

EXAMPLE 2: $\frac{X^{3p}\,X^{2p-4}}{X^{6-3p}}$

$= \frac{X^{3p\,+\,2p-4}}{X^{6-3p}}$ ADD EXPONENTS!

$= X^{5p-4\,-\,(6-3p)}$ SUBTRACT EXPONENTS!

$= X^{5p-4-6+3p}$

$= X^{8p-10}$

45. $2^{3X} \cdot 2^{2Y}$ = ____________

46. $\frac{2^{3X}}{2^{2Y}}$ = ____________

47. $2^{X-4} \cdot 2^{X+6}$ = ____________

48. $\frac{2^{3X}}{2^{X+4}}$ = ____________

49. $\frac{2^X\,2^Y}{2^Z}$ = ____________

= ____________

50. $\frac{2^X}{2^Y\,2^Z}$ = ____________

= ____________

51. $\frac{X^{3p+2}\,X^{4p-6}}{X^{2p+4}}$ = ____________

= ____________

= ____________

= ____________

52. $\frac{Y^{2q-5}\,Y^{6-3q}}{Y^{2-4q}}$ = ____________

= ____________

= ____________

= ____________

ANSWERS 1.06

p. 80-82:

1. $(-\infty, -4]$; 2. $(-\infty, 1)$; 3. $(2, \infty)$; 4. $(-\infty, 3]$; 5. $[5,\infty)$;
6. $(-\infty, -4)$; 7. ϕ; 8. $(-\infty,\infty)$; 9. $(-\infty,\infty)$; 10. $(-\infty,-9]$;
11. $(-1, 5]$; 12. $(-4,-2)$; 13. $(-6, 12]$; 14. $(-9,-3)$;
15. $[-2,4]$; 16. $[-3,7)$.

p. 84-85:

1. Add prop for eq; 2. Add prop for eq; 3. Add prop for ineq;
4. Add prop for ineq; 5. Mult prop for eq; 6. Mult prop for ineq; 7. Mult ineq by positive; 8. Mult ineq by negative;
9. Mult prop for eq; 10. Mult ineq by negative;
11. Transitive, equations; 12. Transitive, inequalities;
13. Transitive, inequalities; 14. Transitive, inequalities;
15. Mult ineq by positive; 16. Transitive, inequalities;
17. Addition prop for ineq; 18. Addition prop for eq;
19. Mult prop for eq; 20. Transitive, equations;
21. Mult ineq by negative; 22. Mult ineq by positive;
23. Mult prop for eq; 24. Add prop for eq;
25. Symmetric for eq; 26. Reflexive for eq;
27. Transitive for ineq; 28. Mult ineq by negative;
29. Add prop for ineq; 30. Mult ineq by positive.

p. 88-90:

1. $(-\infty, -2]$; 2. $(-\infty, 4)$; 3. No Solution; 4. $(-\infty, -2]\cup(4,\infty)$;
5. $(-\infty, \infty)$; 6. $[-2,4)$; 7. $[-2, \infty)$; 8. $[4,\infty)$;
9. $(-\infty, -4)\cup[-2,\infty)$; 10. No Solution; 11. $(-5,-2]$; 12. $(-\infty, \infty)$;
13. $(-\infty, \infty)$; 14. $(-1,5]$; 15. $(-\infty, 2]$; 16. No Solution;
17. $(-1,\infty)$; 18. $(2,\infty)$; 19. $(-2,2]$; 20. $(-\infty, -2)\cup[0,\infty)$.