1.05 Applications

After solving equations and inequalities, the question "What good is this?" inevitably arises! Answers to questions such as "What good is math?" usually involve applications--that is, "word problems!" Word problems range in difficulty from simple to absurd. While it is helpful to arrange them in "categories" for study, it will be even more helpful to see that the "different" categories or types of problems are actually more similar than they are different. It may also be helpful to identify five steps in setting up and solving word problems:

STEP 1: ***IDENTIFY THE VARIABLE.*** State exactly what it is that the variable represents. For example, "Let X = the number of dimes" or "Let X = rate of the first plane" or "Let X = amount of pure alcohol to be added." Then express all other quantities to be used in the problem in terms of X. This is the most important, often the most difficult, and usually the most overlooked step of the problem.

STEP 2: ***WRITE THE EQUATION.*** Having completed step 1, use this step in writing the equation. This is often no more than translating a sentence of the problem into an equation. Read the problem carefully.

STEP 3: ***SOLVE THE EQUATION.*** This is usually the easy part!

STEP 4: ***ANSWER THE QUESTION.*** After solving for X, there may be other quantities to be determined. Be sure you have answered the question before going on to the next exercise.

STEP 5: ***CHECK.*** Check the answers in the worded problem itself and make sure the solution actually works. Reject any extraneous or "inappropriate" answers.

In this section, the following "categories" of applications will be considered:

- **I. Number problems, consecutive number problems**
- **II. Perimeter problems**
- **III. Coin problems**
- **IV. Interest problems**
- **V. Mixture problems**
- **VI. Distance, rate, time problems**
- **VII. Digit problems (From the Florida CLAST!)**

I. NUMBER PROBLEMS, CONSECUTIVE NUMBER PROBLEMS.

When finding consecutive numbers (integers), as 7, 8, 9 or 29, 30, 31, it is usually convenient to let X represent the first number. Then since the difference between the numbers is 1, it follows that X+1 represents the second number, X+2 represents the third number, etc.

When finding consecutive odd numbers (integers), as 7, 9, 11 or 29, 31, 33, notice that the difference between the numbers is 2. For this reason, after letting X represent the first number, the second number will be 2 more than the first (that is, X+2), the third number will be 2 more than the second (that is, X+4), and so on. Likewise for consecutive even numbers, such as 8, 10, 12 or 22, 24, 26, the difference between consecutive numbers is 2. Therefore, if X represents the first, then X+2 and X+4 represent the second and third numbers respectively.

In summary, for consecutive numbers:

Let X = first number
X+1 = second number
X+2 = third number.

For consecutive even numbers <u>or</u> consecutive odd numbers:

Let X = first number
X+2 = second number
X+4 = third number.

The numbers represented by X, X+2, and X+4 will all be even, or they will all be odd, depending upon whether the value of X turns out to be even or odd.

EXAMPLE 1:

Three numbers are such that the second is six less than the first, and the third is ten more than the sum of the first two numbers. The sum of the numbers is 38. Find the numbers.

SOLUTION: Let

$$X = \text{first number}$$
$$X-6 = \text{second number}$$
$$X + X-6 + 10 = \text{third number}$$
$$\text{or } 2X+4$$

Equation:

$$X + (X-6) + (2X+4) = 38$$
$$4X - 2 = 38$$
$$4X = 40$$

Answer the question:

$X = 10$ first number
$X-6 = 4$ second number
$2X+4 = 24$ third number

Check: $10 + 4 + 24 = 38$

EXAMPLE 2:

Find three consecutive numbers whose sum is 189.

SOLUTION: Let

$$X = \text{first number}$$
$$X+1 = \text{second number}$$
$$X+2 = \text{third number}$$

Equation:

$$X + X+1 + X+2 = 189$$
$$3X + 3 = 189$$
$$3X = 186$$
$$X = 62$$

Answer the question:

$X = 62$ first number
$X+1 = 63$ second number
$X+2 = 64$ third number

Check: $62 + 63 + 64 = 189$

EXAMPLE 3:

Find three consecutive <u>odd</u> numbers whose sum is 189.

SOLUTION: Let X = first number
$X+2$ = second number
$X+4$ = third number

Equation:
$$X + X+2 + X+4 = 189$$
$$3X + 6 = 189$$
$$3X = 183$$
$$X = 61$$

Answer the question:
X = 61 first number
X+2 = 63 second number
X+4 = 65 third number

Check: $61 + 63 + 65 = 189$

EXAMPLE 4:

Find three consecutive <u>even</u> numbers such that the first plus twice the second plus three times the third is equal to 100.

SOLUTION: Let X = first number
$X+2$ = second number
$X+4$ = third number

Equation:
$$X + 2(X+2) + 3(X+4) = 100$$
$$X + 2X + 4 + 3X + 12 = 100$$
$$6X + 16 = 100$$
$$6X = 84$$
$$X = 14$$

Answer the question:
X = 14 first number
X+2 = 16 second number
X+4 = 18 third number

Check:
$$14 + 2(16) + 3(18) = 100$$
$$14 + 32 + 54 = 100$$

EXERCISES:

1. Three numbers are such that the second number is 5 more than the first, and the third number is 4 more than three times the second. The sum of the three numbers is 134. Find the numbers.

2. Three numbers are such that the second number is 3 less than the first, and the third is twice the second. The sum of the numbers is 91. Find the numbers.

3. Three numbers are such that the second number is 4 more than three times the first, and the third number is 12 less than the sum of the first two numbers. The sum of the three numbers is 44. Find the numbers.

4. Three numbers are such that the first number is 10 less than twice the second, and the second number is 4 more than three times the third. Twice the second number is equal to the sum of the first and third numbers. Find the numbers.

5. Find three consecutive integers whose sum is 432.

6. Find three consecutive <u>even</u> integers whose sum is 432.

7. Find two consecutive <u>odd</u> integers such that twice the second plus the first is 121.

8. Find three consecutive integers such that the first plus twice the second plus three times the third is equal to 200.

9. Find three consecutive even integers such that three times the first plus six times the second is equal to ten times the third.

10. Find three consecutive integers such that twice the first minus the third is 3 less than the second.

II. PERIMETER PROBLEMS.

The perimeter of a geometric figure (shape) is the total distance around the outside of that figure. For a rectangle, the perimeter consists of two widths and two lengths. For a triangle, the perimeter is just the sum of the three sides. For a circle, the perimeter (for circles it is called the **"circumference"**) is π times the diameter of the circle.

Rectangle:	**$P = 2W + 2L$**
Triangle:	**$P = a + b + c$**
Circle:	**$C = \pi d$ (also $C=2\pi r$)**

EXAMPLE 5: The length of a rectangle is 8 more than twice the width. The perimeter of the rectangle is 106 centimeters. Find the dimensions of the rectangle.

SOLUTION: Let X = width of rectangle
$2X+8$ = length of rectangle

Equation: **$2W + 2L = $ Perimeter**

$$2(X) + 2(2X+8) = 106$$
$$2X + 4X + 16 = 106$$
$$6X = 90$$
$$X = 15$$

Answer question: **$X = $ 15 cm width of rectangle**
$2X + 8 = $ 38 cm length of rectangle

Check:

$$2W + 2L = P$$
$$2(15) + 2(38) = 106$$
$$30 + 76 = 106$$

EXERCISES:

11. The length of a rectangle is 5 meters longer than the width. The perimeter is 50 meters. Find the dimension of the rectangle.

SOLUTION: Let X = width of the rectangle
_____ = length of the rectangle

Equation: **2W + 2L = Perimeter**
2( ) + 2( ) = _____

Check:

12. The length of a rectangle is 5 meters more than twice the width. The perimeter is 130 meters. Find the dimensions.

13. The length of a rectangle is 5 meters less than twice the width. The perimeter is 50 meters. Find the length and width of the rectangle.

14. The length of a rectangle is 50 feet less than three times the width. If the perimeter is 500 feet, find the length and width of the rectangle.

15. The length of a rectangle is 50 meters less than twice the width. The perimeter is 1100 meters. Find the dimensions.

16. The width of a rectangle is 50 feet less than the length. The perimeter is 400 feet. Find the dimensions of the rectangle.

17. The length of a rectangle is 3 less than 5 times the width. The perimeter is 10 times the width. Find the dimensions and the perimeter of the rectangle.

18. **EXTRA CHALLENGE:**
The perimeter of a rectangle is 46 meters. Twice the length is 4 more than 5 times the width. Find the length and width of the rectangle.

SOLUTION: Let X = width

= two lengths

Equation: Two widths + two lengths = Perimeter

+ = 46

III. COIN PROBLEMS.

EXAMPLE 6: A box contains nickels, dimes, and quarters worth a total of $2.10. There are twice as many dimes as quarters, and the number of nickels is two less than the number of dimes. How many of each coin are there?

SOLUTION:

	No. Coins	Each (¢)	Values
Q	X	25	25(X)
D	2X	10	10(2X)
N	2X - 2	5	5(2X - 2)
			210¢

Equation:

$$\begin{aligned} 25X + 20X + 10X - 10 &= 210 \\ 55X - 10 &= 210 \\ 55X &= 220 \\ \mathbf{X} &= \mathbf{4} \quad \textbf{Quarters} \end{aligned}$$

Answer the question:

$$\begin{aligned} \mathbf{2X} &= \mathbf{8} \quad \textbf{Dimes} \\ \mathbf{2X - 2} &= \mathbf{6} \quad \textbf{Nickels} \end{aligned}$$

Check:

4(.25) = $ 1.00 Quarters
8(.10) = .80 Dimes
6(.05) = .30 Nickels
$2.10Total

EXERCISES:

19. A certain number of quarters and four times as many pennies ar worth $1.45. How many of each coin are there?

SOLUTION:

	No. Coins	Each (¢)	Values
Q	X	25	
P	4X	1	

20. A certain number of quarters and three times as many dimes are worth $1.65. How many of each coin are there?

21. A certain number of quarters, four times as many pennies as quarters, and 6 more dimes than pennies are worth $3.36. How many of each coin are there?

22. A box contains 30 coins, in nickels and dimes, worth $2.40. How many of each coin are there?

SOLUTION: Let X = number of nickels, then

30 - X = number of dimes.

	No. Coins	Each (¢)	Values
N	X	5	
D	30 - X	10	

23. A box contains 20 coins in quarters and dimes worth \$2.90. How many of each coin are there?

24. A box contains 20 coins in quarters and dimes worth \$3.80. How many of each coin are there?

25. A box contains \$6.60 in nickels, dimes, and quarters. There are three times as many nickels as quarters, and the number of dimes is 4 less than the number of nickels. How many of each coin are there?

26. A box contains \$8.00 in nickels, dimes, and quarters. There are three times as many nickels as quarters, and the number of dimes is 4 less than the number of nickels. How many of each coin are there?

27. A certain number of pennies, four times as many dimes as pennies, and a number of quarters which is 16 less than twice the number of dimes, are worth \$24.92. How many of each coin are there?

28. A sum of money consists of nickels, dimes, and quarters amounting to \$1.90. If there are twice as many nickels as quarters and three less dimes than nickels, how many of each coin are there?

29. A box contains nickels, dimes, and quarters worth \$12.60. The number of dimes is 2 less than three times the number of nickels, and the number of quarters is 4 less than twice the number of dimes. How many of each coin are there?

30. A box contains nickels, dimes, and quarters worth \$69.50. The number of nickels is 10 more than twice the number of dimes. There are as many quarters as nickels and dimes combined. How many of each coin are there?

IV. INTEREST PROBLEMS.

EXAMPLE 7: A woman invests a sum of money at 6% and $3000 more than this at 9%. If the total interest earned in one year is $4170, how much was invested at each rate?

SOLUTION:

	Principle	X Rate	= Interest
6%	X	0.06	0.06(X)
9%	X + $3000	0.09	0.09(X+3000)
			$4170

Equation:

$$
\begin{aligned}
0.06X + 0.09(X + 3000) &= \$4170 \\
0.06X + 0.09X + 270 &= \$4170 \\
0.15X + 270 &= \$4170 \\
0.15X &= \$3900 \\
X &= \$3900/0.15 \\
\mathbf{X} &= \mathbf{\$26000 \quad @ \quad 6\%}
\end{aligned}
$$

Answer the question: $\mathbf{X + 3000 = \$29000 \quad @ \quad 9\%}$

Check:

$$
\begin{aligned}
26000(0.06) &= \$\ 1560.00 \\
29000(0.09) &= \underline{\ \ 2610.00} \\
&\ \$\ 4170.00 \text{ Total}
\end{aligned}
$$

EXERCISES:

31. A sum of money was invested at 8% simple interest, and three times as much at 10%. The total interest earned for the year was $190. How much was invested at each rate?

SOLUTION:

	Principle	X Rate	= Interest
8%	X	0.08	
10%	3X	0.10	

32. A sum of money was invested at 12% simple interest, and $1000 less than this at 10%. The total interest earned for the year was $1000. How much was invested at each rate?

33. A total of $10,000 was invested, some at 12% and the rest at 10% simple interest. The total interest earned for the year was $1060. How much was invested at each rate?

34. A total of $2500 was invested, part at 12% simple interest, and the rest at 10%. The total interest earned for the year was $260. How much was invested at each rate?

35. A man has $10,000 to invest, some in a relatively safe account earning 5% interest per year, and the rest in more speculative investments earning 12% per year. If the total interest earned for the year was $955, how much was invested at each rate?

36. A sum of money was invested at 5% annual interest, and $500 less than twice this amount was invested at 12%. If the total interest earned for the year was $375, how much was invested at each rate?

V. MIXTURE PROBLEMS.

EXAMPLE 8: Some 10% alcohol solution is to be mixed with some 30% alcohol solution to make 20 liters of 16% solution. How much of each must be used?

SOLUTION:

	Amt. Sol. X	**Strength**	= **Pure Stuff**
10%	X	0.10	0.10(X)
30%	20 - X	0.30	0.30(20 - X)
16%	20	0.16	0.16(20)

Equation:
$$
\begin{aligned}
0.10(X) + 0.30(20 - X) &= 0.16(20)\\
.10X + 6 - 0.30X &= 3.20\\
-0.20X + 6 &= 3.20\\
-0.20X &= -2.80\\
X &= -2.80/(-0.20)\\
\mathbf{X} &= \mathbf{14 \text{ liters of } 10\% \text{ solution}}
\end{aligned}
$$

Answer the question: $\mathbf{20 - X = 6 \text{ liters of } 30\% \text{ solution}}$

Check:
14(0.10) = 1.4 liters alcohol
6(0.30) = 1.8 liters alcohol
20(0.16) = 3.2 liters Total

EXAMPLE 9: How much liquid must be drained from a 20 liter radiator at 20% antifreeze and replaced with pure antifreeze to bring the strength up to 50%?

SOLUTION:

	Amt. Sol.	X **Strength**	= **Pure Stuff**
Begin	20	0.20	0.20(20)
Drain	**(-)** X	0.20	-0.20(X)
Add	**(+)** X	1.00	+1.00(X)
End Up	**(=)**20	0.50	0.50(20)

Equation:
$$
\begin{aligned}
0.20(20) - 0.20(X) + 1.00(X) &= 0.50(20)\\
4 + 0.80X &= 10\\
0.80X &= 6\\
X &= 6/(0.80)\\
\mathbf{X} &= \mathbf{7.5 \text{ liters of antifreeze}}
\end{aligned}
$$

EXAMPLE 10 : How much water must be added to 60 liters of 20% acid solution in order to dilute the solution to 8%?

SOLUTION:

	Amt. Sol. X	**Strength**	**= Pure Stuff**
20%	60	0.20	0.20(60)
Water	X	0.00	0.00(X)
8%	X + 60	0.08	0.08(X + 60)

Equation:

$$
\begin{aligned}
.20(60) + .00(X) &= 0.08(X + 60)\\
12 + 0 &= 0.08X + 4.8\\
7.2 &= 0.08X\\
X &= 7.2/(0.08)\\
\mathbf{X} &= \mathbf{90 \text{ liters of water}}
\end{aligned}
$$

Check:

$$
\begin{aligned}
60(0.20) &= 12 \text{ liters acid}\\
+\underline{90 \text{ Water}} &= \underline{\text{No acid}}\\
150(0.08) &= 12 \text{ liters acid}
\end{aligned}
$$

EXAMPLE 11: Twenty kilograms of nuts consisting of cashews worth $6.00 per kg, pecans worth $2.50 per kg, and peanuts worth $1.50 per kg are mixed. If there are twice as many pecans as cashews, and the total value of the nuts is $56, how many of each are there? **[HINT: Let X = kg cashews; 2X = kg pecans**
3X = kg cashews and pecans combined
20-3X = kg peanuts]

SOLUTION:

	No. Kg. X	**Each Kg.**	**= Total Value**
Cashews	X	6.00	6.00(X)
Pecans	2X	2.50	2.50(2X)
Peanuts	20-3X	1.50	1.50(20-3X)
Total	20		56.00

Equation:

$$
\begin{aligned}
6.00(X) + 2.50(2X) + 1.50(20-3X) &= 56.00\\
6.00X + 5.00X + 30.00 - 4.50X &= 56.00\\
6.50X + 30.00 &= 56.00\\
6.50X &= 26.00\\
X &= 26.00/(6.50)\\
\mathbf{X} &= \mathbf{4 \text{ kg cashews}}\\
\mathbf{2X} &= \mathbf{8 \text{ kg pecans}}\\
\mathbf{20-3X} &= \mathbf{8 \text{ kg peanuts}}
\end{aligned}
$$

EXERCISES:

37. How much 10% alcohol solution must be added to 20 liters of 50% solution to make a 20% solution?

SOLUTION:

	Amt. Sol. X	Strength =	Pure Stuff
10%	X	0.10	
50%	20	0.50	
20%	X + 20	0.20	

38. Some 80% acid solution is to be mixed with some 35% acid solution to make 300 liters of 50% solution. How much of each acid should be used?

39. How much water must be added to 20 liters of 50% alcohol solution to dilute it to 10%?

SOLUTION:

	Amt. Sol. X	Strength =	Pure Stuff
Water 0%	X	0	
50%	20	.50	
10%	X + 20	.10	

40. How much water must be added to 50% alcohol solution to obtain 100 liters of 10% solution?

41. How much pure alcohol must be added to 20 liters of 10% alcohol solution to create a 50% solution?

SOLUTION:

	Amt. Sol.	**X Strength**	**= Pure Stuff**
Pure 100%	X	1.00	
10%	20	0.10	
50%	X + 20	0.50	

42. How much pure alcohol must be added to 100 liters of 10% alcohol solution to create an 80% solution?

43. How much liquid must be drained from an 18 liter radiator that is 10% antifreeze and replaced with pure antifreeze in order to bring the strength up to 50%?

SOLUTION:

	Amt. Sol.	**X Strength**	**= Pure Stuff**
Begin	18	0.10	
Drain	**(-)** X	0.10	
Add	**(+)** X	1.00	
End Up	**(=)** 18	0.50	

44. How much liquid must be drained from a 24 liter radiator that is 25% antifreeze and replaced with pure antifreeze in order to bring the strength up to 50%?

45. A merchant mixes some candy worth \$3.50 per pound with cheap stuff worth \$1.00 per pound. There are 10 more pounds of cheap stuff than the more expensive candy. If the total value of the mixture is \$28, how many pounds of each are there?

46. A merchant mixes a total of 50 pounds of candy, some worth \$2 per pound, the rest worth \$4 per pound. If the total value of the mixture is \$160, how many pounds of each are there?

47. **EXTRA CHALLENGE**
Fifty tickets were sold to a chicken barbeque for a total of \$219. Children's tickets sold for \$2.50, youth tickets sold for \$3.50, and adult's tickets sold for \$5.00. There were 10 more youth tickets than children's tickets. How many of each ticket were sold?

48. EXTRA CHALLENGE

A total of 180 tickets are sold, some at $3, some at $5, and some at $10 each. The total value of the tickets was $1100. The number of $5 tickets was 20 more than the number of $3 tickets. How many of each ticket were sold?

VI. DISTANCE, RATE, TIME PROBLEMS.

EXAMPLE 12: Two bicycles start at the same point traveling in the opposite direction. The speed of the second bike in miles per hour is 12 less than three times the first. At the end of 6 hours, the bicycles are 144 miles apart. Find the speed of each bicycle.

SOLUTION:

	Rate	**X Time**	**= Distance**
1st	X	6	6(X)
2nd	3X - 12	6	6(3X-12)
			144

Equation:

$$6X + 6(3X - 12) = 144$$
$$6X + 18X - 72 = 144$$
$$24X = 216$$
$$\mathbf{X = 9}$$ **mph 1st bicycle**

Answer the question: **3X - 12 = 15 mph 2nd bicycle**

EXAMPLE 13: A train leaves the Sanford terminal averaging 45 mph. A second train leaves the same terminal two hours later averaging 60 mph. How long will it take the second train to catch the first train? **[HINT: Let X = time of second train to catch first. The distances are equal.]**

SOLUTION:

	Rate	**X Time**	**= Distance**
1st	45	X + 2	45(X+2)
2nd	60	X	60(X)

Equation:

$$\begin{aligned} 45(X + 2) &= 60(X) \\ 45X + 90 &= 60X \\ 90 &= 15X \\ \mathbf{X} &= \mathbf{6} \textbf{ hours for second train} \end{aligned}$$

EXERCISES:

49. Two boys are riding bicycles in the opposite direction. One travels 15 mph faster than the other. At the end of 3 hours they are 102 miles apart. How fast is each boy riding?

50. Two cars are driving in opposite directions, one at 55 mph and the other at 65 mph (on the interstate!). How long will it take before the cars are 300 miles apart?

VII. DIGIT PROBLEMS (Remember! This is on the Florida CLAST!!)

Remember that the value of a two digit number is 10 times the ten's digit **(t)**, plus the unit's digit **(u)**.

VALUE of 2 digit number = 10t + u

EXAMPLE 14: The ten's digit of a number is 5 more than the unit's digit. The value of the number is 8 times the sum of its digits. Find the digits and value of the number.

SOLUTION: Let **u = unit's digit**
u + 5 = ten's digit (t)

Value of the number = 10t + u = 8(t + u)

Equation:

$$\mathbf{10(u + 5) + u = 8(u + u+5)}$$
$$10u + 50 + u = 8u + 8u + 40$$
$$11u + 50 = 16u + 40$$
$$-5u = -10$$
$$\mathbf{u = 2} \quad \textbf{Unit's digit}$$

$$\mathbf{u + 5 = 7} \quad \textbf{Ten's digit}$$

The number is 72.

51. The ten's digit of a two digit number is 2 less than the unit's digit. The value of the number is 7 times the unit's digit. Find the digits and the value of the number.

52. The ten's digit of a two digit number is 2 more than the unit's digit. If the number itself is 16 times the unit's digit, find the digits and the value of the number.

ANSWERS 1.05

p.55-76:

1. 22,27,85; 2. 25,22,44; 3. 6,22,16; 4. 58,34,10;
5. 143,144,145; 6. 142,144,146; 7. 39,41; 8. 32,33,34;
9. -28,-26,-24; 10. All integers; 11. W=10m, L=15m;
12. W=20m,L=45m; 13. W=10m, L=15m; 14. W=75ft,L=175ft;
15. W=200m, L=350m; 16. W=75ft,L=125ft; 17. W=3, L=12 P=30;
18. W=6, L=17; 19. 5Q, 20P; 20. 3Q, 9D; 21. 4Q, 16P, 22D;
22. 12N, 18D; 23. 6Q, 14D; 24. 12Q, 8D; 25. 10Q, 26D, 30N;
26. 12Q, 36N, 32D; 27. 12P, 48D, 80Q; 28. 4Q, 8N, 5D;

29. 8N, 22D, 40Q; **30.** 70D, 150N, 220Q;
31. \$500 @ 8%, \$1500 @ 10%; **32.** \$5000 @ 12%, \$4000 @ 10%;
33. \$3000 @ 12%, \$7000 @ 10%; **34.** \$500 @ 12%, \$2000 @ 10%;
35. \$3500 @ 5%, \$6500 @ 12%; **36.** \$1500 @ 5%, \$2500 @12%;
37. 60 l; **38.** 100 l@ 80%, 200 l @ 35%; **39.** 80 l; **40.** 80 l;
41. 16 l; **42.** 350 l; **43.** 8 l; **44.** 8 l;
45. 4 lb. @ \$3.50, 14 lb. @ \$1.00; **46.** 20 lb.@\$2, 30 lb.@\$4;
47. 4 children, 14 youth, 32 adult; **48.** 50@\$3, 70@\$5, 60@\$10;
49. 9.5mph, 24.5mph; **50.** 2.5 hrs; **51.** u=5, t=3, 35;
52. u=4, t=6, 64.