

1.07 Laws of Exponents

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

We will begin with a summary of the laws of exponents. You are probably familiar with these laws from your previous algebra background.

LAWS OF EXPONENTS	GENERALIZATION
1. When you multiply (with the same base number), you add exponents.	$X^m \cdot X^n = X^{(m+n)}$
2. When you divide (with the same base number), you subtract exponents.	$\frac{X^m}{X^n} = X^{(m-n)}$
3. When you raise a power to a power, you multiply exponents.	$(X^m)^n = X^{mn}$
4. When a product or a quotient is raised to a power, you raise each factor to the power.	$(XY)^m = X^m \cdot Y^m$ $\left(\frac{X}{Y}\right)^m = \frac{X^m}{Y^m}$
5. Any non-zero number raised to the zero power is 1.	$X^0 = 1$
6. Any number raised to a negative power is 1 divided by that number raised to the positive power.	$X^{-n} = \frac{1}{X^n}$
7. One (1) divided by any number raised to a negative power is that number raised to the positive power.	$\frac{1}{X^{-n}} = X^n$
8. A fraction raised to a negative power is the reciprocal of the fraction raised to the positive power.	$\left(\frac{X}{Y}\right)^{-n} = \left(\frac{Y}{X}\right)^n$

"QUICKIES"

Simplify each of the following. Express without negative exponents.

1. $X^4 \cdot X^7 =$ _____ 2. $\frac{X^8}{X^2} =$ _____ 3. $(X^4)^7 =$ _____

4. $(X^3)^0 =$ _____ 5. $\frac{X^{10}}{X^5} =$ _____ 6. $X^4 \cdot X^0 =$ _____

7. $2^4 \cdot 2^6 =$ _____ 8. $(2^3)^6 =$ _____ 9. $\frac{2^{10}}{2^5} =$ _____

10. $\frac{X^3}{X^{-2}} =$ _____ 11. $\frac{1}{X^{-3}} =$ _____ 12. $\left(\frac{X^2}{Y^3}\right)^4 =$ _____

13. $(X^3 Y^4)^3 =$ _____ 14. $\left(\frac{X^4 Y^2}{Z^5}\right)^2 =$ _____ 15. $\left(\frac{X^2}{Y^3}\right)^0 =$ _____

16. $X^{-3} =$ _____ 17. $Y^{-5} =$ _____ 18. $2^{-3} =$ _____

19. $3^{-2} =$ _____ 20. $3X^0 =$ _____ 21. $(3X)^0 =$ _____

22. $(3X)^{-1} =$ _____ 23. $3X^{-1} =$ _____ 24. $(3X)^{-2} =$ _____

25. $(3X)^{-4} =$ _____ 26. $3X^{-3} =$ _____ 27. $(3X^{-1})^{-2} =$ _____

"TWO-STEP"

28. $(X^4 \cdot X^3)^3 =$ _____
= _____

29. $(X^6 \cdot X^{-2})^3 =$ _____
= _____

30. $(X^5 \cdot X^{-2})^6 =$ _____
= _____

31. $\left(\frac{X^8}{X^2}\right)^5 =$ _____
= _____

32. $\left(\frac{X^4}{X^{-2}}\right)^7 =$ _____
= _____

33. $\frac{X^4 \cdot X^{10}}{X^{-6}} =$ _____
= _____

34. $\frac{X^{-4} \cdot X^{10}}{X^{-6}} =$ _____
= _____

35. $\frac{X^4 \cdot X^{-10}}{X^{-6}} =$ _____
= _____

36. $\frac{X^{-4} \cdot X^{-10}}{X^{-6}} =$ _____
= _____

"WATCH YOUR STEP!"

37. $(2X^3)^4 \cdot (X^4Y^{-3})^2$
= _____
= _____
= _____
= _____

38. $(3X^3Y^{-2})^2 \cdot (2X^{-4}Y^5)^2$
= _____
= _____
= _____
= _____

$$39. (3^{-1}X^3Y^{-2})^{-2} \cdot (2X^{-4}Y^5)^{-2}$$

= _____

= _____

= _____

= _____

$$41. \frac{X^{-11} (X^{-2})^4}{(X^2)^{-6}}$$

= _____

= _____

= _____

= _____

$$43. \frac{(3X^{-3}Y^2)^{-2}}{(3^{-1}X^4Y^{-5})^{-2}}$$

= _____

= _____

= _____

= _____

$$40. (3X^{-3}Y^2)^2 \cdot (2^{-1}X^4Y^{-5})^{-2}$$

= _____

= _____

= _____

= _____

$$42. \frac{2^{-7} \cdot X^4}{2^{-9} \cdot X^{-4}}$$

= _____

= _____

= _____

= _____

$$44. \frac{(3^{-1}X^4Y^{-5})^{-2}}{(3X^{-3}Y^2)^{-2}}$$

= _____

= _____

= _____

= _____

In the next exercises remember, the rules are exactly the same--but the level of abstraction has increased. Now there are variables in the exponents, which give the exercises a slightly different (more complicated) appearance. Just know and obey the laws of exponents, and combine like terms when possible (in the exponents).

EXAMPLE 1:

$$\frac{X^{3a} X^{2b}}{X^{3c}}$$

$$= \frac{X^{3a+2b}}{X^{3c}}$$

$$= X^{3a+2b-3c}$$

EXAMPLE 2:

$$\frac{X^{3p} X^{2p-4}}{X^{6-3p}}$$

ADD EXPONENTS!

$$= \frac{X^{3p+2p-4}}{X^{6-3p}}$$

SUBTRACT EXPONENTS!

$$= X^{5p-4-(6-3p)}$$

$$= X^{5p-4-6+3p}$$

$$= X^{8p-10}$$

45. $2^{3X} \cdot 2^{2Y}$

= _____

46. $\frac{2^{3X}}{2^{2Y}}$

= _____

47. $2^{X-4} \cdot 2^{X+6}$

= _____

48. $\frac{2^{3X}}{2^{X+4}}$

= _____

49. $\frac{2^X 2^Y}{2^Z}$

= _____

= _____

50. $\frac{2^X}{2^Y 2^Z}$

= _____

= _____

51. $\frac{X^{3p+2} X^{4p-6}}{X^{2p+4}}$

= _____

= _____

= _____

= _____

52. $\frac{Y^{2q-5} Y^{6-3q}}{Y^{2-4q}}$

= _____

= _____

= _____

= _____

ANSWERS 1.07

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1. X^{11} ; 2. X^6 ; 3. X^{28} ; 4. 1; 5. X^5 ; 6. X^4 ; 7. 2^{10} ; 8. 2^8 ;
9. 2^5 or 32; 10. X^5 ; 11. X^3 ; 12. $\frac{X^8}{Y^{12}}$; 13. X^9Y^{12} ; 14. $\frac{X^8 Y^4}{Z^{10}}$;
15. 1; 16. $\frac{1}{X^3}$; 17. $\frac{1}{Y^5}$; 18. $1/8$; 19. $1/9$; 20. 3; 21. 1;
22. $\frac{1}{3X}$; 23. $\frac{3}{X}$; 24. $\frac{1}{9X^2}$; 25. $\frac{1}{81X^4}$; 26. $\frac{3}{X^3}$; 27. $\frac{X^2}{9}$;
28. X^{21} ; 29. X^{12} ; 30. X^{18} ; 31. X^{30} ; 32. X^{42} ; 33. X^{20} ; 34. X^{12} ;
35. 1; 36. $\frac{1}{X^8}$; 37. $\frac{16X^{20}}{Y^6}$; 38. $\frac{36Y^6}{X^2}$; 39. $\frac{9X^2}{4Y^6}$; 40. $\frac{36Y^{14}}{X^{14}}$;
41. $\frac{1}{X^7}$; 42. $4X^8$; 43. $\frac{X^{14}}{81Y^{14}}$; 44. $\frac{81Y^{14}}{X^{14}}$; 45. 2^{3X+2Y} ;
46. 2^{3X-2Y} ; 47. 2^{2X+2} ; 48. 2^{2X-4} ; 49. 2^{X+Y-Z} ; 50. 2^{X-Y-Z} ; 51. X^{5p-8} ;
52. Y^{3q-1} .