2.04 *Multiplication and Division of Fractions*

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When multiplying two fractions $\frac{a}{b}$ and $\frac{b}{c}$, simply multiply the numerator times numerator and denominator times denominator. Of course, the denominators cannot equal zero. That is:

$$PRINCIPLE: \quad \frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}, \quad b \neq 0, \ d \neq 0$$

When dividing two fractions, you must remember to invert the second number (the divisor) and multiply. Again, remember that the denominators cannot equal zero, and you can't divide by zero. That is:

$$PRINCIPLE: \quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}, \quad b \neq 0, \ c \neq 0, \ d \neq 0$$

In both cases, remember to reduce all fractions completely.

In each of the following, perform the indicated operations, and reduce the fractions completely. If your calculator has "fractions" capabilities, check your answers with the calculator. [If your calculator has an "a b/c" button try entering: "numerator", "a b/c", "denominator", "times" (or "divide"), "numerator", "a b/c", "denominator", "enter".]

1. $\frac{3}{7} \cdot \frac{9}{5} = $ _____

2. $\frac{4}{5} \cdot \frac{3}{13} = $ _____

[Note: Improper fractions are preferred, mixed fractions accepted.]

3. $\dfrac{13}{5} \cdot \dfrac{7}{2}$ = _____

4. $\dfrac{7}{2} \cdot \dfrac{5}{13}$ = _____

[Note: Remember, you may divide any factor of any numerator with any factor of any denominator. Be sure to reduce all fractions. (You may need one or more steps!)]

5. $\dfrac{14}{5} \cdot \dfrac{15}{63}$ = _____

= _____

6. $\dfrac{20}{13} \cdot \dfrac{26}{25}$ = _____

= _____

7. $\dfrac{13}{20} \cdot \dfrac{16}{52}$ = _____

= _____

8. $\dfrac{19}{57} \cdot \dfrac{7}{28}$ = _____

= _____

9. $\dfrac{32}{108} \cdot \dfrac{120}{80}$ = _____

= _____

10. $\dfrac{45}{85} \cdot \dfrac{102}{76}$ = _____

= _____

[Remember: When dividing your first step is to invert the second fraction, then multiply as in the problems above!]

11. $\dfrac{3}{10} \div \dfrac{5}{7} = \dfrac{3}{10} \cdot \dfrac{7}{5}$

= _____

12. $\dfrac{8}{3} \div \dfrac{7}{20}$ = _____

= _____

13. $\dfrac{27}{25} \div \dfrac{9}{35}$ = _____

= _____

14. $\dfrac{42}{13} \div \dfrac{7}{26}$ = _____

= _____

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15. $\dfrac{13X^3}{12Y^2} \cdot \dfrac{48X^5}{39Y^3} =$ _____

16. $\dfrac{24X^3}{36Y^5} \cdot \dfrac{39X^4}{8Y^4} =$ _____

17. $\dfrac{38X^6}{19Y^4} \cdot \dfrac{36Y^{10}}{16X^6} =$ _____

18. $\dfrac{52X^3}{39X^6} \cdot \dfrac{74X^4}{37Y^2} =$ _____

19. $\dfrac{42X^7}{27Y^3} \div \dfrac{14X^6}{9Y^5} =$

20. $\dfrac{24X^5}{15X^{10}} \div \dfrac{3Y^3}{40Y^6} =$

$=$

$=$

PRINCIPLE: When multiplying two fractions, you may divide out <u>any</u> factor of <u>any</u> numerator with <u>any</u> factor of <u>any</u> denominator. The key step, then (if possible), is to FACTOR each numerator and denominator! When you divide, invert the <u>second</u> fraction and multiply!

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21. $\dfrac{X^2 - 4X}{16} \cdot \dfrac{8X}{X^2 - 2X - 8} = \dfrac{X(X - 4)}{16} \cdot \dfrac{8X}{(X - 4)(X + 2)}$

$=$ _____

22. $\dfrac{X^2 - 16}{X^2 - 3X - 4} \cdot \dfrac{X^2 - 1}{X^2 + 5X + 4} = \dfrac{(\quad)(\quad)}{(\quad)(\quad)} \cdot \dfrac{(\quad)(\quad)}{(\quad)(\quad)}$

$=$ _____

23. $\dfrac{X^2 - 8X}{X^2 - 7X + 12} \cdot \dfrac{X^2 - 4X + 3}{X^2 - 9X + 8} = \dfrac{X(\quad)}{(\quad)(\quad)} \cdot \dfrac{(\quad)(\quad)}{(\quad)(\quad)}$

$=$

24. $\dfrac{X^2 - X}{X^2 - X - 12} \cdot \dfrac{X^2 - 3X - 4}{X^2 - 1} =$

$=$

25. $\dfrac{X^2 - 49Y^2}{X^2 + 12XY + 35Y^2} \cdot \dfrac{X^2 - 3XY - 10Y^2}{X^2 - 5XY - 14Y^2} =$

$=$

26. $\dfrac{X^2 - 8XY + 16Y^2}{X^2 - 3XY - 10Y^2} \cdot \dfrac{X^2 - 4Y^2}{X^2 - 5XY + 4Y^2} =$

$=$

27. $\dfrac{4X - 16Y}{X^2Y} \div \dfrac{X^2 - 3XY - 4Y^2}{XY^2} = \dfrac{4(\quad)}{X^2Y} \cdot \dfrac{XY^2}{(\quad)(\quad)}$

$= \underline{\hspace{4cm}}$

28. $\dfrac{X^3Y^2}{6XY + 12X} \div \dfrac{Y^3}{Y^2 - 4} =$

$=$

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29. $\dfrac{6X^2 - X - 7}{12X^2 + 16X - 35} \div \dfrac{X^2 - X - 2}{2X^2 + X - 10} = \dfrac{(6X\quad)(X\quad)}{(6X\quad)(2X\quad)} \cdot \dfrac{(2X\quad)(X\quad)}{(\quad)(\quad)}$

$=$

30. $\dfrac{3X^2 - X - 2}{X^4 - X^3} \div \dfrac{3X^2 + 5X + 2}{X^3 - 2X^2} =$

31. $\dfrac{9X^2 - 4Y^2}{9X^2 - 12XY + 4Y^2} \div \dfrac{X^2 - 2XY - 8Y^2}{3X^2 - 14XY + 8Y^2} =$

32. $\dfrac{4X^2 - 9Y^2}{4X^2 - 4XY - 3Y^2} \div \dfrac{4X^2 + 8XY + 3Y^2}{4X^2 - Y^2} =$

Frequently it is helpful (necessary) to factor a negative from one of the factors in order to help things "match-up".

33. $\dfrac{5 - X}{6XY} \cdot \dfrac{2XY^2}{X^2 - 25} = \dfrac{-1(X - 5)}{6XY} \cdot \dfrac{2XY^2}{(X - 5)(X + 5)}$

$$= \rule{4cm}{0.4pt}$$

34. $\dfrac{6 - X}{6XY^5} \cdot \dfrac{9X^3Y^2}{X^2 - 7X + 6} =$

35. $\dfrac{25 - X^2}{4X^3Y} \div \dfrac{X^2 - 10X + 25}{12XY^3}$

36. $\dfrac{4 - X^2}{12X^3Y^4} \div \dfrac{X^2 - 2X - 8}{4XY^2}$

In the next exercises, watch out for factoring by difference/sum of squares and grouping. Be sure you factor completely before reducing.

37. $\dfrac{X^3 - 27}{X^2 - 6X + 9} \cdot \dfrac{X^2 - 9}{X^2 + 3X + 9} = \dfrac{(X-3)(\rule{1.5cm}{0.4pt})}{(\rule{0.7cm}{0.4pt})(\rule{0.7cm}{0.4pt})} \cdot \dfrac{(\rule{0.7cm}{0.4pt})(\rule{0.7cm}{0.4pt})}{X^2 + 3X + 9}$

$$=$$

38. $\dfrac{X^3 + 27}{X^2 + 6X + 9} \cdot \dfrac{X^2 - 9}{X^2 - 3X + 9} =$

$=$

39. $\dfrac{X^3 - 27}{X^3 + 3X^2 - 9X - 27} \cdot \dfrac{(X - 3)^3}{X^2 + 3X + 9} = \dfrac{(X-3)(X^2 + 3X + 9)}{X^2(X+3) - 9(X+3)} \cdot \dfrac{(X - 3)^3}{X^2 + 3X + 9}$

$= \dfrac{(X-3)(X^2 + 3X + 9)}{(X+3)(X^2 - 9)} \cdot \dfrac{(X - 3)^3}{X^2 + 3X + 9}$

$= \dfrac{(X-3)(X^2 + 3X + 9)}{(X+3)(X - 3)(X + 3)} \cdot \dfrac{(X - 3)^3}{X^2 + 3X + 9}$

$=$

40. $\dfrac{X^2 - 25}{X^3 - 125} \div \dfrac{X^2 - 10X + 25}{(X - 5)^3} =$

EXTRA CHALLENGE:

41. $\dfrac{X^4 - 81}{X^4 + 5X^2 - 36} \div \dfrac{X^4 - 3X^3 - 27X + 81}{X^3 - 2X^2 - 9X + 18}$

42. $\dfrac{X^4 - 16}{X^4 - 8X^2 + 16} \div \dfrac{X^4 - 1}{X^4 - 5X^2 + 4}$

43. $\dfrac{X^4 - 81}{X^4 - 13X^2 + 36} \div \dfrac{X^4 - 16}{X^4 - 8X^2 + 16}$

44. $\dfrac{X^8 - 16}{X^8 - 8X^4 + 16} \div \dfrac{X^8 - 1}{X^8 - 5X^4 + 4}$

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1. 27/35; **2.** 12/65; **3.** 91/10; **4.** 35/26; **5.** 2/3; **6.** 8/5;

7. 1/5; **8.** 1/12; **9.** 4/9; **10.** 27/38; **11.** 21/50; **12.** 160/21;

13. 21/5; **14.** 12; **15.** $\dfrac{4X^8}{3Y^5}$; **16.** $\dfrac{13X^7}{4Y^9}$; **17.** $\dfrac{9Y^6}{2}$; **18.** $\dfrac{8X}{3Y^2}$;

19. XY^2 ; **20.** $\dfrac{64Y^3}{3X^5}$; **21.** $\dfrac{X^2}{2(X+2)}$; **22.** $\dfrac{X-1}{X+1}$; **23.** $\dfrac{X}{X-4}$;

24. $\dfrac{X}{X+3}$; 25. $\dfrac{X-5Y}{X+5Y}$; 26. $\dfrac{(X-4Y)(X-2Y)}{(X-5Y)(X-Y)}$; 27. $\dfrac{4Y}{X(X+Y)}$;

28. $\dfrac{X^2(Y-2)}{6Y}$; 29. 1; 30. $\dfrac{X-2}{X(X+1)}$; 31. $\dfrac{3X+2Y}{X+2Y}$; 32. $\dfrac{2X-Y}{2X+Y}$;

33. $\dfrac{-Y}{3(X+5)}$; 34. $\dfrac{-3X^2}{2Y^3(X-1)}$; 35. $\dfrac{-3Y^2(X+5)}{X^2(X-5)}$; 36. $\dfrac{2-X}{3X^2Y^2(X-4)}$;

37. X+3; 38. X-3; 39. $\dfrac{(X-3)^3}{(X+3)^2}$; 40. $\dfrac{(X-5)(X+5)}{X^2+5X+25}$;

41. $\dfrac{(X+3)^2}{(X+2)(X^2+3X+9)}$; 42. $\dfrac{X^2+4}{X^2+1}$; 43. $\dfrac{X^2+9}{X^2+4}$; 44. $\dfrac{X^4+4}{X^4+1}$.

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