2.06 *Complex Fractions*

Complex fractions are fractions whose numerator and/or denominator contain fractions. Compare and contrast the complex fractions: $\dfrac{\frac{a}{b}}{c}$ and $\dfrac{a}{\frac{b}{c}}$. In the first case, the fraction $\dfrac{\frac{a}{b}}{c}$ means the numerator $\frac{a}{b}$ is divided by the denominator **"c"**. This may be written

$$\frac{\frac{a}{b}}{c} = \frac{a}{b} \div \frac{c}{1}$$
$$= \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc}.$$

Now compare this to the complex fraction $\dfrac{a}{\frac{b}{c}}$. In this case, the numerator **"a"** is divided by the fraction $\dfrac{b}{c}$.

This means:
$$\frac{a}{\frac{b}{c}} = \frac{a}{1} \div \frac{b}{c}$$

$$= \frac{a}{1} \cdot \frac{c}{b} = \frac{ac}{b}.$$

The answer is not the same. If the fraction lines are equal length, then the fraction is ambiguous.

In general, $\dfrac{\frac{a}{b}}{\frac{c}{d}}$ means $\dfrac{a}{b} \div \dfrac{c}{d}$ or $\dfrac{a}{b} \cdot \dfrac{d}{c}$.

EXERCISES. Simplify each of the following:

1. $\dfrac{\frac{3}{2}}{\frac{12}{7}} = \dfrac{3}{2} \div \dfrac{12}{7}$

   =

   =

2. $\dfrac{\frac{3}{25}}{\frac{9}{20}} =$

   =

   =

3. $\dfrac{\frac{7}{8}}{\frac{21}{10}} =$

   =

   =

4. $\dfrac{\dfrac{X}{4}}{\dfrac{8}{Y}}$ =

5. $\dfrac{\dfrac{X}{12}}{\dfrac{Y}{4}}$ =

6. $\dfrac{\dfrac{3X^3}{14Y^2}}{\dfrac{21X^5}{8Y}}$ =

7. $\dfrac{\dfrac{4X}{X-4}}{\dfrac{8Y}{X^2-16}}$ = $\dfrac{4X}{X-4} \div \dfrac{8Y}{X^2-16}$

$\qquad = \dfrac{4X}{(X-4)} \cdot \dfrac{(X-4)(X+4)}{8Y}$

$\qquad =$

8. $\dfrac{\dfrac{12X}{X^2-9}}{\dfrac{4X}{X+3}}$ =

9. $\dfrac{\dfrac{X^3-8}{X+2}}{\dfrac{X^2+2X+4}{X^2-4}}$ =

10. $\dfrac{\dfrac{X^2-2X-3}{X^3+1}}{\dfrac{X^2-3X}{X^2-X+1}}$ =

When the numerator and/or denominator of a complex fraction consists of two or more fractions, there are two methods of simplifying. Consider the problem: $\dfrac{\dfrac{1}{x}-\dfrac{1}{y}}{\dfrac{1}{x}+\dfrac{1}{y}}$.

METHOD I. **Rewrite the problem in the form of a <u>division</u> problem.**

$$\frac{\left(\dfrac{1}{x}-\dfrac{1}{y}\right)}{\left(\dfrac{1}{x}+\dfrac{1}{y}\right)} = \textbf{(NUMERATOR)} \div \textbf{(DENOMINATOR)}$$

$$= \left(\frac{1}{x}-\frac{1}{y}\right) \div \left(\frac{1}{x}+\frac{1}{y}\right)$$

$$= \frac{y-x}{xy} \div \frac{y+x}{xy}$$

$$= \frac{y-x}{xy} \bullet \frac{xy}{y+x} = \frac{y-x}{y+x}$$

METHOD II. **Find a common denominator for the entire problem. Multiply the numerator and denominator by this LCD. (LCD = XY!)**

$$\frac{xy\left(\dfrac{1}{x}-\dfrac{1}{y}\right)}{xy\left(\dfrac{1}{x}+\dfrac{1}{y}\right)} \quad \textbf{(USE DISTRIBUTIVE PROPERTY!)}$$

$$= \frac{xy\bullet\dfrac{1}{x} - xy\bullet\dfrac{1}{y}}{xy\bullet\dfrac{1}{x} + xy\bullet\dfrac{1}{y}} \quad \textbf{(You should do this step mentally!)}$$

$$= \frac{y\,x}{y+x}$$

Which method is easier? Sometimes one method is easier; sometimes the other is easier. Sometimes it is a toss up! There will be more explanation on this later.

In 11-13, use method I.

11. $$\dfrac{\dfrac{5}{X} - \dfrac{3}{Y}}{\dfrac{2}{X} + \dfrac{3}{Y}}$$

12. $$\dfrac{\dfrac{X}{5} - \dfrac{5}{X}}{1 + \dfrac{5}{X}}$$

13. $$\dfrac{1 - \dfrac{3}{X}}{\dfrac{X}{3} - \dfrac{3}{X}}$$

In 14 - 16, use method II.

14.
$$\frac{(\dfrac{5}{X} - \dfrac{3}{Y})}{(\dfrac{2}{X} + \dfrac{3}{Y})}$$

15.
$$\frac{\dfrac{X}{5} - \dfrac{5}{X}}{1 + \dfrac{5}{X}}$$

16.
$$\frac{1 - \dfrac{3}{X}}{\dfrac{X}{3} - \dfrac{3}{X}}$$

For exercises in which the common denominator for the numerator is the same as the common denominator for the denominator, Method II will be easier. For more complex exercises, especially when the LCD for the numerator differs greatly from the LCD for the denominator, Method I will be easier. The great advantage for Method I is that you can separate the problem into two smaller problems. In this way, you can ignore half of the problem, while you work on the other half.

In 17 - 24, use either method:

17. $\dfrac{\dfrac{1}{X} + \dfrac{1}{2}}{\dfrac{4}{X^2} - 1}$.

18. $\dfrac{\dfrac{1}{X^2} - 1}{\dfrac{1}{X} - 1}$

(METHOD II works nicely in #19 - 24. Why??)

19. $\dfrac{3 - \dfrac{6}{X - 2}}{8 - \dfrac{16}{X - 2}}$

20. $\dfrac{1 - \dfrac{5}{X} + \dfrac{6}{X^2}}{1 - \dfrac{3}{X} + \dfrac{2}{X^2}}$

21. $\dfrac{4 + \dfrac{4}{X} - \dfrac{3}{X^2}}{4 - \dfrac{8}{X} + \dfrac{3}{X^2}}$

22. $\dfrac{1 + \dfrac{1}{X - 1}}{1 - \dfrac{1}{X - 1}}$

23.
$$\frac{3 + \dfrac{9}{X-2}}{3 - \dfrac{3}{X-2}}$$

24.
$$\frac{\dfrac{1}{X} + \dfrac{1}{X+2}}{\dfrac{1}{X} - \dfrac{1}{X+2}}$$

In 25 - 28, where common denominators are different, Method I is probably easier.

25.
$$\frac{1 - \dfrac{2}{X+2}}{1 + \dfrac{2}{X-2}}$$

26.
$$\frac{\dfrac{X}{X+1} + 1}{\dfrac{2X+1}{X-1}}$$

$$(\qquad) \div (\qquad)$$

27.
$$\frac{\dfrac{X}{X-1} + 2}{\dfrac{6X-4}{X+2}}$$

28.
$$\frac{\dfrac{6x}{X-1} - 3}{4 - \dfrac{12}{X+4}}$$

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29. $$\dfrac{\dfrac{X+2}{X} + \dfrac{X}{X+2}}{\dfrac{X+2}{X} - \dfrac{X}{X+2}}$$

30. $$\dfrac{\dfrac{X+2}{X} + \dfrac{X}{X-2}}{\dfrac{X+2}{X} - \dfrac{X}{X-2}}$$

31. $$\dfrac{\dfrac{X+2}{X} + \dfrac{X}{X+2}}{\dfrac{X+2}{X} - \dfrac{X}{X-2}}$$

32. $$\dfrac{\dfrac{4}{X} - \dfrac{1}{X-2}}{\dfrac{4}{X+2} - \dfrac{4}{X}}$$

33. $$\dfrac{\dfrac{4}{X-4} - \dfrac{2}{X}}{\dfrac{-6}{X+1} + \dfrac{8}{X}}$$

34. $$\dfrac{\dfrac{4}{X+Y} - \dfrac{2}{X}}{\dfrac{4}{X-Y} - \dfrac{2}{X}}$$

In 35 - 54, eliminate negative exponents and simplify:

35. $\dfrac{X^{-2} + Y^{-2}}{X^{-2} - Y^{-2}} = \dfrac{\dfrac{1}{X^2} + \dfrac{1}{Y^2}}{\dfrac{1}{X^2} - \dfrac{1}{Y^2}}$

$= \dfrac{(X^2Y^2)\left(\dfrac{1}{X^2} + \dfrac{1}{Y^2}\right)}{(X^2Y^2)\left(\dfrac{1}{X^2} - \dfrac{1}{Y^2}\right)}$

$=$ _____

36. $\dfrac{2X^{-1} - (2Y)^{-1}}{2XY^{-1}} = \dfrac{2 \cdot \dfrac{1}{X} - \dfrac{1}{2Y}}{2X \cdot \dfrac{1}{Y}}$

$= \dfrac{(2XY)\left(2 \cdot \dfrac{1}{X} - \dfrac{1}{2Y}\right)}{(2XY)\left(2X \cdot \dfrac{1}{Y}\right)}$

$=$ _____

37. $\dfrac{X^{-1} - Y^{-1}}{X^{-1} + Y^{-1}} =$

38. $\dfrac{(3X)^{-1} + 3X^{-1}}{X^{-1} + 3^{-1}} =$

39. $\dfrac{(4X)^{-2} + 4^{-2}}{4X^{-2} - 4^{-2}} =$

40. $\dfrac{X^{-1} - Y^{-1}}{X^{-2} - Y^{-2}} =$

41. $\dfrac{2X - (2X)^{-1}}{1 + (2X)^{-1}} =$

42. $\dfrac{1 + (2X)^{-1}}{2X + (2X)^{-2}} =$

43. $\dfrac{2^{-1}X - 2X^{-1}}{4^{-1}X^2 - 2X^{-1}} =$

44. $\dfrac{2X^{-2} + (2X)^{-2}}{3X^{-1} + 3X^{-2}} =$

45. $(X^{-1} + Y^{-1})^{-1}$

$= \left(\dfrac{1}{X} + \dfrac{1}{Y}\right)^{-1}$

$= \left(\dfrac{Y + X}{XY}\right)^{-1}$

$= \underline{\hspace{2cm}}$

46. $(X^{-1} - Y^{-1})^{-1}$

47. $(X^{-2} - Y^{-2})^{-1}$

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48. $(X^{-1} + Y^{-1})^{-2}$ **49.** $(X^{-1} - Y^{-1})^{-2}$ **50.** $(3X^{-1} + 3Y^{-1})^{-1}$

51. $(3X^{-1} - 3Y^{-1})^{-1}$ **52.** $[(3X)^{-1} + (3Y)^{-1}]^{-1}$

53. $[(3X)^{-1} - (3Y)^{-1}]^{-1}$ **54.** $[(3X)^{-2} - 3Y^{-2}]^{-1}$

p.190-200:

1. $\frac{7}{8}$; 2. $\frac{4}{15}$; 3. $\frac{5}{12}$; 4. $\frac{XY}{32}$; 5. $\frac{X}{3Y}$; 6. $\frac{4}{49X^2Y}$; 7. $\frac{X(X+4)}{2Y}$;

8. $\frac{3}{X-3}$; 9. $(X-2)^2$; 10. $\frac{1}{X}$; 11. $\frac{5Y-3X}{2Y+3X}$; 12. $\frac{X-5}{5}$; 13. $\frac{3}{X+3}$;

14. $\frac{5Y-3X}{2Y+3X}$; 15. $\frac{X-5}{5}$; 16. $\frac{3}{X+3}$; 17. $\frac{X}{2(2-X)}$; 18. $\frac{1+X}{X}$; 19. $\frac{3}{8}$;

20. $\frac{X-3}{X-1}$; 21. $\frac{2X+3}{2X-3}$; 22. $\frac{X}{X-2}$; 23. $\frac{X+1}{X-3}$; 24. $X+1$; 25. $\frac{X-2}{X+2}$;

26. $\frac{X-1}{X+1}$; 27. $\frac{X+2}{2(X-1)}$; 28. $\frac{3(X+4)}{4(X-1)}$; 29. $\frac{X^2+2X+2}{2(X+1)}$; 30. $\frac{2-X^2}{2}$;

31. $\frac{(X-2)(X^2+2X+2)}{-2(X+2)}$; 32. $\frac{(3X-8)(X+2)}{-8(X-2)}$; 33. $\frac{X+1}{X-4}$; 34. $\frac{(X-Y)^2}{(X+Y)^2}$;

35. $\frac{Y^2+X^2}{Y^2-X^2}$; 36. $\frac{4Y-X}{4X^2}$; 37. $\frac{Y-X}{Y+X}$; 38. $\frac{10}{3+X}$; 39. $\frac{1+X^2}{64-X^2}$;

40. $\frac{XY}{Y+X}$; 41. $2X-1$; 42. $\frac{2X}{4X^2-2X+1}$; 43. $\frac{2(X+2)}{X^2+2X+4}$; 44. $\frac{3}{4(X+1)}$;

45. $\dfrac{XY}{X+Y}$; **46.** $\dfrac{XY}{Y-X}$; **47.** $\dfrac{X^2Y^2}{Y^2-X^2}$; **48.** $\dfrac{X^2Y^2}{(Y+X)^2}$; **49.** $\dfrac{X^2Y^2}{(Y-X)^2}$; **50.** $\dfrac{XY}{3(Y+X)}$;

51. $\dfrac{XY}{3(Y-X)}$; **52.** $\dfrac{3XY}{Y+X}$; **53.** $\dfrac{3XY}{Y-X}$; **54.** $\dfrac{9X^2Y^2}{Y^2-27X^2}$.