2.05 *Addition and Subtraction of Fractions*

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FINDING THE COMMON DENOMINATOR

Addition and subtraction of fractions requires a completely different procedure from the multiplication and division of the last section. Fractions cannot be added (or subtracted) unless they have a **common denominator.** Fractions that have a common denominator are added in the same way that you combine like terms. Just like adding 3 apples plus 2 apples (which equals 5 apples), adding 3 sevenths plus 2 sevenths equals 5 sevenths. This is written in math symbols:

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}.$$

When you add or subtract fractions **with a common denominator,** you add or subtract the numerators, and place that answer over the common denominator. And of course remember, you are still not allowed (and never will be allowed!) to divide by zero!

PRINCIPLE: $\dfrac{a}{d} + \dfrac{b}{d} - \dfrac{c}{d} = \dfrac{a+b-c}{d}, \quad d \neq 0$

PRINCIPLE: Before you can add or subtract fractions, you must have a common denominator.

If the fractions do not have a common denominator, then the first order of business must be to find a common denominator. (It must really be lost! People have been looking for it for decades!)

A common denominator is a number or an expression such that each denominator divides into the common denominator evenly. For example, to add $\frac{1}{2} + \frac{1}{3}$, the common denominator of **6** may be used, since both 2 and 3 divide evenly into 6. However, notice that any multiple of 6, like 12, 18, 24, 36, etc., are also divisible by 2 and 3, so these could also be considered "common denominators" for this addition problem. Finally, we can say that of all these common denominators, **6 is the <u>least (lowest) common denominator</u> (LCD)**, and in <u>most</u> cases the **least common denominator is the best one to use.**

DEFINITIONS: **A <u>common denominator</u> is a number or quantity such that each denominator divides evenly into it.**

The <u>least (lowest) common denominator</u> (LCD) is the smallest of all the common denominators.

Many of "least common denominators" are rather obvious. While there are many explanations and methods of finding LCDs, a good approach is to begin with "obvious" examples (intuitive method). From these examples, you will discover a strategy that will allow you to quickly and easily find most LCDs. Then, after further probing, you will discover a method that works to find <u>all</u> LCDs, even those you do not intuitively know. This method will not only guarantee your ability to find LCDs regardless of how complicated it may be--it will ensure your understanding of the entire concept of LCDs--the what, the how, and the why!

In the following exercises, by trial and error, find the least common denominators (LCD). Remember you are trying to find the smallest possible number that each of the denominators divides into evenly. For your convenience, the answers to this page are provided at the bottom of the page:

1. $\frac{1}{2}, \frac{1}{4}$ LCD = ____ 2. $\frac{1}{2}, \frac{1}{6}$ LCD = ____ 3. $\frac{1}{2}, \frac{1}{8}$ LCD = ____

4. $\frac{1}{3}, \frac{1}{6}$ LCD = ____ 5. $\frac{1}{3}, \frac{1}{12}$ LCD = ____ 6. $\frac{1}{2}, \frac{1}{12}$ LCD = ____

7. $\frac{1}{4}, \frac{1}{12}$ LCD = ____ 8. $\frac{1}{6}, \frac{1}{12}$ LCD = ____ 9. $\frac{1}{5}, \frac{1}{10}$ LCD = ____

10. $\frac{1}{5}, \frac{1}{20}$ LCD = ____ 11. $\frac{1}{20}, \frac{1}{60}$ LCD = ____ 12. $\frac{1}{6}, \frac{1}{24}$ LCD = ____

13. $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ LCD = ____ 14. $\frac{1}{2}, \frac{1}{3}, \frac{1}{12}$ LCD = ____

15. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{12}$ LCD = ____ 16. $\frac{1}{2}, \frac{1}{6}, \frac{1}{8}, \frac{1}{24}$ LCD = ____

17. $\frac{1}{2}, \frac{1}{3}$ LCD = ____ 18. $\frac{1}{3}, \frac{1}{4}$ LCD = ____ 19. $\frac{1}{2}, \frac{1}{5}$ LCD = ____

20. $\frac{1}{3}, \frac{1}{5}$ LCD = ____ 21. $\frac{1}{4}, \frac{1}{5}$ LCD = ____ 22. $\frac{1}{5}, \frac{1}{7}$ LCD = ____

23. $\frac{1}{2}, \frac{1}{3}, \frac{1}{5}$ LCD = ____ 24. $\frac{1}{2}, \frac{1}{5}, \frac{1}{7}$ LCD = ____

ANSWERS:
1.4; **2.**6; **3.**8; **4.**6; **5.**12; **6.**12; **7.**12; **8.**12;
9.10; **10.**20; **11.**60; **12.**24; **13.**6; **14.**12; **15.**12; **16.**24;
17.6; **18.**12; **19.**10; **20.**15; **21.**20; **22.**35; **23.**30; **24.**70.

What is the pattern in #1-16? ______________________

What is the pattern in #17-24? ______________________

25. $\frac{1}{4}, \frac{1}{6}$ LCD = _____ 26. $\frac{1}{6}, \frac{1}{8}$ LCD = _____ 27. $\frac{1}{10}, \frac{1}{15}$ LCD = _____

28. $\frac{1}{6}, \frac{1}{15}$ LCD = _____ 29. $\frac{1}{10}, \frac{1}{25}$ LCD = _____ 30. $\frac{1}{10}, \frac{1}{8}$ LCD = _____

#31 - 34 are harder to do by trial and error. This underscores the need for systematic strategies that will follow these exercises. (Before getting frustrated, you may want to sneak a peek ahead!)

31. $\frac{1}{9}, \frac{1}{60}$ LCD = _____ 32. $\frac{1}{25}, \frac{1}{40}$ LCD = _____

33. $\frac{1}{27}, \frac{1}{36}$ LCD = _____ 34. $\frac{1}{50}, \frac{1}{45}$ LCD = _____

From these exercises you probably noticed that sometimes the LCD is the larger (or largest) of the numbers (as in **#1-16**). Sometimes the LCD is the product of the numbers (as in **#17-24**). Sometimes you just have to use trial and error looking at multiples of the denominators (as in **#25-34**). Sometimes you could spend hours looking (as in **#35-40**)!

An easy way to find the LCD is to begin with the largest denominator--see if that is the LCD. If it is not, then take multiples of that largest denominator, trying each one in order, until you find one that "works"--that is, until you find one that each of the other denominators divides into evenly.

For example, if the denominators are **6, 12,** and **9,** begin with the largest number which is **12.** Since 12 is <u>not</u> divisible by 9, try multiples of 12: **24, 36, 48, 60, etc.** Notice that 24 does not work either, but **36 does work, since it is divisible by 6, 12, and 9.** Therefore, **the LCD is 36.** This is a quick and easy way to find most LCDs. Unfortunately, some problems could take a long time to do this way.

Perhaps you also correctly noticed that if the denominators have no common factors, then the LCD is the product of the denominators.

Most importantly, perhaps you noticed that every LCD is built using the prime factors of the denominators involved. The following examples should add details to this idea, and enable you to develop a systematic way of finding the LCD, based upon the factors of the denominators. Begin by factoring each denominator into its prime factors.

EXAMPLE 1: $\frac{1}{2}, \frac{1}{10} = \frac{1}{2}, \frac{1}{2 \cdot 5}$

In the LCD, you need factors of **2 and 5,** but the extra 2 is not needed. **LCD = $2 \cdot 5$ = 10.**

EXAMPLE 2: $\frac{1}{5}, \frac{1}{7}, \frac{1}{2}$

The denominators consist of prime factors **2, 5, and 7.** Therefore the **LCD = $2 \cdot 5 \cdot 7$ = 70.**

EXAMPLE 3: $\frac{1}{6}, \frac{1}{9} = \frac{1}{2 \cdot 3}, \frac{1}{3^2}$

In this case the prime factors are 2 and 3. Since you probably already know the LCD is 18, you will need factors of **2** and of **3^2 (the highest power of the 3 factors).**

EXAMPLE 4: $\frac{1}{8}, \frac{1}{20} = \frac{1}{2^3}, \frac{1}{2^2 \cdot 5}$

The prime factors are 2 and 5. If you take **2^3 (the highest power of 2)** that will include the other **2^2**. Therefore, **LCD = $2^3 \cdot 5$ = 40.**

EXAMPLE 5: $\frac{1}{24}, \frac{1}{18} = \frac{1}{2^3 \cdot 3}, \frac{1}{2 \cdot 3^2}$

This time the highest power of 2 is 3, and the highest power of 3 is 2. Therefore, **LCD = $2^3 \cdot 3^2$ = $8 \cdot 9$ = 72.**

PRINCIPLE: To find the LCD, factor each denominator into prime factors. The LCD is the product of the factors of the denominators, *each factor being raised to the highest power* of the factor.

Find the LCDs by factoring each denominator into prime factors.

35. $\frac{1}{30}, \frac{1}{25}$ LCD = ______

36. $\frac{1}{27}, \frac{1}{60}$ LCD = ______

37. $\frac{1}{54}, \frac{1}{32}$ LCD = ______

38. $\frac{1}{24}, \frac{1}{52}$ LCD = ______

39. $\frac{1}{21}, \frac{1}{98}$ LCD = ______

40. $\frac{1}{72}, \frac{1}{40}$ LCD = ______

This technique is particularly useful where variables are involved.

EXAMPLE 6. $\frac{1}{3X^2}, \frac{1}{8X^3}$

The LCD for 3 and 8 is obviously 24. The highest power of X is X^3. **LCD=24 X^3.**

EXAMPLE 7. $\frac{1}{3X^3Y}, \frac{1}{3X^2Y^4}$

LCD = $3X^3.Y^4$.

EXAMPLE 8. $\frac{1}{3X}, \frac{1}{X+3}$

The factors are **3, X, and X+3.** Notice that X+3 is a distinct factor. **LCD = 3X(X+3).**

EXAMPLE 9. $\frac{1}{X^2+3X+2}, \frac{1}{X^2+5X+6}$

$= \frac{1}{(X+2)(X+1)}, \frac{1}{(X+2)(X+3)}$

First factor each denominator. Then the factors are **(X+2), (X+1), and (X+3).** (Any order!) **LCD = (X+1)(X+2)(X+3).**

EXAMPLE 10. $\frac{1}{X^2+5X+6}, \frac{1}{X^2+6X+9}$

$= \frac{1}{(X+2)(X+3)}, \frac{1}{(X+3)^2}$

The prime factors are **(X+2) and (X+3).** **(X+3)** to the highest power is **$(X+3)^2$.**
LCD = $(X+2)(X+3)^2$.

In the following exercises, find the LCD.

41. $\frac{1}{16}, \frac{1}{18}, \frac{1}{24}$

42. $\frac{1}{16}, \frac{1}{24}, \frac{1}{30}$

43. $\frac{1}{36}, \frac{1}{20}, \frac{1}{25}$

44. $\frac{1}{72}, \frac{1}{30}, \frac{1}{20}$

45. $\frac{1}{5X^2}, \frac{1}{10X^3}, \frac{1}{2X^4}$

46. $\frac{1}{8X^3}, \frac{1}{10X}, \frac{1}{20X^2}$

47. $\frac{1}{25X^2Y^3}, \frac{1}{10XY^2}, \frac{1}{2X^4Y}$

48. $\frac{1}{18Y^3}, \frac{1}{10XY^2}, \frac{1}{25X^2Y^5}$

49. $\frac{1}{12X^3}, \frac{1}{20X^7(X+3)}$

50. $\frac{1}{12X^3(X+4)}, \frac{1}{15X(X+4)^2}$

51. $\frac{1}{15X^3(X-5)^3}, \frac{1}{20X^2(X-5)^2}$

52. $\frac{1}{9X^4(X-2)^2}, \frac{1}{15X^5(X-2)^6}$

53. $\dfrac{1}{X^2 + 3X}$, $\dfrac{1}{X^2 + 5X + 6}$

$= \dfrac{1}{X(\qquad)}$, $\dfrac{1}{(\qquad)(\qquad)}$

LCD = ______________

54. $\dfrac{1}{X^2 - 5X}$, $\dfrac{1}{X^2 - 25}$

$= \dfrac{1}{\_\_\_(\qquad)}$, $\dfrac{1}{(\qquad)(\qquad)}$

LCD = ______________

55. $\dfrac{1}{X^2 + 5X - 6}$, $\dfrac{1}{X^2 - 36}$

$= \dfrac{1}{(\qquad)(\qquad)}$, $\dfrac{1}{(\qquad)(\qquad)}$

LCD = ______________

56. $\dfrac{1}{X^2 - 4}$, $\dfrac{1}{X^2 - 5X + 6}$

$= \dfrac{1}{(\qquad)(\qquad)}$, $\dfrac{1}{(\qquad)(\qquad)}$

LCD = ______________

57. $\dfrac{1}{X^2 + 9X + 20}$, $\dfrac{1}{X^2 + 6X + 8}$

=

LCD = ______________

58. $\dfrac{1}{X^2 - 4X}$, $\dfrac{1}{X^2 + 4X}$

=

LCD = ______________

59. $\dfrac{1}{X^2 - 2X + 1}$, $\dfrac{1}{X^2 - X}$

=

LCD = ______________

60. $\dfrac{1}{X^2 - 4X + 4}$, $\dfrac{1}{X^2 - 4}$

=

LCD = ______________

61. $\dfrac{1}{X^2 - 8X + 16}$, $\dfrac{1}{X^2 - 5X + 4}$

=

LCD = ______________

62. $\dfrac{1}{X^2 + 10X + 25}$, $\dfrac{1}{X^2 + 3X - 10}$

=

LCD = ______________

ADDING AND SUBTRACTING FRACTIONS

Before beginning the algebra of adding and subtracting fractions, it might be helpful to do a few "simple" fractions, especially since you have a calculator to help out. As always, when using the calculator, it is good to begin with simple exercises to check it out first.

Solve each of the following by calculator (especially if you have fractions capabilities) and check by regular LCD methods.

EXAMPLE 1. $\frac{1}{8} + \frac{3}{8}$

CALCULATOR:
If you have **a b/c** button, type **"1"**, **"a b/c"**, **"8"**, **"+"**, **"3"**, **"a b/c"**, **"8"**, **"="**, _____ Your calculator should say "1 ⌋ 2", which means 1/2.

Without the fractions key, you can enter "1", "÷", "8", "+", "3", "÷", "8", "=", _____ The calculator gives the decimal 0.5 which is 1/2.

IN YOUR HEAD: you add to get 4/8, which reduces to 1/2.

EXAMPLE 2. $\frac{3}{4} + \frac{1}{2}$

CALCULATOR:
If you have **a b/c** button, type **"3"**, **"a b/c"**, **"4"**, **"+"**, **"1"**, **"a b/c"**, **"2"**, **"="**, _____ The calculator says "1 1 ⌋ 4", which means a mixed fraction "1 1/4", which is 5/4.

Without the fractions key, you can enter "3", "÷", "4", "+", "1", "÷", "2", "=", _____ The calculator gives the decimal 1.25 which is 1 1/4 or 5/4.

IN YOUR HEAD: you must find LCD of 4. Then 3/4 + 2/4 gives you 5/4, or 1 1/4.

EXAMPLE 3. $\frac{5}{12} + \frac{3}{16}$

CALCULATOR:
If you have **a b/c** button, type **"5"**, **"a b/c"**, **"12"**, **"+"**, **"3"**, **"a b/c"**, **"16"**, **"="**, _____ Did you get "29 ⌋ 48" (i.e. 29/48)?

Without fractions key, you get a decimal you may or may not be able to convert to a fraction.

IN YOUR HEAD: It is much harder. Find LCD = 48. We'll save the rest of the check for later.

EXAMPLE 4. $\frac{1}{3} + \frac{1}{8} + \frac{3}{4} + \frac{7}{24}$

CALCULATOR:
If you have **a b/c** button, you can easily enter the numbers and get "1 1 ⌋ 2", which means 1 ½ or 3/2.

Even without the fractions key, you should easily get the decimal value 1.5 which is 1 ½ or 3/2.

IN YOUR HEAD: The LCD = 24, and with some work (later!) the answer reduces to 3/2.

Use a calculator to add or subtract the following fractions:

1. $\frac{2}{3} + \frac{3}{5}$ 2. $\frac{3}{8} + \frac{5}{6}$ 3. $\frac{7}{6} - \frac{2}{3}$ 4. $\frac{5}{24} - \frac{1}{8}$

5. $\frac{2}{13} + \frac{5}{17}$ 6. $\frac{23}{36} - \frac{14}{23}$ 7. $\frac{5}{6} + \frac{17}{18}$ 8. $\frac{25}{28} - \frac{13}{42}$

The first question that must be answered when adding and subtracting fractions without a calculator is this: "Is there a common denominator?" If the fractions already have a common denominator, then just put down the common denominator as THE denominator of the answer. To get the numerator of the answer just add (or subtract) the numerators. Then, of course, try to reduce the fractions, if possible, as you did in an earlier section.

9. $\frac{2X}{3} + \frac{10Y}{3}$ 10. $\frac{3X}{5} - \frac{13Y}{5}$ 11. $\frac{2X}{3} + \frac{10X}{3}$ 12. $\frac{3Y}{5} - \frac{13Y}{5}$

$= \frac{\qquad}{3}$ $= \underline{\qquad}$ $=$ $=$

$= \underline{\qquad}$ (Reduce!) $= \underline{\qquad}$

13. $\frac{3}{4X} - \frac{9}{4X}$ 14. $\frac{2}{7X^2} - \frac{9}{7X^2}$

15. $\frac{X^2}{X+4} + \frac{4X}{X+4} = \frac{\qquad}{X+4}$ 16. $\frac{Y^2}{Y-4} - \frac{4Y}{Y-4} =$

FACTOR: $=$

REDUCE: $=$

17. $\dfrac{X^2 + 4}{X+2} + \dfrac{4X}{X+2} = \dfrac{\qquad\qquad}{X+2}$

$= \dfrac{X^2 + 4X + 4}{X+2}$

$=$

$=$

18. $\dfrac{Y^2 + 9}{Y-3} - \dfrac{6Y}{Y-3} =$

Notice that in the next few exercises, the primary concept in adding or subtracting fractions is putting down the LCD and then adding or subtracting numerators. Whether the <u>numerators</u> can be factored or not is usually irrelevant. Don't forget that if you were multiplying or dividing the fractions, the first step would be to multiply everything, both numerators and denominators. However, when adding or subtracting fractions, <u>***there is usually no need to factor numerators***</u>! (Even if they do factor, it is <u>usually</u> not a good idea!)

19. $\dfrac{X^2 - 12X}{X - 6} + \dfrac{X^2 - 5X + 30}{X - 6}$

$= \dfrac{\qquad + \qquad}{X - 6}$

$=$

$=$

$=$

20. $\dfrac{Y^2 + 4Y}{3Y + 5} + \dfrac{2Y^2 + 4Y + 5}{3Y + 5} =$

In the next exercises, be careful of the signs:

21. $\dfrac{3Y^2 + 4Y}{Y + 2} - \dfrac{2Y^2 - 4}{Y + 2}$

22. $\dfrac{5X^2}{X - 2} - \dfrac{4X^2 + 3X - 2}{X - 2}$

$= \dfrac{-2Y^2 \quad + \quad 4}{Y + 2}$

=

=

=

23. $\dfrac{3X^2 - 4X}{X^2 - 4} - \dfrac{2X^2 - 3X + 6}{X^2 - 4}$

24. $\dfrac{2X^3 - 6X^2 - 3}{X^3 + X^2 - 6X} - \dfrac{X^3 - 8X - 3}{X^3 + X^2 - 6X}$

=

=

=

=

=

NOW, if the fractions do **__not__** have a common denominator, then you must factor each denominator completely and form the common denominator from the denominator factors, as in previous exercises. Next compare each denominator of the problem to the LCD and in each case decide **"WHAT'S MISSING?"** You must multiply the numerator and denominator of each fraction by the "missing factors." Don't forget! Always multiply the **__numerator__** ***__AND__*** **__denominator__!** Then combine numerators adding or subtracting like terms, as illustrated in the next examples. Between the examples and the exercises, you will find an outline summary of the entire procedure.

EXAMPLE 5. $\dfrac{3}{8X^2Y} + \dfrac{5}{6X^3}$ The LCD is $24X^3Y$.

1st Denom missing: 3X

2nd Denom missing: 4Y

Multiply numerator and denominator of each fraction by "What's Missing":

$$= \frac{3}{8X^2Y} \cdot \frac{(\quad)}{(\quad)} + \frac{5}{6X^3} \cdot \frac{(\quad)}{(\quad)}$$

$$= \frac{3}{8X^2Y} \cdot \frac{(3X)}{(3X)} + \frac{5}{6X^3} \cdot \frac{(4Y)}{(4Y)}$$

$$= \frac{9X + 20Y}{24X^3Y}$$

EXAMPLE 6. $\dfrac{2}{X^2+3X} + \dfrac{3}{X^2+4X+3}$ Factor denominators

$= \dfrac{2}{X(X+3)} + \dfrac{3}{(X+3)(X+1)}$ The LCD is X(X+3)(X+1).

1st Denom missing: (X+1)

2nd Denom missing: (X)

Multiply numerator and denominator of each fraction by "What's Missing":

$$= \frac{2}{X(X+3)} \cdot \frac{(\quad)}{(\quad)} + \frac{3}{(X+3)(X+1)} \cdot \frac{(\quad)}{(\quad)}$$

$$= \frac{2}{X(X+3)} \cdot \frac{(X+1)}{(X+1)} + \frac{3}{(X+3)(X+1)} \cdot \frac{(X)}{(X)}$$

$$= \frac{2(X+1) + 3(X)}{X(X+3)(X+1)} = \frac{2X + 2 + 3X}{X(X+3)(X+1)} = \frac{5X + 2}{X(X+3)(X+1)}$$

EXAMPLE 7. $\dfrac{X}{X^2-6X+9} - \dfrac{4}{X^2+2X-15}$ Factor denominators

$= \dfrac{X}{(X-3)^2} - \dfrac{4}{(X-3)(X+5)}$ The LCD is $(X-3)^2(X+5)$.

1st Denom missing: (X+5)

2nd Denom missing: (X-3)

Multiply numerator and denominator of each fraction by "What's Missing":

$$= \frac{X}{(X-3)^2} \cdot \frac{(\quad)}{(\quad)} - \frac{4}{(X-3)(X+5)} \cdot \frac{(\quad)}{(\quad)}$$

$$= \frac{X}{(X-3)^2} \cdot \frac{(X+5)}{(X+5)} - \frac{4}{(X-3)(X+5)} \cdot \frac{(X-3)}{(X-3)}$$

$$= \frac{X^2 + 5X - 4X + 12}{(X-3)^2(X+5)} = \frac{X^2 + X + 12}{(X-3)^2(X+5)}$$

ADDITION AND SUBTRACTION OF FRACTIONS

Summary

I. FIND THE LEAST COMMON DENOMINATOR (LCD).

A. Factor each denominator to determine what factors are needed for the common denominator.

B. For each of the denominator factors, determine the highest power of each factor. The **LCD** is the product of each factor raised to its highest power.

C. The LCD becomes the denominator of the fraction.

II. PLAY "WHAT'S MISSING."

A. Compare each denominator to the LCD, and determine the **missing factors** for each denominator.

B. Multiply each **numerator and denominator** by **"What's missing!"**

III. ADD OR SUBTRACT NUMERATORS.

A. Add (or subtract) numerators, and place over the common denominator.

B. Combine like terms and reduce the resulting fraction, if possible.

EXERCISES: Add or subtract the fractions as indicated.

25. $\frac{7}{6XY^2} + \frac{5}{4X^3Y}$ **The LCD is ________.**

1st Denom missing: ______
2nd Denom missing: ______

MULTIPLY NUMERATOR AND DENOMINATOR OF EACH FRACTION BY "WHAT'S MISSING":

$$= \frac{7}{6XY^2} \cdot \frac{(\quad)}{(\quad)} + \frac{5}{4X^3Y} \cdot \frac{(\quad)}{(\quad)}$$

$=$

26. $\frac{4}{3Y^2} - \frac{8}{5X^2Y}$ The LCD is ________.

1st Denom missing: ______
2nd Denom missing: ______

MULTIPLY NUMERATOR AND DENOMINATOR OF EACH FRACTION BY "WHAT'S MISSING":

$$= \frac{4}{3Y^2} \cdot \frac{(\qquad)}{(\qquad)} - \frac{8}{5X^2Y} \cdot \frac{(\qquad)}{(\qquad)}$$

$=$

27. $\frac{6}{25X^3Y^2} - \frac{7}{5XY}$ The LCD is ________.

1st Denom missing: ______
2nd Denom missing: ______

MULTIPLY NUMERATOR AND DENOMINATOR OF EACH FRACTION BY "WHAT'S MISSING":

$$= \frac{6}{25X^3Y^2} - \frac{7}{5XY} \cdot \frac{(\qquad)}{(\qquad)}$$

$=$

[Note: What you are doing is fixing the denominators to look like the common denominator. With regard to the first fraction in #27, if it ain't broke don't fix it!]

28. $\frac{5}{9Y^2} - \frac{7}{18XY}$ 29. $\frac{5}{24Y} - \frac{8}{9XY^3}$ 30. $\frac{7}{5XY^2} + \frac{8}{45X^4Y^3}$

31. $\dfrac{4X}{X^2 - 5X + 6} + \dfrac{3}{X^2 - 3X + 2}$

$= \dfrac{4X}{(\quad)(\quad)} + \dfrac{3}{(\quad)(\quad)}$ The LCD is________________.

$= \dfrac{4X}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)} + \dfrac{3}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)}$

$= \dfrac{\quad + \quad}{(\quad)(\quad)(\quad)} =$

$=$

32. $\dfrac{5}{X^2 - 5X + 6} - \dfrac{3}{X^2 - 9}$

$= \dfrac{5}{(\quad)(\quad)} - \dfrac{3}{(\quad)(\quad)}$ The LCD is________________.

$= \dfrac{5}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)} - \dfrac{3}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)}$

$= \dfrac{\quad}{(\quad)(\quad)(\quad)}$

$=$

33. $\dfrac{4X}{X^2 - 6X + 9} - \dfrac{3}{X^2 - 9}$

$= \dfrac{4X}{(\quad)^2} - \dfrac{3}{(\quad)(\quad)}$ The LCD is ________________.

$= \dfrac{4X}{(\quad)^2} \cdot \dfrac{(\quad)}{(\quad)} - \dfrac{3}{(\quad)(\quad)} \cdot \dfrac{(\quad)}{(\quad)}$

$=$

$=$

34. $\dfrac{X}{X^2 + 4X + 3} - \dfrac{4}{X^2 - 3X - 4}$

35. $\dfrac{2X}{X^2 - 4} - \dfrac{3}{X^2 + X - 6}$

36. $\dfrac{4X}{X^2 + 6X + 9} - \dfrac{3}{X^2 - 9}$

37. $\dfrac{5}{X^2 - 10X + 25} - \dfrac{3}{X^2 - 5X}$

IN #38-40, REMEMBER, FACTORING THE NUMERATOR, IF POSSIBLE, IS USUALLY IRRELEVANT! DON'T FORGET, AFTER SIMPLIFYING, REDUCE ANSWERS BY FACTORING.

38. $\dfrac{3X^2 - 4}{2X^2 - 4X} - \dfrac{X}{X - 2} - \dfrac{3}{2X}$

39. $\dfrac{2a^2 - 7ab - 12b^2}{2a(3a - 4b)} + \dfrac{2a + 4b}{3a - 4b}$

40. $\dfrac{Y + X}{Y - 2X} + \dfrac{Y^2 - 7XY}{2Y^2 - 3XY - 2X^2} + \dfrac{-Y + X}{2Y + X}$

41. $\dfrac{X}{X^3 - 8} + \dfrac{4}{X - 2}$

$= \dfrac{X}{(\quad - \quad)(\quad + \quad + \quad)} + \dfrac{4}{(\quad - \quad)} \cdot \dfrac{(\quad + \quad + \quad)}{(\quad + \quad + \quad)}$

$=$

$=$

42. $\dfrac{X}{X^3 - 8} + \dfrac{4}{X^2 + 2X + 4}$

$= \dfrac{X}{(\quad - \quad)(\quad + \quad + \quad)} + \dfrac{4}{(\quad + \quad + \quad)} \cdot \dfrac{(\qquad)}{(\qquad)}$

$=$

$=$

43. $\frac{X}{X^3 + 8} + \frac{4}{X + 2}$

44. $\frac{X}{X^3 - 64} + \frac{4}{X - 4}$

45. $\frac{X}{X^3 - 125} + \frac{10}{X^2 + 5X + 25}$

46. $\frac{X}{X^3 + 27} + \frac{9}{X^2 - 3X + 9}$

47. $\frac{X}{X^3 - 125} + \frac{10}{X^2 - 25}$

48. $\frac{X}{X^3 + 27} + \frac{9}{X^2 - 9}$

49. $\dfrac{X+4}{4X^2-16X+15} - \dfrac{X-4}{4X^2-4X-15}$

$= \dfrac{X+4}{(\quad - \quad)(\quad - \quad)} - \dfrac{X-4}{(\quad - \quad)(\quad + \quad)}$ The LCD is __________________.

$= \dfrac{(X+4)}{(\qquad)(\qquad)} \cdot \dfrac{(\qquad)}{(\qquad)} - \dfrac{(X-4)}{(\qquad)(\qquad)} \cdot \dfrac{(\qquad)}{(\qquad)}$

$= \dfrac{(\qquad\qquad) - (\qquad\qquad)}{(\qquad)(\qquad)(\qquad)}$

$=$

$=$

50. $\dfrac{X+4}{4X^2-16X+15} - \dfrac{X-4}{4X^2-9}$

NOTE: Problems #51-54 are from *New School Algebra* (1898), by G.A. Wentworth.

51. $\dfrac{3}{10a^2+a-3} - \dfrac{4}{2a^2+7a-4}$

52. $$\frac{1}{X-2} + \frac{1}{X^2-3X+2} - \frac{2}{X^2-4X+3}$$

53. $$\frac{1}{a^2-7a+12} + \frac{2}{a^2-4a+3} - \frac{3}{a^2-5a+4}$$

54. $$\frac{a+b}{a-b} - \frac{a-b}{a+b} - \frac{4ab}{a^2-b^2}$$

55. $$\frac{X}{X + 4} + \frac{2X}{X^2 - 4} - \frac{2}{(X + 2)(X + 4)}$$

56. $$\frac{X}{X^2 - 5X + 6} - \frac{X}{X^2 - 6X + 8} - \frac{12 - X^2}{(X - 2)(X - 3)(X - 4)}$$

FACTORS of "X - Y" and "Y - X"

Frequently it is necessary to work with factors and their negatives. This may occur in reducing fractions, multiplying/ dividing fractions, or adding/subtracting fractions. Sometimes it is helpful to factor a "-1" from one of the factors, in order to make them "match up." Another helpful hint is to remember that any number (except zero, of course!) divided by its negative is "-1". Consider: $\frac{-6}{6} = -1$; $\frac{3}{-3} = -1$; $\frac{-125}{125} = -1$; $\frac{-X}{X} = -1$; $\frac{Y}{-Y} = -1$; and $\frac{-3Z}{3Z} = -1$.

Likewise, since "4-X" is the negative of "X-4", and "Y-X" is the negative of "X-Y": $\frac{X - 4}{4 - X} = -1$ and $\frac{X - Y}{Y - X} = -1$.

1. $\frac{X - 6}{6 - X} =$ ______ 2. $\frac{3X - 8Y}{8Y - 3X} =$ ______ 3. $\frac{8Y - 3X}{3X - 8Y} =$ ______ 4. $\frac{3X - 8Y}{-8Y + 3X} =$ ______

5. $\frac{X^2 - 16}{4 - X} = \frac{(X - 4)(X + 4)}{4 - X}$

$= -1(X + 4)$

or $- X - 4$

6. $\frac{X^2 - 36}{6 - X} =$

7. $\frac{X^3 - 125}{5 - X} =$

8. $\frac{X^3 - 8}{2 - X} =$

9. $\frac{X^3 - 64}{16 - X^2} =$

10. $\frac{X^3 - 27}{9 - X^2} =$

When adding and subtracting fractions such as $\frac{X}{X-6} + \frac{4}{6-X}$, the first reaction may be to use the product (X-6)(6-X) as the least common denominator. True, it is a common denominator, but it is not the least common denominator, and the answer you get will need to be reduced. It is much easier to multiply the numerator and denominator of one of the fractions (either one--your choice!) by "-1". Consider the following examples:

EXAMPLE 1: $\frac{X}{X-6} + \frac{4}{6-X}$

$$= \frac{X}{X-6} + \frac{(-1)}{(-1)} \cdot \frac{4}{6-X}$$

$$= \frac{X}{X-6} + \frac{-4}{X-6}$$

$$= \frac{X-4}{X-6}$$

EXAMPLE 2: $\frac{X}{X^2-9} - \frac{3}{3-X}$

$$= \frac{X}{(X-3)(X+3)} - \frac{(-1)}{(-1)} \cdot \frac{3}{3-X}$$

$$= \frac{X}{(X-3)(X+3)} + \frac{3}{X-3}$$

$$= \frac{X}{(X-3)(X+3)} + \frac{3}{X-3} \cdot \frac{(X+3)}{(X+3)}$$

$$= \frac{X + 3X + 9}{(X-3)(X+3)}$$

$$= \frac{4X + 9}{(X-3)(X+3)}$$

11. $\frac{5}{X-5} + \frac{5}{5-X}$

12. $\frac{X}{X-2} - \frac{X}{2-X}$

13. $\frac{16}{X-8} + \frac{2X}{8-X}$

14. $\frac{2X}{X-10} + \frac{20}{10-X}$

15. $\frac{X^2}{X-2} + \frac{4}{2-X}$

16. $\frac{X^2}{X-2} + \frac{2X}{2-X}$

17. $\dfrac{X^3}{X - 2} + \dfrac{4X}{2 - X}$

18. $\dfrac{X^3}{X - 2} + \dfrac{8}{2 - X}$

19. $\dfrac{X^3}{X - 4} + \dfrac{64}{4 - X}$

20. $\dfrac{X^3}{X - 3} + \dfrac{9X}{3 - X}$

21. $\dfrac{X^2}{3X - 9} + \dfrac{3}{3 - X}$

22. $\dfrac{X^2}{5X - 25} + \dfrac{5}{5 - X}$

23. $\dfrac{X}{X^2 - 25} - \dfrac{5}{5 - X}$

24. $\dfrac{-50}{X^2 - 25} - \dfrac{5}{5 - X}$

ANSWERS 2.05

p.167-172:

1. 4; **2.** 6; **3.** 8; **4.** 6; **5.** 12; **6.** 12; **7.** 12; **8.** 12; **9.** 10; **10.** 20; **11.** 60; **12.** 24; **13.** 6; **14.** 12; **15.** 12; **16.** 24; **17.** 6; **18.** 12; **19.** 10; **20.** 15; **21.** 20; **22.** 35; **23.** 30; **24.** 70; #1-16, LCD is the largest denominator; #17-24, LCD is the product of the denominators; **25.** 12; **26.** 24; **27.** 30; **28.** 30; **29.** 50; **30.** 40; **31.** 180; **32.** 200; **33.** 108; **34.** 450; **35.** 150; **36.** 540; **37.** 864; **38.** 312; **39.** 294; **40.** 360; **41.** 144; **42.** 240; **43.** 900; **44.** 360; **45.** $10X^4$; **46.** $40X^3$; **47.** $50X^4Y^3$; **48.** $450X^2Y^5$; **49.** $60X^7(X+3)$; **50.** $60X^3(X+4)^2$; **51.** $60X^3(X-5)^3$; **52.** $45X^5(X-2)^6$; **53.** $X(X+2)(X+3)$; **54.** $X(X-5)(X+5)$; **55.** $(X-1)(X-6)(X+6)$; **56.** $(X-2)(X+2)(X-3)$; **57.** $(X+4)(X+5)(X+2)$; **58.** $X(X-4)(X+4)$; **59.** $X(X-1)^2$; **60.** $(X-2)^2(X+2)$; **61.** $(X-4)^2(X-1)$; **62.** $(X+5)^2(X-2)$.

p.174-186:

1. 1 4/15; 2. 1 5/24; 3. 1/2; 4. 1/12; 5. 99/221; 6. 25/828;

7. 1 7/9; 8. 7/12; 9. $\frac{2X+10Y}{3}$; 10. $\frac{3X-13Y}{5}$; 11. 4X; 12. -2Y;

13. $-\frac{3}{2X}$; 14. $-\frac{1}{X^2}$; 15. X; 16. Y; 17. X+2; 18. Y-3;

19. 2X-5; 20. Y+1; 21. Y+2; 22. X-1; 23. $\frac{X-3}{X-2}$; 24. $\frac{X-4}{X+3}$;

25. $\frac{14X^2+15Y}{12X^3Y^2}$; 26. $\frac{20X^2-24Y}{15X^3Y^2}$; 27. $\frac{6-35X^2Y}{25X^3Y^3}$; 28. $\frac{10X-7Y}{18XY^2}$;

29. $\frac{15XY^2-64}{72XY^3}$; 30. $\frac{63X^3Y+8}{45X^4Y^3}$; 31. $\frac{4X^2-X-9}{(X-3)(X-2)(X-1)}$;

32. $\frac{2X+21}{(X-2)(X-3)(X+3)}$; 33. $\frac{4X^2+9X+9}{(X-3)^2(X+3)}$; 34. $\frac{X^2-8X-12}{(X+3)(X+1)(X-4)}$;

35. $\frac{2X^2+3X-6}{(X-2)(X+2)(X+3)}$; 36. $\frac{4X^2-15X-9}{(X+3)^2(X-3)}$; 37. $\frac{2X+15}{X(X-5)^2}$; 38. $\frac{X-1}{2X}$;

39. $\frac{2a+3b}{2a}$; 40. $\frac{Y-X}{Y-2X}$; 41. $\frac{4X^2+9X+16}{(X-2)(X^2+2X+4)}$; 42. $\frac{5X-8}{(X-2)(X^2+2X+4)}$;

43. $\frac{4X^2-7X+16}{(X+2)(X^2-2X+4)}$; 44. $\frac{4X^2+17X+64}{(X-4)(X^2+4X+16)}$; 45. $\frac{11X-50}{(X-5)(X^2+5X+25)}$;

46. $\frac{10X+27}{(X+3)(X^2-3X+9)}$; 47. $\frac{11X^2+55X+250}{(X-5)(X+5)(X^2+5X+25)}$; 48. $\frac{10X^2-30X+81}{(X-3)(X+3)(X^2-3X+9)}$;

49. $\frac{22X}{(2X-5)(2X-3)(2X+3)}$; 50. $\frac{24X-8}{(2X-5)(2X-3)(2X+3)}$; 51. $\frac{-17a}{(5a+3)(2a-1)(a+4)}$;

52. $\frac{X-4}{(X-2)(X-3)}$; 53. 0; 54. 0; 55. $\frac{X^2+2}{(X+4)(X-2)}$; 56. $\frac{X+3}{(X-2)(X-3)}$.

p.187-189:

1. -1; **2.** -1; **3.** -1; **4.** 1; **5.** $-(X+4)$ or $-X-4$; **6.** $-(X+6)$ or $-X-6$; **7.** $-(X^2+5X+25)$ or $-X^2-5X-25$; **8.** $-(X^2+2X+4)$ or $-X^2-2X-4$;

9. $\frac{-(X^2+4X+16)}{X+4}$ or $\frac{-X^2-4X-16}{X+4}$; **10.** $\frac{-(X^2+3X+9)}{X+3}$ or $\frac{-X^2-3X-9}{X+3}$;

11. 0; **12.** $\frac{2X}{X-2}$; **13.** -2; **14.** 2; **15.** $X+2$; **16.** X; **17.** $X(X+2)$;

18. X^2+2X+4; **19.** $X^2+4X+16$; **20.** $X(X+3)$; **21.** $\frac{X+3}{3}$; **22.** $\frac{X+5}{5}$;

23. $\frac{6X+25)}{(X-5)(X+5)}$; **24.** $\frac{5}{X+5}$.

Dr. Robert J. Rapalje

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE