2.09 *Ratio and Proportion Variation*

Some of the best answers to the age-old question, "What good is math?" may be found in this section on ratio and proportion. The concept of ratio and proportion is a convenient way of organizing given information, setting up a simple equation, and using this to determine an unknown quantity. The applications to everyday life are innumerable.

First, what is a **ratio**, and what is a **proportion**? **A ratio is simply the quotient of two numbers.** This is where we get the word "**rati**onal numbers." A **rational number** is any number that can be expressed as the "**ratio**" or quotient of two **integers (denom ≠ 0).** Everytime you write a fraction, you have written a ratio. **A proportion is simply the equating of two ratios. Whenever one ratio** (or fraction) **equals another ratio** (or fraction)**, this is a proportion.** In fact, the first concept and the first 14 exercises in the last section were **proportions.** Do you remember the **definition of equality of fractions?** Two fractions, $\frac{a}{b}$ and $\frac{c}{d}$,

are equal if and only if **a·d = b·c.**

$$\frac{a}{b} = \frac{c}{d} \text{ means that } a \cdot d = b \cdot c$$

By the way, this section could be sub-titled "applications of fractional equations."

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EXAMPLE 1: If 5 packages of a product sell for $6.00, how much should 8 packages cost?

SOLUTION: One obvious solution is to find the cost of one package by dividing $6 by 5 (which is $1.20), then multiply by the number of packages that you want (8 pkg) which is $9.60. With a calculator, this is all very easy.

However, as an intoduction to a larger and very useful method, let's set up a proportion. Remember in this that the expression **"how much"** always means **"X"**. While there are many ways to set up a proportion, begin with the

ratio: $\dfrac{packages}{price}$. The proportion will be

$\dfrac{packages}{price} = \dfrac{packages}{price}$. However you set it up, it is very

important that you be consistent. This is the ratio that

you need: $\dfrac{packages}{price}$: $\dfrac{5\ packages}{\$6} = \dfrac{8\ packages}{\$X}$

From this you can see:

$$5 \cdot X = \$6 \cdot 8$$
$$5X = \$48$$
$$X = \$48/5 \text{ or } \$9.60$$

ALTERNATE SOLUTION: $\dfrac{price}{packages}$: $\dfrac{\$6}{5\ packages} = \dfrac{\$X}{8\ packages}$

Notice--equation is same as before:

$$5 \cdot X = \$6 \cdot 8$$
$$5X = \$48$$
$$X = \$48/5 \text{ or } \$9.60$$

EXAMPLE 2: If 5 packages of a product sell for $6.00, how many packages can you buy for $20?

SOLUTION: Set up a ratio $\dfrac{cost}{packages}$: $\dfrac{\$6}{5\ packages} = \dfrac{\$20}{X\ packages}$

$$6X = 100$$
$$X = 100/6 \text{ or } 16.7 \text{ pkg}$$

(Technically, you could only buy 16 packages, since they come in packages, and $20 is not enough to buy the 17th package.)

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1. If a 9-oz. bag of potato chips costs $1.50, what would you expect to pay for a 16 oz. bag?

2. If 8 pounds of dog food costs $6.50, what would you expect to pay for 25 pounds?

3. If a 20-oz. bottle of ketchup costs $0.90, what would you expect to pay for a 44 oz. bottle?

4. If 6 packages of a product cost $2.50, what would you expect to pay for 40 packages?

5. If on the interstate it takes 3 hours to travel 200 miles, how far can you travel at this rate in 16 hours?

6. If on the interstate it takes 3 hours to travel 200 miles, how long will it take to travel 750 miles at this rate?

7. If it takes 3 hours to drive 65 miles in the mountains, how long will it take to drive 100 miles at the same rate?

8. If it takes 35 minutes to drive 15 miles in the mountains, how far can you drive at this rate in 2 hours?

9. If a typist can type 5 pages in 13 hours, how long will it take him to type a 14 page report?

10. If a typist can type 13 pages in 5 hours, how many pages can she type in 14 hours?

11. If an author can complete 5 sections in 2 days, how many sections can she complete in 17 days?

12. If an author can complete 2 sections in 5 days, how long will it take him to complete 17 sections?

13. A gardening chemical is to be applied at 5 teaspoons per 300 sq. ft. How many teaspoons should be applied for 10,000 sq.ft.?

14. A gardening chemical is to be applied at 7 teaspoons per 300 sq. ft. How many sq. ft. can be treated by 100 teaspoons of the chemical?

15. A salt water brine is to be made at the rate of 3 pounds of salt for each 8 gallons of water. How much salt should be used for 25 gallons of water?

16. A salt water brine is to be made at the rate of 3 pounds of salt for each 8 gallons of water. How much water should be used with 25 pounds of salt?

VARIATION

One very simple application of fractional equation-solving and ratio and proportion is **variation**. The root word here is "**vary**", from which we get the word "**variable.**" Remember that **variables vary** (or change), and **constants** stay the same. Frequently variables are related one to another, and as one variable changes, it affects the other variables. There are three vocabulary words to learn--**three types of** "variation:"

"Y varies _directly_ as X" or "Y is proportional to X" means that whatever happens to X, the same happens to Y. If X gets larger, then Y gets larger, and if X gets smaller, then Y gets smaller. The equation for this is "Y = kX", where X and Y are variables, and "k" represents a constant that will be determined later. This constant is called the "constant of variation (or proportionality)." As an example of **direct variation**, when you are traveling at a variable speed or velocity, the faster you drive, the greater the distance you travel. According to this, distance d varies directly as velocity, so "**d=k·v**".

"Y varies _inversely_ (or indirectly) as X" means that whatever happens to X, the opposite happens to Y. If X gets larger, then Y gets smaller, and if X gets smaller, then Y gets larger. The equation for this is "Y = k/X" or "Y = k·1/X", where again X and Y are variables, and "k" represents a constant. As an example of **inverse variation**, if you are traveling a certain distance, the faster you drive, the less time it will take you to arrive. According to this, the time that it takes to arrive varies inversely with the velocity, so "**t=k/v.**"

"Y varies _jointly_ as X and Z" means that Y varies as two or more variables, in this case, X and Z. The equation for this is "Y = kXZ", where X, Y, and Z are variables, and "k" represents a constant. As an example of joint variation, if you are traveling a distance at variable speed and for a variable time, the longer and faster you drive, the greater the distance traveled. Therefore, distance varies jointly as velocity and time, so "**d=k·vt.**" (In this case, k=1!)

The expression "Y varies jointly as X and the square of Z and inversely as the cube of W" could be written in the equation

$$Y = \frac{kXZ^2}{W^3} \ .$$

VARIATION

Y varies *directly* as X	Y = kX
Y varies *inversely* as X	Y = k/X
Y varies *jointly* as X and Z	Y = kXZ

While the exercises that follow can be made to look intimidating (see the last ones!), they are really quite simple, and they may be solved in three steps.

STEP 1: Write the equation for the given statement. Don't forget to use "k".

STEP 2: Whatever variables are involved in the problems, you will always be given one complete set of information. Use that complete set of information to find the value of "k."

STEP 3: Once you have "k", use the value of "k" and the other information that is given in the problem to answer the given question.

One final thought, notice that while the title of this section is *"variation"*, the *"k"* is a constant. This means that while all the **variables** in the problems **vary** (change) throughout the problem, the value of *"k"* is **constant**—it does not change. Once you have determined the value of *"k"*, it remains **constant** throughout the entire problem.

EXERCISES: Write the equation for each of the following statements.

1. Y varies directly as the square of X.

$$Y = k \left(\quad \right)^2$$

2. Y varies directly as the square of t.

$$Y = $$

3. Y varies inversely as the cube of X.

$$Y = \frac{k}{\left(\quad \right)^3}$$

4. Y varies inversely as the cube of t.

$$Y = $$

5. Y varies jointly as q and the square of p.

$$Y = k \left(\quad \right)\left(\quad \right)$$

6. Y varies jointly as the square of X and the cube of W.

$$Y = \underline{\hspace{3cm}}$$

7. Y varies directly as z and inversely as the cube of W.

$$Y = \frac{k\left(\quad \right)}{\left(\quad \right)^3}$$

8. Y varies directly as the cube of Z and inversely as the square root of X.

$$Y = \underline{\hspace{3cm}}$$

9. The distance that a body falls varies directly as the square of the time t.

10. The force F between two bodies in space varies jointly as the product of their masses "M_1" and "M_2," and inversely as the square of the distance "d" between them

EXAMPLE 1.

Y varies directly as X^2, and Y=48 when X=4. Find Y when X=2.

STEP 1: $Y = \kappa X^2$

STEP 2: $48 = k \ 4^2$
$48 = 16 \ k$
$k = 3$

STEP 3: $Y = k \ X^2$
$Y = 3 \cdot 2^2$
$Y = 12$

EXERCISES:

11. Y varies directly as X^2, and Y=8 when X=2. Find Y when X=3.

STEP 1: $Y = \underline{\hspace{2cm}}$

STEP 2: $\underline{\hspace{1cm}} = k \cdot (\quad)^2$

$k = \underline{\hspace{1cm}}$

STEP 3: $Y = \underline{\hspace{2cm}}$

$= \underline{\hspace{2cm}}$

13. Y varies directly as the square root of X, and Y=12 when X=9. Find Y when X=25.

EXAMPLE 2.

Y varies inversely as the square root of X, and Y=12 when X=4. Find Y when X=9.

STEP 1: $Y = \dfrac{k}{\sqrt{X}}$

STEP 2: $12 = \dfrac{k}{\sqrt{4}}$

$12 = \dfrac{k}{2}$

$k = 24$

STEP 3: $Y = \dfrac{24}{\sqrt{X}}$

$Y = \dfrac{24}{\sqrt{9}} = \dfrac{24}{3} = 8$

12. Y varies directly as X^3, and Y=4 when X=2. Find Y when X=3.

14. Y varies inversely as X, and Y=4 when X=2. Find Y when X=3.

STEP 1: $Y = \dfrac{k}{(\quad)}$

STEP 2:

STEP 3:

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15. Y varies inversely as the square of X, and Y=3 when X=2. Find Y when X=6.

16. Y varies inversely as the square root of X, and Y=6 when X=25. Find Y when X=9.

17. Y varies jointly as the square of X and the square root of Z, and Y=24 when X=2 and Z=9. Find Y when X=3 and Z=25.

18. Y varies jointly as the square of X and the square root of Z, and Y=100 when X=2 and Z=25. Find Y when X=5 and Z=16.

19. Y varies directly as X and inversely as the square of Z. If Y=12 when X=8 and Z=2, then find Y when X=5 and Z=3.

20. Y varies directly as X and inversely as the square of Z. If Y=20 when X=12 and Z=3, then find Y when X=2 and Z=5.

21. Z varies jointly as X and Y and inversely as W. If Z=2 when X=6, Y=2, and W=18, find Z when X=4, Y=3, and W=6.

22. Z varies jointly as X and the square root of W and inversely as the square of Y. If Z=4 when X=2, Y=1, and W=16, find Z when X=6, Y=9, and W=25.

23. Distance varies directly as the square of the time. If distance is 50 feet when time is 5 seconds, find distance when time is 10 seconds.

24. A certain force varies inversely as distance from an object. If force is 10 pounds when distance is 6 feet, find the force when distance is 4 feet.

25. The illumination from a light varies inversely as the square of the distance from the light. If illumination is 25 units when distance is 4 feet, find the illumination when distance is a) 2 feet; b) 10 feet.

26. The illumination from a light varies inversely as the square of the distance from a light. If illumination is 10 units when distance is 6 feet, find the illumination when distance is 4 feet.

NOTE: DO NOT BE INTIMIDATED BY THE VOCABULARY IN #27-28.

27. The moladah of a shemei varies directly as the square of the tolad. If the moladah is 12 etams when the tolad is 2 ashans, find the moladah when the tolad is 5 ashans.

28. The moladah of a shemei varies inversely as the tolad. If the moladah is 12 etams when the tolad is 2 ashans, find the moladah when the tolad is 5 ashans.

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p.218-219:

      1. \$2.67; **2.** \$20.31; **3.** \$1.98; **4.** \$16.67; **5.** 1066.67 mi;
      6. 11.25 hr; **7.** 4.62 hr; **8.** 51.43 mi; **9.** 36.4 hr; **10.** 36.4 pg;
      11. 42.5 sec; **12.** 42.5 days; **13.** 166.67 tsp; **14.** 4285.71 sq ft;
      15. 9.375 lb; **16.** 66.67 gal.

p.222-225:

    1. $Y = kX^2$; **2.** $Y = kt^2$; **3.** $Y = \dfrac{k}{X^3}$; **4.** $Y = \dfrac{k}{t^3}$; **5.** $Y = kqp^2$;

    6. $Y = kX^2W^3$; **7.** $Y = \dfrac{kZ}{W^3}$; **8.** $Y = \dfrac{kZ^3}{\sqrt{X}}$; **9.** $d = kt^2$; **10.** $F = \dfrac{kM_1M_2}{d^2}$;

    11. 18; **12.** 27/2; **13.** 20; **14.** 8/3; **15.** 1/3; **16.** 10; **17.** 90; **18.** 500;
    19. 10/3; **20.** 6/5; **21.** 6; **22.** 5/27; **23.** 200 ft; **24.** 15 pounds;
    25a) 100 units; **b)** 4 units; **26.** 22.5 units; **27.** 75 etams;
    28. 24/5 etams.