

2.07 Fractional and Literal Equations

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

When solving a fractional equation, first find the least common denominator (LCD). Then multiply both sides of the equation by the LCD, and divide out all denominator factors. However, if you multiply both sides of an equation by a variable, you must check the answers to be sure no denominators equal zero. If an answer that you get ever makes a denominator equal zero, then that answer must be rejected. The answer thus obtained was not a "legal" answer. Like evidence that is illegally obtained and cannot be allowed in court, such answers must be thrown out, and if no other solution can be found, then there is no solution for the problem. In such cases, the answer is "No Solution" or the empty set, which is denoted by " ϕ " or "{ }".

PRINCIPLE

Whenever an equation is solved by multiplying both sides of that equation by a variable, the solution must be checked to be sure no denominators equal zero.

Before beginning the exercises, one more principle will be useful in this section. This is the **definition of equality of fractions**. Two fractions, $\frac{a}{b}$ and $\frac{c}{d}$, are equal if and only if $a \cdot d = b \cdot c$.

$$\frac{a}{b} = \frac{c}{d} \text{ means that } a \cdot d = b \cdot c$$

Solve the equations. Be sure to check all denominators.

EXAMPLE 1

EXERCISES 1.

2.

$$\frac{4}{x} = \frac{x-2}{2} \quad (x \neq 0)$$

$$4 \cdot 2 = x \cdot (x-2)$$

$$8 = x^2 - 2x$$

$$0 = x^2 - 2x - 8$$

$$0 = (x-4)(x+2)$$

$$x=4 \quad x=-2$$

$$\frac{x}{x-2} = \frac{4}{5}$$

$$\frac{6}{3x+6} = \frac{4}{x-1}$$

3. $\frac{5x-3}{2} = \frac{15x-2}{8}$

4. $\frac{2y+8}{5} = \frac{10y+4}{15}$

5. $\frac{x-8}{x+4} = \frac{1}{4}$

6. $\frac{x+6}{x-1} = \frac{1}{8}$

$$7. \quad \frac{4}{X+1} = \frac{6}{X+1}$$

$$8. \quad \frac{3}{X-2} = \frac{8}{3X-6}$$

$$9. \quad \frac{X-2}{X+4} = \frac{X}{X+2}$$

$$10. \quad \frac{X+6}{X} = \frac{X-2}{X-4}$$

$$11. \quad \frac{4}{X} = \frac{X+2}{2}$$

$$12. \quad \frac{X}{X+4} = \frac{6}{X-4}$$

$$13. \quad \frac{X}{X-4} = \frac{-6}{X+4}$$

$$14. \quad \frac{4}{X} = \frac{X-2}{2}$$

EXAMPLE 2

$$\frac{2X}{3} + \frac{3(X-4)}{2} = \frac{1}{2}$$

MULT. BOTH SIDES OF EQ. BY 6

$$\cancel{2} \frac{2X}{\cancel{3}} + \frac{\cancel{3} 3(X-4)}{\cancel{2}} = \frac{\cancel{3} 1}{\cancel{2}}$$

$$4X + 9X - 36 = 3$$

$$13X - 36 = 3$$

$$13X = 39$$

$$X = 3$$

15.

$$\frac{3(X-2)}{4} - \frac{X}{2} = 2$$

$$16. \quad \frac{X}{3} - \frac{X+2}{2} = 1$$

$$17. \quad \frac{X+3}{5} - \frac{X-4}{4} = 2$$

EXAMPLE 3

$$\cancel{6} \frac{X(X-1)}{\cancel{6}} + \frac{\cancel{2} X}{\cancel{2}} = \frac{6}{1}$$

$$X^2 - X + 2X = 6$$

$$X^2 + X - 6 = 0$$

$$(X+3)(X-2) = 0$$

$$X = -3 \text{ or } X = 2$$

18.

$$\frac{X(X+1)}{6} - \frac{X}{3} = 1$$

EXAMPLE 4 LCD = (x-1)(x-5) 19.

$$\frac{\cancel{(x-1)}(x-5)_x}{\cancel{x}1} + \frac{\cancel{(x-1)}(x-5)_2}{\cancel{x}5} = \frac{\cancel{(x-1)}(x-5)_{-4}}{\cancel{(x-5)}\cancel{(x-1)}}$$

$$\frac{x}{x-5} - \frac{13}{x-2} = \frac{15}{(x-5)(x-2)}$$

$$x(x-5) + 2(x-1) = -4$$

$$x^2 - 5x + 2x - 2 = -4$$

$$x^2 - 3x + 2 = 0 \quad \begin{matrix} x \neq 1 \\ x \neq 5 \end{matrix}$$

$$(x-2)(x-1) = 0$$

$$x = 2 \quad \cancel{x = 1}$$

Reject! Denom $\neq 0$.

20. $\frac{5}{x-2} - \frac{5}{x+2} = 4$

21. $\frac{3}{x-3} - \frac{7}{x+3} = 2$

EXAMPLE 5

$$\frac{1}{x^2-4x+3} - \frac{1}{x^2+4x-5} = \frac{1}{x^2+2x-15} \quad \text{LCD} = (x-3)(x-1)(x+5)$$

$$\frac{\cancel{(x-3)}\cancel{(x-1)}(x+5)}{\cancel{(x-3)}\cancel{(x-1)}} - \frac{(x-3)\cancel{(x-1)}(x+5)}{\cancel{(x+5)}\cancel{(x-1)}} = \frac{\cancel{(x-3)}\cancel{(x-1)}(x+5)}{\cancel{(x+5)}\cancel{(x-3)}}$$

$$x+5 - x+3 = x-1$$

$$8 = x-1$$

$$\boxed{9 = x}$$

$$22. \quad \frac{X}{X^2 - X - 6} + \frac{3}{X^2 - 5X + 6} + \frac{2}{X^2 - 4} = 0$$

$$23. \quad \frac{1}{X^2 - 4X + 3} - \frac{2}{X^2 + 4X - 5} = \frac{4}{X^2 + 2X - 15}$$

$$24. \quad \frac{X + 2}{X^2 - X - 6} - \frac{X}{X^2 - 4} = \frac{1}{X^2 - 5X + 6}$$

LITERAL EQUATIONS

Back in the first chapter, the topic of "Literal Equations" was introduced in a section on equation solving. Now that you have studied fractions and fractional equations, you will be able to understand **literal equations** that involve **fractions**. This is an appropriate time to mention these equations. Since it has been some time since Chapter 1 literal equations, a brief review is in order. You may wish to refer back to your work in Chapter 1. You will probably recognize some of the formulas from that chapter.

The procedure for solving literal equations is similar to that of solving other equations. Usually, you must separate all variable terms on one side of the equation, and non-variable terms on the other side. The key step is to factor the variable as a common factor, so as to get the variable in one place, then divide both sides of the equation by the resulting factor. This may leave a strange-looking fraction, and the answers will probably be very abstract. Just do what you know is correct algebraically and have confidence in your work!

EXAMPLE 1. Solve for x :

$$axy + bx = cy + d \quad \text{Variable (x) terms are all on left side.}$$

$$x(ay + b) = cy + d \quad \text{Factor out the variable (x).}$$

$$x = \frac{cy + d}{ay + b} \quad \text{Divide both sides by } ay + b.$$

EXAMPLE 2. Solve for y :

$$axy + bx = cy + d \quad \text{Get variable (y) terms on left side.}$$

$$axy - cy = d - bx \quad \text{Get "non-y" terms on right side.}$$

$$y(ax - c) = d - bx \quad \text{Factor out the variable (y).}$$

$$y = \frac{d - bx}{ax - c} \quad \left(\text{or } \frac{bx - d}{c - ax} \right) \quad \text{Divide both sides by } ax - c.$$

Exercises. Solve for X:

1. $aX + b = cX + d$

2. $a(X - b) = c(d - X)$

3. $Y = mX + b$

4. $Y - a = m(X - b)$

5. $AX + BY = C$

Solve for the indicated variable:

6. $A = \frac{1}{2}bh$, for h

7. $V = \frac{1}{3}\pi r^2 h$, for h

8. $V = \frac{1}{3}\pi r^2 h$, for r

9. $A = \frac{1}{2}(B + b)h$, for h

10. $A = \frac{1}{2}(B + b)h$, for B

11. $F = \frac{9}{5} C + 32$, for C

12. $C = \frac{5}{9} (F - 32)$, for F

$$13. \quad A = \frac{XY}{X + Y}, \text{ for } X$$

$$A(X + Y) = XY$$

$$AX + AY = XY$$

$$AY = XY - AX$$

$$AY = X(Y - A)$$

$$\frac{AY}{Y - A} = X$$

$$14. \quad A = \frac{XY}{X + Y}, \text{ for } Y$$

$$15. \quad P = \frac{XY}{a + bX}, \text{ for } a$$

$$16. \quad P = \frac{XY}{a + bX}, \text{ for } b$$

$$17. \quad P = \frac{XY}{a + bX}, \text{ for } X$$

$$18. \quad P = \frac{XY}{a + bX}, \text{ for } Y$$

$$19. \quad \frac{1}{F} = \frac{1}{S} + \frac{1}{U}, \text{ for } S$$

$$20. \quad \frac{1}{X} = \frac{1}{Y} + \frac{1}{Z}, \text{ for } X$$

LCD = _____

ANSWERS 2.07

p.202-206:

1. -8; 2. -5; 3. 2; 4. 5; 5. 12; 6. -7; 7. No Sol; 8. No Sol;
 9. -1; 10. 6; 11. -4, 2; 12. 12, -2; 13. -12, 2; 14. 4, -2; 15. 14;
 16. -12; 17. -8; 18. 3, -2; 19. 10 (Reject 5); 20. 3, -3; 21. -6, 4;
 22. 0, -3; 23. No sol (Reject 3); 24. No sol (Reject 3).

p.208-209:

1. $\frac{d-b}{-c}$; 2. $\frac{ab+cd}{a+c}$; 3. $\frac{y-b}{m}$. 4. $\frac{y-a+mb}{m}$; 5. $\frac{C-BY}{A}$;
6. $2A$; 7. $\frac{3V}{\pi r^2}$ 8. $\sqrt{\frac{3V}{\pi h}}$ 9. $\frac{2A}{B+b}$; 10. $\frac{2A-bh}{h}$ or $\frac{2A}{h} - b$
11. $\frac{5}{9}(F-32)$ 12. $\frac{9}{5}C+32$ or $\frac{9C+160}{5}$ 13. $Y-A$; 14. $X-A$;
15. $\frac{XY-PbX}{P}$ 16. $XY-Pa$ 17. $Y-Pb$; 18. $\frac{Pa+PbX}{X}$
19. $\frac{UF}{U-F}$ 20. $\frac{YZ}{Z+Y}$

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