

## 2.03 Fractional Expressions

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**ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE**

For many students, the greatest challenges in mathematics are "fractions" (or "**rational expressions**") and "word problems." In both of these topics, the basic concepts have been taught in your very first algebra course. In this chapter there will be five types of fractions problems: **reducing fractions, multiplying and dividing fractions, adding and subtracting fractions, complex fractions, and solving fractional equations.** Many of these topics are probably familiar to you from your previous mathematics background. As you might expect, the factoring becomes more difficult and the common denominators become more complicated.

A fraction can be "**reduced**" whenever a **factor** can be found that divides evenly into both the **numerator** and **denominator** of the fraction. For example, the fraction  $8/12$  can be reduced because 4 is a factor of the numerator and also the denominator. Dividing the numerator and denominator by 4 reduces the fraction to  $2/3$ .

**PRINCIPLE:** If the numerator and denominator of a fraction can be multiplied or divided by the same (non-zero) number, then the resulting fraction is equivalent..

**FORMAL STATEMENT:**  $\frac{a \cdot c}{b \cdot c} = \frac{a}{b}, \quad b \neq 0, c \neq 0$

**INTERPRETATION:** In reducing fractions, you will be dividing the numerators and denominators by factors. It is often helpful to factor the numerator and denominator to see what factors are common to both.

In each of the following, reduce the fractions completely. If your calculator has "fractions" capabilities, check your answers with the calculator. [If your calculator has an "a b/c" button try entering: "numerator", "a b/c", "denominator", "enter". You may want to try "numerator", "÷", "denominator", and convert the decimal to a fraction (if your calculator will do this!)].

1.  $\frac{75}{100} = \underline{\hspace{2cm}}$  Divide numerator and denominator by 25.
2.  $\frac{35}{75} = \underline{\hspace{2cm}}$  Divide numerator and denominator by 5.
3.  $\frac{34}{51} = \underline{\hspace{2cm}}$  Divide numerator and denominator by 17.
4.  $\frac{30}{135} = \underline{\hspace{2cm}}$  Divide numerator and denominator by 15.
5.  $\frac{38}{95} = \underline{\hspace{2cm}}$  Divide numerator and denominator by \_\_\_\_.
6.  $\frac{58}{87} = \underline{\hspace{2cm}}$  Divide numerator and denominator by \_\_\_\_.
7.  $\frac{112}{140} = \underline{\hspace{2cm}}$  Divide numerator and denominator by \_\_\_\_.
8.  $\frac{140}{112} = \underline{\hspace{2cm}}$  Divide numerator and denominator by \_\_\_\_.
9.  $\frac{95}{38} = \underline{\hspace{2cm}}$  Divide numerator and denominator by \_\_\_\_.
10.  $\frac{234}{387} = \underline{\hspace{2cm}}$  Divide numerator and denominator by \_\_\_\_.

11.  $\frac{9000}{40500} = \underline{\hspace{2cm}}$  Divide numerator and denominator by what-  
 $= \underline{\hspace{2cm}}$  ever factors you can until it is reduced  
 $= \underline{\hspace{2cm}}$  completely. [Hint: one to three steps!]

12.  $\frac{3240}{13320} = \underline{\hspace{2cm}}$  Divide numerator and denominator by what-  
 $= \underline{\hspace{2cm}}$  ever factors you can until it is reduced  
 $= \underline{\hspace{2cm}}$  completely. [Hint: one to three steps!]

13.  $\frac{306}{1071} = \underline{\hspace{2cm}}$  Divide numerator and denominator by what-  
 $= \underline{\hspace{2cm}}$  ever factors you can until it is reduced  
 $= \underline{\hspace{2cm}}$  completely. [Hint: one to three steps!]

14.  $\frac{5400}{13500} = \underline{\hspace{2cm}}$  Divide numerator and denominator by what-  
 $= \underline{\hspace{2cm}}$  ever factors you can until it is reduced  
 $= \underline{\hspace{2cm}}$  completely. [Hint: one to three steps!]

15.  $\frac{91X^3Y^2}{49X^2Y} = \underline{\hspace{2cm}}$  Divide numerator and denominator by  $7X^2Y$ .  
 [Remember: when you divide, you subtract exponents!]

16.  $\frac{34X^5Y^8}{17X^3Y^2} = \underline{\hspace{2cm}}$  Divide numerator and denominator by  $\underline{\hspace{2cm}}$ .

17.  $\frac{98X^2Y^8}{14X^6Y^6} = \underline{\hspace{2cm}}$       18.  $\frac{105X^3Y^5}{10X^7Y^3} = \underline{\hspace{2cm}}$  .

Divide out 14,  $X^2$ , and  $Y^6$ .  
 Avoid negative exponents!

$$19. \frac{26X^8Y^4}{39X^4Y^{12}} = \underline{\hspace{2cm}}$$

$$20. \frac{77X^9Y^5}{121X^4Y^4} = \underline{\hspace{2cm}} .$$

$$21. \frac{48X^3Y^3}{32X^5Y^6} = \underline{\hspace{2cm}}$$

$$22. \frac{39X^4Y^4}{117X^9Y^{12}} = \underline{\hspace{2cm}} .$$

$$23. \frac{4(X - 5)}{(X - 5)(X + 5)} = \underline{\hspace{2cm}} \quad \text{Divide numerator and denominator by } (X - 5) .$$

$$24. \frac{(X - Y)(X + Y)}{9(X + Y)} = \underline{\hspace{2cm}} \quad \text{Divide numerator and denominator by } (X + Y) .$$

$$25. \frac{9(X - 6)}{12(X - 6)(X + 3)} = \underline{\hspace{2cm}} \quad \text{Divide numerator and denominator by } 3(X - 6) .$$

$$26. \frac{40(X - 4)(X + 10)}{75(X + 10)(X - 8)} = \underline{\hspace{2cm}} \quad \text{Divide numerator and denominator by } 5(X + 10) .$$

[Notice that in these exercises, only FACTORS can be divided out!]

$$27. \frac{9(X - 6)(X + 6)}{54(X + 2)(X - 6)} = \underline{\hspace{2cm}} \quad 28. \frac{5(X - 5)(X + 8)}{75(X + 8)(X + 5)} = \underline{\hspace{2cm}}$$

$$29. \frac{24(X + 3)(X - 8)}{16(X + 3)(X + 8)} = \underline{\hspace{2cm}} \quad 30. \frac{140(X - 3Y)(X + Y)}{49(X + Y)(X + 3Y)} = \underline{\hspace{2cm}}$$

In the next exercises, remember you must factor the numerator and denominator (if possible!) first. Divide out only factors that are common to the numerator and denominator.

$$31. \frac{X^2 - 4}{16X + 32} = \frac{(\quad)(\quad)}{16(\quad)} \quad 32. \frac{X^2 - 9}{9X - 27} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$33. \frac{X^2 - 4X}{X^2 - 16} = \frac{X(\quad)}{(\quad)(\quad)} \quad 34. \frac{X^2 - 9X}{X^2 - 81} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$35. \frac{X - 8}{X^2 - 64} = \underline{\hspace{2cm}} \quad 36. \frac{X^2 - 25}{X - 5} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$37. \frac{X^2 - 5X + 4}{X^2 - 3X - 4} = \frac{(\quad)(\quad)}{(\quad)(\quad)} \quad 38. \frac{X^2 + 7X + 6}{X^2 + 12X + 36} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$39. \frac{X^2 - 49}{X^2 - 14X + 49} = \underline{\hspace{2cm}} \quad 40. \frac{X^2 + 8X + 16}{X^2 - 16} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$41. \frac{8X^3 - 8X^2}{X^2 - 1} = \underline{\hspace{2cm}} \quad 42. \frac{8X^2 - 16X}{X^2 - 4X + 4} = \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}} \quad = \underline{\hspace{2cm}}$$

$$43. \frac{X^2 - 16}{X^2 - 8X + 16} =$$

$$44. \frac{X^2 - 25}{X^2 - 4X - 5} =$$

$$45. \frac{X^3 - 27}{X^2 - 6X + 9} = \frac{(X - 3)(\quad)}{(\quad)(\quad)}$$

$$46. \frac{X^2 - 10X + 25}{X^3 - 125}$$

$$47. \frac{X^3 - 27}{X^2 + 3X + 9}$$

$$48. \frac{X^2 - 5X + 25}{X^3 + 125}$$

[Hint: Denom does not factor]

$$49. \frac{X^2 - 25}{X^3 - 125}$$

$$50. \frac{X^3 - 64}{X^2 - 16}$$

In the next exercises, watch out for factoring by grouping!

$$51. \frac{X^2-2X-XY+2Y}{X^2+2X-XY-2Y} = \frac{X(X-2) - Y(X-2)}{X(X+2) - Y(X+2)}$$

$$= \frac{(\quad)(\quad)}{(\quad)(\quad)}$$

=

$$52. \frac{X^2+4X-2XY-8Y}{X^2-2X-2XY+4Y} =$$

$$53. \frac{X^3 - 125}{X^3 - 5X^2 + 5X - 25}$$

$$54. \frac{X^3 - 8}{X^3 - 2X^2 - 2XY + 4Y}$$

$$55. \frac{X^3 + 3X^2 - 9X - 27}{X^3 + 27}$$

$$56. \frac{X^3 + 4X^2 - XY - 4Y}{X^3 + 64}$$

## ANSWERS 2.03

p. 151-156:

1.  $3/4$ ; 2.  $7/15$ ; 3.  $2/3$ ; 4.  $2/9$ ; 5.  $2/5$ ; 6.  $2/3$ ; 7.  $4/5$ ;  
8.  $5/4$  or  $1\ 1/4$ ; 9.  $5/2$  or  $2\ 1/2$ ; 10.  $26/43$ ; 11.  $2/9$ ;

12.  $9/37$ ; 13.  $2/7$ ; 14.  $2/5$ ; 15.  $\frac{13XY}{7}$ ; 16.  $2X^2Y^6$ ; 17.  $\frac{7Y^2}{X^4}$ ;

18.  $\frac{21Y^2}{2X^4}$ ; 19.  $\frac{2X^4}{3Y^8}$ ; 20.  $\frac{7X^8Y}{11}$ ; 21.  $\frac{3}{2X^2Y^3}$ ; 22.  $\frac{1}{3X^6Y^8}$ ;

23.  $\frac{4}{X+5}$ ; 24.  $\frac{X-Y}{9}$ ; 25.  $\frac{3}{4(X+3)}$ ; 26.  $\frac{8(X-4)}{15(X-8)}$ ; 27.  $\frac{X+6}{6(X+2)}$ ;



28.  $\frac{X-5}{15(X+5)}$  ; 29.  $\frac{3(X-8)}{2(X+8)}$  ; 30.  $\frac{20(X-3Y)}{7(X+3Y)}$  ; 31.  $\frac{X-2}{16}$  ;  
 32.  $\frac{X+3}{9}$  ; 33.  $\frac{X}{X+4}$  ; 34.  $\frac{X}{X+9}$  ; 35.  $\frac{1}{X+8}$  ; 36.  $X+5$  ;  
 37.  $\frac{X-1}{X+1}$  ; 38.  $\frac{X+1}{X+6}$  ; 39.  $\frac{X+7}{X-7}$  ; 40.  $\frac{X+4}{X-4}$  ; 41.  $\frac{8X^2}{X+1}$  ;  
 42.  $\frac{8X}{X-2}$  ; 43.  $\frac{X+4}{X-4}$  ; 44.  $\frac{X+5}{X+1}$  ; 45.  $\frac{X^2+3X+9}{X-3}$  ;  
 46.  $\frac{X-5}{X^2+5X+25}$  ; 47.  $X-3$  ; 48.  $\frac{1}{X+5}$  ; 49.  $\frac{X+5}{X^2+5X+25}$  ;  
 50.  $\frac{X^2+4X+16}{X+4}$  ; 51.  $\frac{X-2}{X+2}$  ; 52.  $\frac{X+4}{X-2}$  ; 53.  $\frac{X^2+5X+25}{X^2+5}$  ;  
 54.  $\frac{X^2+2X+4}{X^2-2Y}$  ; 55.  $\frac{(X-3)(X+3)}{X^2-3X+9}$  ; 56.  $\frac{X^2-Y}{X^2-4X+16}$

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