3.02 *Operations with Radicals*

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After simplification of radicals, the next step is **operations with radicals**--that is, addition, subtraction, multiplication, and division of radicals. Addition and subtraction of radicals is just like combining like terms. Even as 3X + 4X = 7X, so it is true that $3\sqrt{2} + 4\sqrt{2} = 7\sqrt{2}$ and $3\sqrt[3]{2} + 4\sqrt[3]{2} = 7\sqrt[3]{2}$. The expression $3\sqrt{2} + 4\sqrt{3}$ cannot be combined because $\sqrt{2}$ and $\sqrt{3}$ are "unlike radicals." Similarly, $(3\sqrt[3]{2} + 4\sqrt[3]{3})$ and $(3\sqrt[3]{5} + 4\sqrt{5})$ cannot be combined since $\sqrt[3]{2}$ and $\sqrt[3]{3}$ are not like radicals and $\sqrt[3]{5}$ and $\sqrt{5}$ are unlike radicals.

What about $3\sqrt{2} + 4\sqrt{8}$? At first glance, it appears that $\sqrt{2}$ and $\sqrt{8}$ are unlike radicals. However, since $\sqrt{8}$ simplifies to $2\sqrt{2}$, the expression <u>can</u> be combined!

Can **6 - 4√2** be simplified to **2√2**? This is a very common error! Even as **6 - 4X** cannot be combined, neither can **6 - 4√2.** It is possible to factor the common factor of **2** from **6 - 4√2** and write **2(3 - 2√2)**. There will be more on factoring later.

Simplify each of the following radical expressions <u>if possible</u>.

1. $3\sqrt{5} + 7\sqrt{5}$
= ________

2. $3\sqrt[3]{2} + 7\sqrt[3]{2}$
= ________

3. $3\sqrt{2} + 5\sqrt{3}$
= ________

4. $6 - 3\sqrt{5}$
= ________

5. $2\sqrt{3} - 23\sqrt{3}$
= ________

6. $3\sqrt{2} + 4\sqrt[3]{2}$
= ________

7. $\sqrt{2} + \sqrt{8}$

= ________

= ________

8. $\sqrt{27} + \sqrt{12}$

= ________

= ________

9. $3\sqrt{2} + 4\sqrt{8}$

= ________

= ________

10. $5\sqrt{18} + 6\sqrt{12}$

11. $5\sqrt{27} + 6\sqrt{12}$

12. $3\sqrt{75} + 4\sqrt{48}$

In 13-18, find a) simplest radical form
b) the calculator value (to nearest hundredth).

13. $\sqrt[3]{16} + \sqrt[3]{54}$

14. $2\sqrt[3]{81} - 3\sqrt[3]{375}$

15. $7\sqrt[3]{40} + 3\sqrt[3]{320}$

16. $5\sqrt[3]{108} - 4\sqrt[3]{32}$

17. $7\sqrt[4]{32} - 3\sqrt[4]{162}$

18. $5\sqrt[5]{5} + 4\sqrt[5]{160}$

19. $7X^2\sqrt{24XY^6} + 8Y^3\sqrt{54X^5}$

20. $5XY\sqrt{20X^7Y^5} - 4\sqrt{45X^9Y^7}$

21. $3X^2Y\sqrt{20XY^4} - 2X\sqrt{45X^3Y^6}$

22. $3X^2Y\sqrt{20XY^4} + 2X\sqrt{45X^3Y^6}$

23. $5X^2Y\sqrt[3]{54X^7Y^5} - 4XY^2\sqrt[3]{16X^{10}Y^2}$

24. $7X\sqrt[3]{16XY^3} + 8Y\sqrt[3]{54X^4}$

When multiplying radicals, use the **product property of radicals** in reverse: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$, where "a" and "b" are non-negative quantities, and "n" represents the **index** or the **order of the radical.**

25. $\sqrt{5} \cdot \sqrt{7}$

26. $\sqrt{3} \cdot \sqrt{11}$

27. $\sqrt{6} \cdot \sqrt{10}$

28. $\sqrt{15} \cdot \sqrt{6}$

29. $\sqrt[3]{5} \cdot \sqrt[3]{7}$ 30. $\sqrt[3]{3} \cdot \sqrt[3]{11}$ 31. $\sqrt[3]{12} \cdot \sqrt[3]{6}$ 32. $\sqrt[3]{50} \cdot \sqrt[3]{5}$

In the next examples (see **#33**), notice that if you just multiply the numbers together (like **35** times **77**), you would get a very large number (like **2695**) that will be difficult to break down and simplify. So, instead of multiplying it out, **why not factor the numbers first?** In the process, for square root problems (as this one is), be looking for **"pairs"** of numbers; for cube root problems, be looking for **"triplets"** or **"three of a kind;"** fourth roots, look for **"four of a kind;"** etc. Remember, "if you ain't got no pair, then you ain't got no square!"

In 33-38, find a) simplest radical form
b) the calculator value (to nearest hundredth).

33. $\sqrt{35} \cdot \sqrt{77}$ **34.** $\sqrt{55} \cdot \sqrt{33}$ **35.** $\sqrt{46} \cdot \sqrt{69}$

$= \sqrt{7 \cdot 5 \cdot 7 \cdot 11}$

$= \sqrt{7^2} \cdot \sqrt{55}$

= ____________

36. $\sqrt{85} \cdot \sqrt{34}$ **37.** $\sqrt{92} \cdot \sqrt{69}$ **38.** $\sqrt{155} \cdot \sqrt{124}$

39. $\sqrt[3]{35} \cdot \sqrt[3]{50}$

$= \sqrt[3]{5 \cdot 7 \cdot 5^2 \cdot 2}$

$= \sqrt[3]{5^3} \cdot \sqrt[3]{7 \cdot 2}$

= ______________

40. $\sqrt[3]{98} \cdot \sqrt[3]{35}$

41. $\sqrt[3]{75} \cdot \sqrt[3]{15}$

42. $\sqrt[3]{105} \cdot \sqrt[3]{45}$

43. $\sqrt[3]{105} \cdot \sqrt[3]{50}$

44. $\sqrt[3]{242} \cdot \sqrt[3]{55}$

45. $4\sqrt{3} \cdot 6\sqrt{15}$

$= 24\sqrt{45}$

= ______________

= ______________

46. $2\sqrt{6} \cdot 9\sqrt{10}$

47. $6\sqrt{35} \cdot 5\sqrt{42}$

48. $8\sqrt[3]{65} \cdot 2\sqrt[3]{50}$

49. $15\sqrt[3]{98} \cdot 4\sqrt[3]{35}$

50. $4\sqrt[3]{21} \cdot 4\sqrt[3]{45}$

Frequently, problems involve the **distributive property,** and the familiar process known as **"F OI L"** is used to find the products of radicals.

51. $8\sqrt{10}\,(2\sqrt{6} - 3\sqrt{2})$

52. $2\sqrt{6}\,(4\sqrt{3} + 5\sqrt{2})$

53. $4\sqrt{10}\,(8\sqrt{15} + 9\sqrt{30})$

54. $3\sqrt{20}\,(5\sqrt{2} - 8\sqrt{15})$

55. $(4 + 5\sqrt{6})\,(8 + 2\sqrt{6})$

56. $(4 - 5\sqrt{6})\,(3 + 2\sqrt{6})$

57. $(5\sqrt{3} + 2\sqrt{6})\,(8\sqrt{3} - 5\sqrt{6})$

58. $(4\sqrt{5} - 5\sqrt{15})\,(3\sqrt{5} + 2\sqrt{15})$

In 59-60, find a) simplest radical form
b) the calculator value (to nearest hundredth).

59. $(6\sqrt{3} - 2\sqrt{15})^2$

60. $(4\sqrt{6} + 5\sqrt{2})^2$

61. $(6\sqrt{3} - 2\sqrt{15})(6\sqrt{3} + 2\sqrt{15})$

62. $(4\sqrt{15} - 5\sqrt{6})(4\sqrt{15} + 5\sqrt{6})$

63. $(4 - \sqrt[3]{2})(16 + 4\sqrt[3]{2} + \sqrt[3]{4})$

64. $(5 + \sqrt[3]{5})(25 - 5\sqrt[3]{5} + \sqrt[3]{25})$

65. $(4 + \sqrt[3]{2})(16 - 4\sqrt[3]{2} + \sqrt[3]{4})$

66. $(\sqrt[3]{3} + \sqrt[3]{2})(\sqrt[3]{9} - \sqrt[3]{6} + \sqrt[3]{4})$

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1. $10\sqrt{5}$; 2. $10\sqrt[3]{2}$; 3. $3\sqrt{2} + 5\sqrt{3}$; 4. $6 - 3\sqrt{5}$;

5. $-21\sqrt{3}$; 6. $3\sqrt{2} + 4\sqrt[3]{2}$; 7. $3\sqrt{2}$; 8. $5\sqrt{3}$; 9. $11\sqrt{2}$;

p. 249-255:

10. $15\sqrt{2}+12\sqrt{3}$; **11.** $27\sqrt{3}$; **12.** $31\sqrt{3}$; **13.** $5\sqrt[3]{2}$, 6.30;

14. $-9\sqrt[3]{3}$, -12.98; **15.** $26\sqrt[3]{5}$, 44.46;

16. $7\sqrt[3]{4}$, 11.11; **17.** $5\sqrt[4]{2}$, 5.95; **18.** $13\sqrt[3]{5}$, 17.94;

19. $38X^2Y^3\ \sqrt{6X}$; **20.** $-2X^4Y^3\ \sqrt{5XY}$; **21.** 0;

22. $12X^2Y^3\ \sqrt{5X}$; **23.** $7X^4Y^2\ \sqrt[3]{2XY^2}$; **24.** $38XY\ \sqrt[3]{2X}$;

25. $\sqrt{35}$; **26.** $\sqrt{33}$; **27.** $2\sqrt{15}$; **28.** $3\sqrt{10}$; **29.** $\sqrt[3]{35}$;

30. $\sqrt[3]{33}$; **31.** $2\sqrt[3]{9}$; **32.** $5\sqrt[3]{2}$; **33.** $7\sqrt{55}$, 51.91;

34. $11\sqrt{15}$, 42.60; **35.** $23\sqrt{6}$, 56.34; **36.** $17\sqrt{10}$, 53.76;

37. $46\sqrt{3}$, 79.67; **38.** $62\sqrt{5}$, 138.64; **39.** $5\sqrt[3]{14}$, 12.05;

40. $7\sqrt[3]{10}$, 15.08; **41.** $5\sqrt[3]{9}$,10.40; **42.** $3\sqrt[3]{175}$,16.78;

43. $5\sqrt[3]{42}$, 17.38; **44.** $11\sqrt[3]{10}$, 23.70; **45.** $72\sqrt{5}$;

46. $36\sqrt{15}$; **47.** $210\sqrt{30}$; **48.** $80\sqrt[3]{26}$; **49.** $420\sqrt[3]{10}$;

50. $48\sqrt[3]{35}$; **51.** $32\sqrt{15}-48\sqrt{5}$; **52.** $24\sqrt{2}+20\sqrt{3}$;

53. $160\sqrt{6}+360\sqrt{3}$; **54.** $30\sqrt{10}-240\sqrt{3}$; **55.** $92+48\sqrt{6}$;

56. $-48-7\sqrt{6}$; **57.** $60-27\sqrt{2}$; **58.** $-90-35\sqrt{3}$;

59. $168-72\sqrt{5}$,7.00; **60.** $146+80\sqrt{3}$, 284.56; **61.** 48;
62. 90; **63.** 62; **64.** 130; **65.** 66; **66.** 5.

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