3.01 *Introduction to Radicals*

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Before beginning a study of **radicals** (or **roots**), it is appropriate to begin with some necessary terminology. The expression $\sqrt{X}$ or $\sqrt[2]{X}$ means the "square root of X" or "what squared would equal X?" The quantity inside the radical sign (or in this case **X)** is called the **radicand**, and the **2** (in this case) is the **index of the radical.** The expression $\sqrt[3]{X}$ is called the "cube root of X," and it asks the question, "What cubed would equal X?" In general, $\sqrt[n]{X}$ means the **"nth root of X,"** where the **radicand** is **X,** and the **index of the radical** is **"n".**

Remember also that the operations of **square root, cube root, fourth root, etc.** are actually **inverse operations** for the operations of **squaring, cubing, raising to the fourth power, etc.** When taking a **square root,** it is essential to be familiar with the **perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144,** and **169.** Also, remember that the **even powers** **(X^2, X^4, X^6, X^8, X^{10}, etc.)** are perfect squares. When taking a **cube root,** it is essential to be familiar with (i.e., **memorize them!!**) the **perfect cubes: 1, 8, 27, 64,** and **125.** Powers that are **divisible by three** **(X^3, X^6, X^9, X^{12}, X^{15}, etc.)** are perfect cubes. When taking **fourth roots, fifth roots, etc,** remember the **perfect fourth powers:** **$2^4 = 16$, $3^4 = 81$,** and powers that are **divisible by four (X^4, X^8, X^{12}, etc.),** and the **perfect fifth powers: $2^5 = 32$,** and powers that are **divisible by five (X^5, X^{10}, X^{15}, etc.).**

In the assignment that follows, it is important to know the perfect squares through **$13^2 = 169$;** the **perfect cubes: 1, 8, 27, 64, 125; perfect 4th powers: 1, 16, 81; perfect 5th powers: 1, 32.** Remember also that **for perfect squares involving variables raised to powers, the power must be even; for perfect cubes, the power must be divisible by 3; for perfect fourth powers, the power must be divisible by 4; etc.**

Simplify each of the following:

1. $\sqrt{25}$ = _____ because 5^2 = 25.

2. $\sqrt{49}$ = _____ because $(\quad)^2$ = 49.

3. $\sqrt{169}$ = _____ because $(\quad)^2$ = _____.

4. $\sqrt{X^4}$ = _____ because $(\quad)^2$ = _____.

5. $\sqrt{X^{10}}$ = _____ because $(\quad)^2$ = _____.

6. $\sqrt[3]{8}$ = _____ because $(\quad)^3$ = _____.

7. $\sqrt[3]{125}$ = _____ because $(\quad)^3$ = _____.

8. $\sqrt[3]{64}$ = _____ because $(\quad)^3$ = _____.

9. $\sqrt[3]{X^6}$ = _____ because _______________.

10. $\sqrt[3]{X^{21}}$ = _____ because _______________.

11. $\sqrt[4]{16}$ = _____ because $(\quad)^4$ = _____.

12. $\sqrt[4]{81}$ = _____ because _______________.

13. $\sqrt[5]{32}$ = _____ because _______________.

14. $\sqrt[4]{X^{12}}$ = _____ because _______________.

15. $\sqrt[4]{X^{20}}$ = _____ because _______________.

16. $\sqrt[5]{X^{20}}$ = _____ because _______________.

17. $\sqrt{25X^6}$ 18. $\sqrt{49X^{12}}$ 19. $\sqrt{16X^{16}}$ 20. $\sqrt{25X^{100}}$

21. $\sqrt[3]{125X^6}$ 22. $\sqrt[3]{8X^{12}}$ 23. $\sqrt[3]{27X^{27}}$ 24. $\sqrt[3]{64X^{51}}$

25. $\sqrt[4]{16X^{16}}$ 26. $\sqrt[4]{81X^{12}}$ 27. $\sqrt[5]{32X^{20}}$ 28. $\sqrt[5]{32X^{60}}$

If the root to be taken is not a "perfect power," then sometimes it can be simplified by using the **product property of radicals.**

Product Property of Radicals

$$\sqrt{a \cdot b} = \sqrt{a} \cdot \sqrt{b}$$
$$\sqrt[3]{a \cdot b} = \sqrt[3]{a} \cdot \sqrt[3]{b}$$
$$\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}$$

Because the product property of radicals is a property of **real numbers,** if the **index** of the radical is **even,** then the **radicands** must be positive.

To simplify a square root by this property, it may be helpful to think of the "radical two-step": 1. Sort; 2. Sqrt. In the first step, you must **"sort"** the radical, placing the **"perfect squares"** in the first radical and the other "leftover" factors in the second radicals. In the second step, you take the square root of the perfect square, and just bring down the "leftover" radical. For higher roots, the process is analogous.

29. $\sqrt{60} = \sqrt{4} \cdot \sqrt{15}$

$= \underline{\qquad\qquad}$

30. $\sqrt{98} = \sqrt{49} \cdot \sqrt{\qquad}$

$= \underline{\qquad\qquad}$

31. $\sqrt{45} = \sqrt{\qquad} \cdot \sqrt{\qquad}$

$= \underline{\qquad\qquad}$

32. $\sqrt[3]{54} = \sqrt[3]{27} \cdot \sqrt[3]{\qquad}$

$= \underline{\qquad\qquad}$

33. $\sqrt[3]{80} = \sqrt[3]{\qquad} \cdot \sqrt[3]{\qquad}$

$= \underline{\qquad\qquad}$

34. $\sqrt[3]{250} = \sqrt[3]{\qquad} \cdot \sqrt[3]{\qquad}$

$= \underline{\qquad\qquad}$

35. $\sqrt[4]{80} = \sqrt[4]{\qquad} \cdot \sqrt[4]{\qquad}$

$= \underline{\qquad\qquad}$

36. $\sqrt[4]{162} = \sqrt[4]{\qquad} \cdot \sqrt[4]{\qquad}$

$= \underline{\qquad\qquad}$

37. $\sqrt[5]{64} = \sqrt[5]{\qquad} \cdot \sqrt[5]{\qquad}$

$= \underline{\qquad\qquad}$

38. $\sqrt{X^3} = \sqrt{X^2} \cdot \sqrt{\qquad}$

$= \underline{\qquad\qquad}$

39. $\sqrt{X^{15}} = \sqrt{X^{14}} \cdot \sqrt{\qquad}$

$= \underline{\qquad\qquad}$

40. $\sqrt{X^8 Y^9} = \sqrt{\qquad} \cdot \sqrt{Y}$

$= \underline{\qquad\qquad}$

41. $\sqrt[3]{X^{14}} = \sqrt[3]{X^{12}} \cdot \sqrt[3]{\qquad}$

$= \underline{\qquad\qquad}$

42. $\sqrt[3]{X^8 Y^{13}} = \sqrt[3]{X^6 Y^{12}} \cdot \sqrt[3]{\qquad}$

$= \underline{\qquad\qquad}$

43. $\sqrt[3]{X^{30} Y^{14}} = \sqrt[3]{\qquad} \cdot \sqrt[3]{\qquad}$

$= \underline{\qquad\qquad}$

44. $\sqrt[4]{X^{13} Y^9} = \sqrt[4]{X^{12} Y^8} \cdot \sqrt[4]{\qquad}$

$= \underline{\qquad\qquad}$

45. $\sqrt[4]{X^{19}Y^{21}} = \sqrt[4]{\quad} \cdot \sqrt[4]{\quad}$

$= \underline{\hspace{3cm}}$

46. $\sqrt[5]{X^{16}} = \sqrt[5]{\quad} \cdot \sqrt[5]{\quad}$

$= \underline{\hspace{3cm}}$

47. $\sqrt{125X^{6}} = \sqrt{\quad} \cdot \sqrt{\quad}$

$= \underline{\hspace{3cm}}$

48. $\sqrt{48X^{13}} = \sqrt{\quad} \cdot \sqrt{\quad}$

$= \underline{\hspace{3cm}}$

49. $\sqrt{72X^{9}} =$

50. $\sqrt{50X^{7}} =$

51. $\sqrt{75X^{8}Y^{9}} =$

52. $\sqrt{40X^{11}Y^{6}} =$

53. $\sqrt{98X^{7}Y^{13}} =$

54. $\sqrt{300X^{15}Y^{25}} =$

55. $\sqrt[3]{54X^6Y^{10}} = \sqrt[3]{27X^6Y^9} \cdot \sqrt[3]{2Y}$

= ______________

56. $\sqrt[3]{16X^7Y^{12}} =$

57. $\sqrt[3]{72X^5Y^8} =$

58. $\sqrt[3]{80X^4Y^{14}} =$

59. $\sqrt[4]{32X^8Y^6} =$

60. $\sqrt[4]{48X^5Y^{16}} =$

61. $\sqrt[4]{162X^9Y^{10}} =$

62. $\sqrt[4]{405X^7Y^{14}} =$

63. $\sqrt[5]{96X^{12}Y^9} =$

64. $\sqrt[5]{64X^{25}Y^{13}} =$

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1. 5; **2.** 7; **3.** 13; **4.** X^2; **5.** X^5; **6.** 2; **7.** 5; **8.** 4;
9. X^2; **10.** X^7; **11.** 2; **12.** 3; **13.** 2; **14.** X^3; **15.** X^5;
16. X^4; **17.** $5X^3$; **18.** $7X^6$; **19.** $4X^8$; **20.** $5X^{50}$; **21.** $5X^2$;
22. $2X^4$; **23.** $3X^9$; **24.** $4X^{17}$; **25.** $2X^4$; **26.** $3X^3$; **27.** $2X^4$;

28. $2X^{12}$; **29.** $2\sqrt{15}$; **30.** $7\sqrt{2}$; **31.** $3\sqrt{5}$; **32.** $3\sqrt[3]{2}$;

33. $2\sqrt[3]{10}$; **34.** $5\sqrt[3]{2}$; **35.** $2\sqrt[4]{5}$; **36.** $3\sqrt[4]{2}$; **37.** $2\sqrt[5]{2}$;

38. $X\sqrt{X}$; **39.** $X^7\sqrt{X}$; **40.** $X^4Y^4\sqrt{Y}$; **41.** $X^4\sqrt[3]{X^2}$;

42. $X^2Y^4\sqrt[3]{X^2Y}$; **43.** $X^{10}Y^4\sqrt[3]{Y^2}$; **44.** $X^3Y^2\sqrt[4]{XY}$;

45. $X^4Y^5\sqrt[4]{X^3Y}$; **46.** $X^3\sqrt[5]{X}$; **47.** $5X^3\sqrt{5}$; **48.** $4X^6\sqrt{3X}$;

49. $6X^4\sqrt{2X}$; **50.** $5X^3\sqrt{2X}$; **51.** $5X^4Y^4\sqrt{3Y}$;

52. $2X^5Y^3\sqrt{10X}$; **53.** $7X^3Y^6\sqrt{2XY}$; **54.** $10X^7Y^{12}\sqrt{3XY}$;

55. $3X^2Y^3\sqrt[3]{2Y}$; **56.** $2X^2Y^4\sqrt[3]{2X}$; **57.** $2XY^2\sqrt[3]{9X^2Y^2}$;

58. $2XY^4\sqrt[3]{10XY^2}$; **59.** $2X^2Y\sqrt[4]{2Y^2}$; **60.** $2XY^4\sqrt[4]{3X}$;

61. $3X^2Y^2\sqrt[4]{2XY^2}$; **62.** $3XY^3\sqrt[4]{5X^3Y^2}$; **63.** $2X^2Y\sqrt[5]{3X^2Y^4}$;

64. $2X^5Y^2\sqrt[5]{2Y^3}$.

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