3.04 *Rationalizing Denominators*

There is a tradition in mathematics of eliminating the radicals from the denominators (or numerators) of fractions. This process is called **rationalizing the denominator (or numerator) of the fraction.** This may be done to simplify the radical expression or to make calculation of the expression easier, especially in days when calculators were not available. For example, knowing the value of $\sqrt{2}$ to be approximately **1.414,** to calculate $\frac{20}{\sqrt{2}}$ without a calculator would require **long division** of 20 divided by 1.414. It is much easier to multiply numerator and denominator by $\sqrt{2}$,

$$\frac{20}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{20\sqrt{2}}{2} = 10\sqrt{2}\ .$$

It is much easier to calculate **10(1.414)**, than to divide $\frac{20}{1.414}$.

When rationalizing a **monomial** square root denominator, multiply numerator and denominator by "something" that makes the denominator result in a perfect square. For monomial cube root denominators, multiply numerator and denominator by "something" that makes the denominator a perfect cube, etc.

In each of the following exercises, rationalize the denominators:

1. $\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{}}{\sqrt{}}$ **2.** $\frac{20}{\sqrt{5}}$ **3.** $\frac{20}{\sqrt{6}}$ **4.** $\frac{6}{\sqrt{10}}$

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Note: In the next exercises, it is usually a good idea to simplify the radical first, then rationalize the denominator.

5. $\frac{6}{\sqrt{18}} = \frac{6}{3\sqrt{2}}$

$= \frac{6}{3\sqrt{2}} \cdot \frac{\sqrt{\quad}}{\sqrt{\quad}}$

$= \frac{6 \cdot \sqrt{2}}{3 \cdot 2}$

$=$

6. $\frac{12}{\sqrt{20}}$

7. $\frac{12}{\sqrt{45}}$

8. $\frac{8}{\sqrt{80}}$

9. $\frac{15}{\sqrt{72}}$

10. $\frac{10}{\sqrt{75}}$

11. $\frac{6}{\sqrt{48X}} = \frac{6}{\sqrt{16}\,\sqrt{3X}}$

$= \frac{6}{4\sqrt{3X}} \cdot \frac{\sqrt{\quad}}{\sqrt{\quad}}$

$= \frac{6 \cdot \sqrt{\quad}}{4 \cdot (3X)}$

$=$

$=$

12. $\frac{20}{\sqrt{12X}}$

13. $\frac{45}{\sqrt{12X^2}}$

14. $\dfrac{30X}{\sqrt{18X^5}} = \dfrac{30X}{\sqrt{9X^4}\,\sqrt{2X}}$

$= \dfrac{30X}{3X^2\sqrt{2X}} \cdot \dfrac{\sqrt{\quad}}{\sqrt{\quad}}$

$= \dfrac{30X \cdot \sqrt{\quad}}{3X^2 \cdot (2X)}$

$=$

$=$

15. $\dfrac{40X^3}{\sqrt{20X^3}}$

16. $\dfrac{18X^5}{\sqrt{72X^7}}$

17. $\dfrac{21X^5}{\sqrt{98X^6}}$

18. $\dfrac{30Y^4}{\sqrt{72Y^3}}$

Consider the problem $\dfrac{6}{\sqrt[3]{2}}$.

A common error is to multiply numerator and denominator by $\sqrt[3]{2}$.

$$\frac{6}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} = \frac{6\sqrt[3]{2}}{\sqrt[3]{4}}$$

This does <u>not</u> help, because it does <u>not</u> eliminate the radical from the denominator! The denominator should end up a **perfect cube**

(like "8"!). To do this, you should multiply numerator and denominator by $\sqrt[3]{4}$ as follows:

19. $\frac{6}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{4}}{\sqrt[3]{4}} = \frac{6\sqrt[3]{4}}{\sqrt[3]{8}}$

$= \frac{6\sqrt[3]{4}}{2}$

$=$ ______

20. $\frac{6}{\sqrt[3]{4}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} =$ ______

$=$ ______

$=$ ______

21. $\frac{6}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{\quad}}{\sqrt[3]{\quad}}$

22. $\frac{6}{\sqrt[3]{9}}$

23. $\frac{10}{\sqrt[3]{25}}$

24. $\frac{10}{\sqrt[3]{5}}$

25. $\frac{35}{\sqrt[3]{49}}$

26. $\frac{35}{\sqrt[3]{7}}$

When the denominator of the fraction involves **binomial** radical expressions, such as $\frac{17}{6 - \sqrt{2}}$, a special procedure is used. Multiplying the numerator and denominator by **$6 + \sqrt{2}$** will eliminate the radicals from the denominator. For the fraction $\frac{6}{\sqrt{6} + \sqrt{2}}$, multiply numerator and denominator by **$\sqrt{6} - \sqrt{2}$.** In general, whatever the binomial denominator may be, you multiply the numerator and denominator by the same quantity as the denominator but with the opposite sign in the middle. This is called the **conjugate** of the denominator.

In each of the following exercises, rationalize the denominators and reduce each fraction to lowest terms:

1. $\frac{17}{(6 - \sqrt{2})} \cdot \frac{(6 + \sqrt{2})}{(6 + \sqrt{2})}$

$= \frac{17\ (6 + \sqrt{2})}{36 + 6\sqrt{2} - 6\sqrt{2} - 2}$

$= \frac{17\ (6 + \sqrt{2})}{34}$

= __________

2. $\frac{6}{(\sqrt{6} + \sqrt{2})} \cdot \frac{(\sqrt{6} - \sqrt{2})}{(\sqrt{6} - \sqrt{2})}$

3. $\frac{15}{\sqrt{5} + 5\sqrt{2}}$

4. $\frac{20}{3\sqrt{6} + 2}$

5. $\dfrac{12}{4 + 2\sqrt{3}}$

6. $\dfrac{12}{6 - 3\sqrt{3}}$

7. $\dfrac{15}{2\sqrt{6} - 3\sqrt{2}}$

8. $\dfrac{6}{3\sqrt{2} + 4\sqrt{3}}$

9. $\dfrac{\sqrt{27}}{\sqrt{6} + \sqrt{3}}$

10. $\dfrac{\sqrt{27}}{\sqrt{6} - \sqrt{3}}$

$$= \frac{3\sqrt{3}}{(\sqrt{6} + \sqrt{3})} \cdot \frac{(\sqrt{6} - \sqrt{3})}{(\sqrt{6} - \sqrt{3})}$$

$$= \frac{3\sqrt{3} \cdot (\sqrt{6} - \sqrt{3})}{(6 - 3)}$$

$$= \frac{3\sqrt{18} - 3 \cdot 3}{3}$$

$$= \frac{9\sqrt{2} - 9}{3}$$

$$= \frac{9(\sqrt{2} - 1)}{3} = \_\_\_\_\_\_\_\_\_\_$$

11. $\dfrac{\sqrt{20}}{6 - \sqrt{6}}$

12. $\dfrac{\sqrt{12}}{6 + \sqrt{6}}$

13. $\dfrac{\sqrt{12}}{6\sqrt{2} + \sqrt{6}}$

14. $\dfrac{\sqrt{27}}{2\sqrt{6} - 3\sqrt{3}}$

15. $\dfrac{(3 + \sqrt{6})}{(3 - \sqrt{3})} \cdot \dfrac{(3 + \sqrt{3})}{(3 + \sqrt{3})}$

$$= \frac{9 + 3\sqrt{3} + 3\sqrt{6} + \sqrt{18})}{9 - 3}$$

$$= \frac{9 + 3\sqrt{3} + 3\sqrt{6} + 3\sqrt{2}}{6}$$

$$= \frac{3(3 + \sqrt{3} + \sqrt{6} + \sqrt{2})}{6}$$

= ____________________

16. $\dfrac{3 - \sqrt{6}}{3 + \sqrt{3}}$

17. $\dfrac{3 + \sqrt{3}}{3 + \sqrt{6}}$

18. $\dfrac{3 - \sqrt{3}}{3 - \sqrt{6}}$

19. $\dfrac{\sqrt{6} + \sqrt{3}}{\sqrt{6} - \sqrt{3}}$

20. $\dfrac{\sqrt{6} - \sqrt{3}}{\sqrt{6} + \sqrt{3}}$

21. $\dfrac{4\sqrt{5} + 5\sqrt{2}}{3\sqrt{2} - 2\sqrt{5}}$

22. $\dfrac{3\sqrt{10} - 2\sqrt{6}}{4\sqrt{10} + 5\sqrt{6}}$

ANSWERS 3.04

p. 259–262:

1. $3\sqrt{2}$; 2. $4\sqrt{5}$; 3. $\frac{10\sqrt{6}}{3}$; 4. $\frac{3\sqrt{10}}{5}$; 5. $\sqrt{2}$; 6. $\frac{6\sqrt{5}}{5}$;

7. $\frac{4\sqrt{5}}{5}$; 8. $\frac{2\sqrt{5}}{5}$; 9. $\frac{5\sqrt{2}}{4}$; 10. $\frac{2\sqrt{3}}{3}$; 11. $\frac{\sqrt{3X}}{2X}$;

12. $\frac{10\sqrt{3X}}{3X}$; 13. $\frac{15\sqrt{3}}{2X}$; 14. $\frac{5\sqrt{2X}}{X^2}$; 15. $4X\sqrt{5X}$;

16. $\frac{3X\sqrt{2X}}{2}$; 17. $\frac{3X^2\sqrt{2}}{2}$; 18. $\frac{5Y^2\sqrt{2Y}}{2}$; 19. $3\sqrt[3]{4}$;

20. $3\sqrt[3]{2}$; 21. $2\sqrt[3]{9}$; 22. $2\sqrt[3]{3}$; 23. $2\sqrt[3]{5}$; 24. $2\sqrt[3]{25}$;

25. $5\sqrt[3]{7}$; 26. $5\sqrt[3]{49}$.

p. 263–266:

1. $\frac{6+\sqrt{2}}{2}$; 2. $\frac{3(\sqrt{6}-\sqrt{2})}{2}$; 3. $\frac{5\sqrt{2}-\sqrt{5}}{3}$; 4. $\frac{2(3\sqrt{6}-2)}{5}$;

5. $12-6\sqrt{3}$ *or* $6(2-\sqrt{3})$ *or* $3(4-2\sqrt{3})$; 6. $4(2+\sqrt{3})$ *or* $8+4\sqrt{3}$;

7. $\frac{5(2\sqrt{6}+3\sqrt{2})}{2}$; 8. $\frac{4\sqrt{3}-3\sqrt{2}}{5}$; 9. $3\sqrt{2}-3$;

10. $3\sqrt{2}+3$ *or* $3(\sqrt{2}+1)$; 11. $\frac{6\sqrt{5}+\sqrt{30}}{15}$; 12. $\frac{2\sqrt{3}-\sqrt{2}}{5}$;

13. $\frac{2\sqrt{6}-\sqrt{2}}{11}$; 14. $-6\sqrt{2}-9$; 15. $\frac{3+\sqrt{3}+\sqrt{6}+\sqrt{2}}{2}$;

16. $\frac{3-\sqrt{3}-\sqrt{6}+\sqrt{2}}{2}$; 17. $3-\sqrt{6}+\sqrt{3}-\sqrt{2}$;

18. $3+\sqrt{6}-\sqrt{3}-\sqrt{2}$; 19. $3+2\sqrt{2}$; 20. $3-2\sqrt{2}$;

21. $-11\sqrt{10}-35$; 22. $\frac{90-23\sqrt{15}}{5}$;

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE