4.01 *Absolute Value Equations and Inequalities*

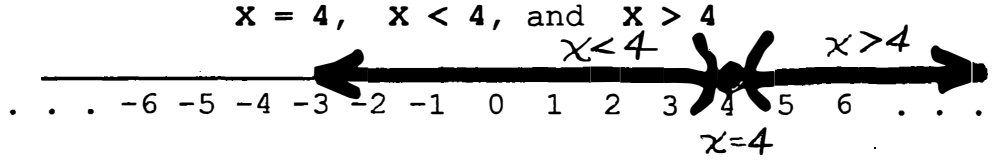
Dr. Robert J. Rapalje

ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

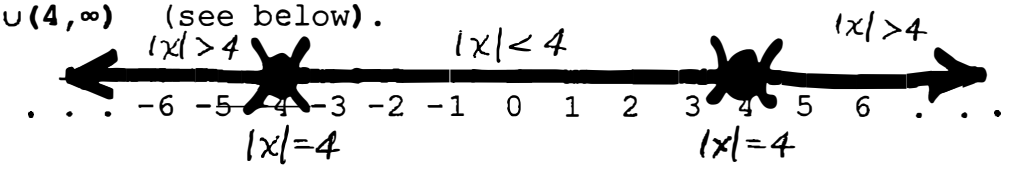
In **Section 1.04** you have already worked with absolute value equations, and in **Section 1.06** you worked with inequalities. In **Section 1.06** the **Trichotomy Axiom** (three possibilities of numerical comparison: **=**, **<**, and **>**) was briefly mentioned. This section combines the concepts of **absolute value** and **inequalities** with special use of the **Trichotomy Axiom.** It should be mentioned at the outset that some instructors prefer a graphical approach to solving these exercises, and the use of the graphics calculator may render the following explanation unnecessary. Even so, there is some merit in applying **"Trichotomy"** to the total concept of absolute value inequalities.

Begin by remembering that according to the **Trichotomy Axiom,** there are **three** ways to compare **X** to a number such as 4:

X = 4, X < 4, and **X > 4**



Likewise, there are **three** ways to compare **|X|** to a **positive** number such as 4: **|X|=4**, **|X|<4**, and **|X|>4.** By trial and error, you can see the solution to **|X|=4** consists of the two numbers **X=-4** and **X =4.** Also by trial and error, you can see that **|X|<4** consists of all values of X **between -4** and **4,** that is **-4<X<4,** or in interval notation: **(-4,4).** What remains for the solution to **|X|>4** are two separate intervals in which **X>4** or **X<-4,** or in interval notation: **(-∞, -4)∪(4,∞)** (see below).



Betweenness	Two points	Extremes
|X| < 4	**|X| = 4**	**|X| > 4**
-4 < X < 4	**X = 4 or X = -4**	**X > 4 or X < -4**
Int.Notation: **(-4, 4)**		**(-∞, -4) ∪ (4, ∞)**

Solve the absolute value equations (see Section 1.04 for review!): Don't forget--absolute value cannot equal a negative number.

1. $|X| = 4$

$X=$____ or $X=$____

2. $|X| = 6$

3. $|X| = 10$

4. $|Y| = 4$

$Y=$____ or $Y=$____

5. $|\$| = 6$

6. $|Junk| = 10$

7. $|X-2| = 4$

$X-2=$ or $X-2=$

$X=$ or $X=$

8. $|X-2| = 6$

9. $|X-2| = 10$

10. $|2X-2| = 4$

$2X-2=$ or $2X-2=$

$2X=$ or $2X=$

$X=$ or $X=$

11. $|2X-2| = 6$

12. $|2X-2| = 10$

13. $|3X+2| = 4$

$3X+2=$ or $3X+2=$

$3X=$ or $3X=$

$X=$ or $X=$

14. $|3X+2| = 6$

15. $|3X+2| = 10$

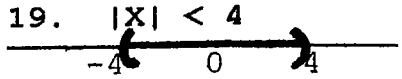
16. $|2X-7| = 5$

17. $|2X-7| = 15$

18. $|3X + 6| = 6$

Each of the next exercises is an **absolute value** with "<" or "≤" a **positive number.** Notice that each of these represents a **"BETWEENNESS"** situation. The first few can be solved by inspection, and then you may generalize to notice a pattern or rule.

19. |X| < 4 **20. |X| < 6** **21. |X| < 10**



Endpts: X= 4; X=-4
X is between -4 and 4
so: -4 < X < 4
INT: (-4, 4)

22. |Y| < 4 **23. |$| < 6** **24. |Junk| < 10**

25. |X-2| < 4 **26. |X-2| < 6** **27. |X-2| < 10**

Endpoints:
X-2=____ ; X-2= ____

X is between ___ and ___
Interval:

In #28-33, "≤" means to include the endpoints (use brackets).

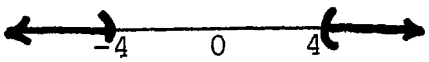
28. |2X-2| ≤ 4 **29. |2X-2| ≤ 6** **30. |2X-2| ≤ 10**

31. $|3X+2| \leq 4$ **32.** $|3X+2| \leq 6$ **33.** $|3X+2| \leq 10$

34. $|2X-7| < 5$ **35.** $|2X-7| < 15$ **36.** $|3X + 6| \leq 6$

Each of the next exercises is an **absolute value** with "**>**" or "≥" a **positive number.** Notice that each of these represents a **"EXTREMES"** situation. Solve the first few by inspection, and then generalize to find a pattern or rule.

37. $|X| > 4$ **38.** $|X| > 6$ **39.** $|X| > 10$



There are two endpoints:

X=____ ; X = ____

"EXTREMES"

Two separate regions:

X > 4 **or** X < -4

INT: $(-\infty,-4) \cup (4, \infty)$

40. $|Y| > 4$ **41.** $|\$| > 6$ **42.** $|Junk| > 10$

Endpoints:

Y= ____ ; Y= ____

43. $|X-2| > 4$ **44.** $|X-2| > 6$ **45.** $|X-2| > 10$

"EXTREMES"

There are two regions and two endpoints:

$X-2= 4$; $X-2= -4$ $X-2=$ ____ ; $X-2=$ ____

46. $|2X-2| \geq 4$ **47.** $|2X-2| \geq 6$ **48.** $|2X-2| \geq 10$

"EXTREMES"

Endpoints:

$2X-2=$ ___ ; $2X-2=$ ____

49. $|3X+2| \geq 4$ **50.** $|3X+2| \geq 6$ **51.** $|3X+2| \geq 10$

52. $|2X-7| > 5$ **53.** $|2X-7| > 15$ **54.** $|3X + 6| \geq 6$

In the more general cases, where **c > 0**,

Betweenness	**Two points**	**Extremes**
|aX+b| < c	**|aX+b| = c**	**|aX+b| > c**
-c < aX+b < c	**aX+b = c or aX+b = -c**	**aX+b > c or aX+b < -c**

Continuing with the idea of **Atrichotomy,"** there are **three** conditions needed to guarantee each of these cases:

Betweenness	**Two points**	**Extremes**
1. Absolute value	**1. Absolute value**	**1. Absolute value**
2. < or #	**2. =**	**2. > or ∃**
3. c > 0	**3. c > 0**	**3. c > 0**

EXAMPLE 1 (TWO POINTS)

|2X + 5| = 7

2X + 5 = 7 or 2X + 5 = -7

2X = 2 2X = -12

X = 1 or X = -6

EXAMPLE 2

|2X + 5| < 7 (BETWEENNESS)

Endpoints from EXAMPLE 1

X = 1; X = -6

The answer: **between -6 and 1.**

-6 < X < 1

(-6, 1)

EXAMPLE 3

|2X + 5| ∃ 7 (EXTREMES)

Endpoints from EXAMPLE 1

X = 1;

X = -6

The answer: **extremes from -6 and 1.**

X ∃ 1 **or** X # -6

(-4, -6] χ [1, 4)

Notice that the solutions outlined above assume you are comparing the absolute value to a positive number. In cases in which the absolute value is compared to a negative, solutions must be reached in another way--you just have to THINK:

1. **First, remember that an absolute value cannot equal a negative number. Therefore, an absolute value equal to a negative has NO SOLUTION.**

2. **Second, remember that an absolute value cannot be less than a negative, so likewise, an absolute value A<≅ or A#≅ a negative number has NO SOLUTION.**

3. **Third, remember that an absolute value must be greater than or equal to zero. Therefore, an absolute value A>≅ or A∃≅ a negative number is automatically true--it is true for all values of X.**

EXERCISES:

55. $|X| = -3$

56. $|X| < -3$

57. $|X| > -3$

58. $|2X-6| = -4$

59. $|2X-6| < -4$

60. $|2X-6| > -4$

61. $|2X-6| = 4$

62. $|2X-6| < 4$

63. $|2X-6| > 4$

64. $|2X-8| = 8$

65. $|2X-8| < 8$

66. $|2X-8| > 8$

67. $|2X-8| \geq -8$

68. $|2X-8| = -8$

69. $|2X-8| \leq -8$

70. $|2X-8| > 4$

71. $|2X-8| \leq 4$

72. $|2X-8| = 4$

73. $|4-X| = 8$

74. $|4-X| < 8$

75. $|4-X| \geq 8$

76. $|6-2X| \leq 8$

77. $|6-2X| > 8$

78. $|6-2X| = 8$

79. $\left|\frac{8+X}{3}\right| = 4$

80. $\left|\frac{8+X}{3}\right| \leq 4$

81. $\left|\frac{8+X}{3}\right| \geq 4$

82. $\left|\frac{6+X}{3}\right| > 3$

83. $\left|\frac{6+X}{3}\right| < 3$

84. $\left|\frac{6+X}{3}\right| = 3$

85. $\left|\frac{8-2X}{3}\right| = 4$

86. $\left|\frac{8-2X}{3}\right| \leq 4$

87. $\left|\frac{8-2X}{3}\right| \geq 4$

88. $\left|\frac{6-X}{3}\right| > 3$

89. $\left|\frac{6-X}{3}\right| < 3$

90. $\left|\frac{6-X}{3}\right| = 3$

91. $\left|\frac{8-2X}{3}\right| \leq -4$

92. $\left|\frac{8-2X}{3}\right| \geq -4$

93. $\left|\frac{8-2X}{3}\right| = -4$

94. $\left|\frac{6-X}{3}\right| = 0$

95. $\left|\frac{6-X}{3}\right| < 0$

96. $\left|\frac{6-X}{3}\right| \geq 0$

ANSWERS 4.01

p.307-314:

1. 4,-4; **2.** 6,-6; **3.** 10,-10; **4.** 4,-4; **5.** 6,-6; **6.** 10,-10;
7. 6,-2; **8.** 8,-4; **9.** 12,-8; **10.** 3,-1; **11.** 4,-2; **12.** 6,-4;
13. 2/3,-2; **14.** 4/3,-8/3; **15.** 8/3,-4; **16.** 6,1; **17.** 11,-4;
18. 0,-4; **19.** (-4,4); **20.** (-6,6); **21.** (-10,10); **22.** (-4,4);
23. (-6,6); **24.** (-10,10); **25.** (-2,6); **26.** (-4,8);
27. (-8,12); **28.** [-1,3]; **29.** [-2,4]; **30.** [-4,6];
31. [-2,2/3]; **32.** [-8/3,4/3]; **33.** [-4,8/3]; **34.** (1,6);
35. (-4,11); **36.** [-4,0]; **37.** $(-\infty,-4)\cup(4,\infty)$; **38.** $(-\infty,-6)\cup(6,\infty)$;
39. $(-\infty,-10)\cup(10,\infty)$; **40.** $(-\infty,-4)\cup(4,\infty)$; **41.** $(-\infty,-6)\cup(6,\infty)$;
42. $(-\infty,-10)\cup(10,\infty)$; **43.** $(-\infty,-2)\cup(6,\infty)$; **44.** $(-\infty,-4)\cup(8,\infty)$;
45. $(-\infty,-8)\cup(12,\infty)$; **46.** $(-\infty,-1]\cup[3,\infty)$; **47.** $(-\infty,-2]\cup[4,\infty)$;
48. $(-\infty,-4]\cup[6,\infty)$; **49.** $(-\infty,-2]\cup[2/3,\infty)$; **50.** $(-\infty,-8/3]\cup[4/3,\infty)$;
51. $(-\infty,-4]\cup[8/3,\infty)$; **52.** $(-\infty,+1)\cup(6,\infty)$; **53.** $(-\infty,-4)\cup(11,\infty)$;
54. $(-\infty,-4]\cup[0,\infty)$; **55.** No Sol; **56.** No Sol; **57.** $(-\infty,\infty)$;
58. No Sol; **59.** No Sol; **60.** $(-\infty,\infty)$; **61.** 5,1; **62.** (1,5);
63. $(-\infty,1)\cup(5,\infty)$; **64.** 8,0; **65.** (0,8); **66.** $(-\infty,0)\cup(8,\infty)$;
67. $(-\infty,\infty)$; **68.** No Sol; **69.** No Sol; **70.** $(-\infty,2)\cup(6,\infty)$;
71. [2,6]; **72.** 6,2; **73.** -4,12; **74.** (-4,12);
75. $(-\infty,-4]\cup[12,\infty)$; **76.** [-1,7]; **77.** $(-\infty,-1)\cup(7,\infty)$; **78.** -1,7;
79. 4,-20; **80.** [-20,4]; **81.** $(-\infty,-20]\cup[4,\infty)$;
82. $(-\infty,-15)\cup(3,\infty)$; **83.** (-15,3); **84.** 3,-15; **85.** -2,10;
86. [-2,10]; **87.** $(-\infty,-2]\cup[10,\infty)$; **88.** $(-\infty,-3)\cup(15,\infty)$;
89. (-3,15); **90.** -3,15; **91.** No Sol; **92.** $(-\infty,\infty)$; **93.** No Sol;
94. 6; **95.** No Sol; **96.** $(-\infty,\infty)$.

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