# 4.03 *Quadratic Formula*

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The general form of the quadratic equation is $aX^2+bX+c = 0$, where **"a", "b", and "c"** represent any real numbers. This equation can be solved by completing the square, and a formula known as the **quadratic formula** can be derived. The derivation of this formula is much more complicated than using the formula, and it is generally beyond the scope of this course. The derivation is presented at the end of the exercises for advanced students and/or for future reference. The solution to the general quadratic equation $aX^2+bX+c = 0$ is given by the

**World Famous Quadratic Formula!!**

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If this is the first time you have ever seen this formula, it can be rather intimidating. However, it is not nearly as bad as it looks. (It is really as easy as "**a, b, c**"!) If you are wondering, "Do I have to memorize this formula?", the answer is "No! You will learn it very well just by doing all the homework." If you are not going to do the homework, then "Yes! You need to memorize it!"

The quadratic formula may be used to solve <u>any</u> quadratic equation, even if the equation can be factored. However, it is usually best to use factoring if the equation factors and use the quadratic formula otherwise. There are some problems in which completing the square may be easier than the quadratic formula, particularly if **a=1** and **b is even.** ***Please note that, while the <u>use</u> of the quadratic formula on the next pages is not as easy as the factoring method, it does work on <u>all</u> problems, and it is <u>much</u> easier than it may at first appear.***

**GUIDELINES FOR USING QUADRATIC FORMULA**

1. The equation should always be set equal to zero.
2. Arrange the terms in descending powers of the variable.
3. Identify the "a", "b", and "c":
   Coefficient of $X^2$ is "a", coefficient of X is "b", and the constant term is "c".
4. It is preferred (not required!) that "a" be positive.
5. Write down the formula.
6. Substitute "a", "b", and "c".
7. Simplify the radical, simplify and reduce the fraction if possible.

**EXAMPLE 1. Solve for X:**

$$X^2 + 5X - 2 = 0$$

Equation is set = 0

a=1 b=5 c=-2

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{-5 \pm \sqrt{5^2 - 4(1)(-2)}}{2(1)}$$

$$X = \frac{-5 \pm \sqrt{25 + 8}}{2}$$

$$X = \frac{-5 \pm \sqrt{33}}{2}$$

**Fraction cannot be reduced!**

**EXAMPLE 2. Solve for X:**

$$X^2 + 5 = 6X$$

Set equal to zero: $X^2 - 6X + 5 = 0$

a=1 b=-6 c=5

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(5)}}{2(1)}$$

$$X = \frac{6 \pm \sqrt{36 - 20}}{2}$$

$$X = \frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2}$$

$$X = \frac{6 + 4}{2} \text{ or } X = \frac{6 - 4}{2}$$

$$X = \frac{10}{2} = 5 \quad or \quad X = \frac{2}{2} = 1$$

Notice that in Example 1, the answer contains a radical, whereas in Example 2, you obtained the square root of a perfect square. This perfect square means that Example 2 could have been solved by factoring, whereas Example 1 (not a perfect square) did not factor.

**EXERCISES. Solve the following quadratic equations.**

**1.** $X^2 + 3X - 2 = 0$

Equation is set = 0

a=_____ b=_____ c=_____

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$

**2.** $X^2 + 5 = 5X$

Set equal to zero: $X^2$ ___ X ___ = 0

a=_____ b=_____ c=_____

Write formula: X = ____________________

$$X = \frac{-(\quad) \pm \sqrt{(\quad)^2 - 4(\quad)(\quad)}}{2(\quad)}$$

**3.** $X^2 - 3X = 5$

Set = 0: ____________________ = 0

a=_____ b=_____ c=_____

Formula:

Substitute:

**4.** $X^2 = 7 + 5X$

Set = 0: ____________________ = 0

a=_____ b=_____ c=_____

Formula:

Substitute:

5. $X^2 + 3X - 4 = 0$

a=____ b=____ c=____

6. $X^2 - 5X - 50 = 0$

a=____ b=____ c=____

7. $X^2 - 4X = 21$

Set = 0: ________________

a=____ b=____ c=____

8. $X^2 + 50 = 12X + 30$

a=____ b=____ c=____

9. $X^2 - 5X = 0$ (No constant!)

10. $X^2 - 25 = 0$ (No X term!)

**EXAMPLE 3. Solve for X:**

$$X^2 + 6X - 2 = 0$$

Equation is set = 0

a=1 b=6 c=-2

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{-6 \pm \sqrt{6^2 - 4(1)(-2)}}{2(1)}$$

$$X = \frac{-6 \pm \sqrt{36 + 8}}{2}$$

$$X = \frac{-6 \pm \sqrt{44}}{2}$$

$$X = \frac{-6 \pm 2\sqrt{11}}{2}$$

$$X = \frac{2(-3 \pm \sqrt{11})}{2}$$

$$\mathbf{X = -3 \pm \sqrt{11}}$$

**EXAMPLE 4. Solve for X:**

$$X^2 + 25 = 6X$$

Set equal to zero: $X^2 - 6X + 25 = 0$

a=1 b=-6 c=25

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(25)}}{2(1)}$$

$$X = \frac{6 \pm \sqrt{36 - 100}}{2}$$

$$X = \frac{6 \pm \sqrt{-64}}{2}$$

$$X = \frac{6 \pm 8i}{2}$$

$$\mathbf{X = 3 \pm 4i}$$

Notice that in Example 3 the radical needed to be simplified, and then the fraction reduced. Pay attention to these steps--traditionally they are difficult for students and a predictable source of errors. Notice that in Example 4, there was a negative in the radical. This negative in the radical means that there were no real solutions--that the solutions were in fact complex numbers.

11. $X^2 - 4X - 6 = 0$

12. $X^2 + 4X + 2 = 0$

13. $X^2 - 6X - 6 = 0$

14. $X^2 = 4X + 8$

15. $2X(X - 4) = -7$

16. $2X(X - 4) = 7$

17. $3X^2 + 2(3 + X) = 4 - 6X$

18. $3X(X - 4) = 7 - 8X$

REMEMBER, IN #19 AND 20, PREFERENCE IS TO MAKE THE VALUE OF "a" POSITIVE!

19. $3(2 - X^2) = 4X$

20. $X(6 - X) = 4$

Watch out for complex numbers:

21. $X(X + 6) + 25 = 0$

22. $X^2 = 2(3X - 5)$

23. $4X(X + 3) = -13$

24. $9X^2 = 4(3X - 2)$

25. $2X(2 - X) = 3$

26. $2X(X + 2) = -5$

27. $4X(X + 5) = -27$

28. $3X^2 = 2(X - 1)$

The following derivation is presented for advanced students and/or for future reference. The solution to the general quadratic equation $aX^2+bX+c = 0$, derived by completing the square, is given by the "quadratic formula."

**PROOF:** **Begin with** $aX^2 + bX + c = 0$

$aX^2 + bX + c = 0$ Subtract c from both sides.

$aX^2 + bX = -c$ Divide both sides by a, $(a \neq 0)$.

$X^2 + \frac{b}{a}X = -\frac{c}{a}$ Complete the square.

*Half of* $\frac{b}{a}$ *is* $\frac{b}{2a}$, *and* $(\frac{b}{2a})^2$ *is* $\frac{b^2}{4a^2}$

$$X^2 + \frac{b}{a}X + (\qquad) = -\frac{c}{a} + (\qquad)$$

$$X^2 + \frac{b}{a}X + (\frac{b^2}{4a^2}) = -\frac{c}{a} + (\frac{b^2}{4a^2})$$

Add $\frac{b^2}{4a^2}$ to both sides.
Find LCD = $4a^2$ on right side.

$$(X + \frac{b}{2a})^2 = -\frac{c}{a}\cdot\frac{4a}{4a} + \frac{b^2}{4a^2}$$

$$(X + \frac{b}{2a})^2 = \frac{-4ac + b^2}{4a^2}$$

Take square root of both sides.

$$X + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

Denom $\mathbf{4a^2}$ is a perfect square.

$$X + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

Add $-\frac{b}{2a}$ to both sides.

$$X = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

LCD = **2a** on right side.

$$\boxed{X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

World Famous **Quadratic Formula!!**

# ANSWERS 4.03

p. 325-330:

1. $\dfrac{-3 \pm \sqrt{17}}{2}$ ; 2. $\dfrac{5 \pm \sqrt{5}}{2}$ ; 3. $\dfrac{3 \pm \sqrt{29}}{2}$ ; 4. $\dfrac{5 \pm \sqrt{53}}{2}$ ;

5. -4,1; 6. 10,-5; 7. 7,-3; 8. 10,2; 9. 0,5; 10. 5,-5;

11. $2 \pm \sqrt{10}$ ; 12. $-2 \pm \sqrt{2}$ ; 13. $3 \pm \sqrt{15}$ ; 14. $2 \pm 2\sqrt{3}$ ;

15. $\dfrac{4 \pm \sqrt{2}}{2}$ ; 16. $\dfrac{4 \pm \sqrt{30}}{2}$ ; 17. $\dfrac{-4 \pm \sqrt{10}}{3}$ ; 18. 7/3,-1;

19. $\dfrac{-2 \pm \sqrt{22}}{3}$ ; 20. $3 \pm \sqrt{5}$ ; 21. $-3 \pm 4i$; 22. $3 \pm i$;

23. $\dfrac{-3 \pm 2i}{2}$ ; 24. $\dfrac{2 \pm 2i}{3}$ ; 25. $\dfrac{2 \pm i\sqrt{2}}{2}$ ; 26. $\dfrac{-2 \pm i\sqrt{6}}{2}$ ;

27. $\dfrac{-5 \pm i\sqrt{2}}{2}$ 28. $\dfrac{1 \pm i\sqrt{5}}{3}$ .

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE