4.06 *Quadratic Applications*

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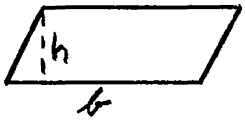
ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

The question "What good is factoring?" was answered in part with the use of factoring to solve quadratic equations. This may have raised more questions than it answered: "What good are quadratic equations?" "What real life problems result in quadratic equations?" Questions like this useually end up in word problems. Some of the very best applications of quadratic equations involve some very important, yet simple concepts. A few basic formulas will be necessary.

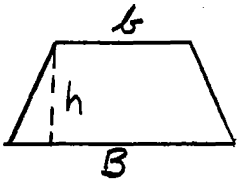
AREA OF RECTANGLE $A = L \cdot W$

AREA OF TRIANGLE $A = \frac{bh}{2}$ *or* $\frac{1}{2}b \cdot h$

AREA OF PARALLELOGRAM $A = b \cdot h$

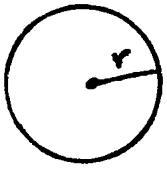


AREA OF TRAPEZOID $A = \frac{1}{2}(B + b) \cdot h$

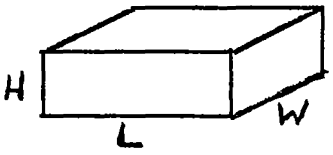


CIRCUMFERENCE OF CIRCLE $C = \pi d$ *or* $C = 2\pi r$

AREA OF CIRCLE $A = \pi r^2$



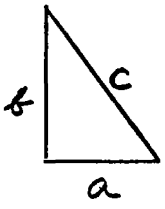
VOLUME OF RECTANGULAR SOLID $V = L \cdot W \cdot H$



and of course,
THEOREM OF PYTHAGORAS

IN ANY RIGHT TRIANGLE, WHERE "a" AND "b" ARE LEGS AND "c" THE IS HYPOTENUSE:

$$a^2 + b^2 = c^2$$



PERIMETER/AREA/VOLUME

PERIMETER	always 1 DIMENSIONAL
LINEAR UNITS	ft, in, m, cm, etc.
AREA	always 2 DIMENSIONAL
SQUARE UNITS	sq ft, sq in, sq m, sq cm, etc.
or	ft^2, in^2, m^2, cm^2, etc.
VOLUME	always 3 DIMENSIONAL
CUBIC UNITS	cu ft, cu in, cu m, cu cm, etc.
or	ft^3, in^3, m^3, cm^3, etc.

Remember the 5 steps in solving word problems from **Section 1.05**:

STEP 1: ***IDENTIFY THE VARIABLE.***
STEP 2: ***WRITE THE EQUATION.***
STEP 3: ***SOLVE THE EQUATION.***
STEP 4: ***ANSWER THE QUESTION.***
STEP 5: ***CHECK.***

EXAMPLE 1. **The length of a rectangle is 3 times the width. If the area of the rectangle is 75 square centimeters, find the dimensions of the rectangle.**

STEP 1: **Let** **X =** width of rectangle
3X = length of rectangle

STEP 2: **Eq:** **Width · Length = Area**
$X \cdot 3X = 75$

STEP 3: **Solve:**
$$3X^2 = 75$$
$$X^2 = 25$$
$$X = \pm 5$$

STEP 4: **Answer:** Since -5 is meaningless for the dimensions of a rectangle, reject X = -5. However, **X = 5 cm** is an acceptable width; the length 3X = **15 cm.** The rectangle is **5 cm. by 15 cm.**

EXERCISES. Complete the following exercises.

1. **The length of a rectangle is 5 times the width. If the area of the rectangle is 245 square centimeters, find the dimensions of the rectangle.**

STEP 1: **Let** **X =** width of rectangle
_____ = length of rectangle

STEP 2: **Eq:** **Width · Length = Area**
_____

STEP 3: **Solve:**

STEP 4: **Answer:**

2. **The length of a rectangle is 3 more than the width. If the area of the rectangle is 10 square centimeters, find the dimensions of the rectangle.**

STEP 1: **Let** **X =** _____
_____ = _____

STEP 2: **Eq:** **Width · Length = Area**
X · ( ) = _____

STEP 3: **Solve:** $X^2 + 3X = 10$ (Quadratic: Set = 0)
(Factor!)

STEP 4: **Answer:**

3. **The length of a rectangle is 5 more than the width. If the area of the rectangle is 50 square meters, find the dimensions of the rectangle.**

EXAMPLE 2: The base of a triangle is 3 more than twice the height. If the area of the triangle is 10 square centimeters, find the dimensions of the triangle.

SOLUTION: **Let X =** height of triangle
2X+3 = base of triangle

Equation: **½bh = Area**

$\frac{1}{2}X(2X+3) = 10$ Mult both sides of equation by 2

$X(2X+3) = 20$ Add -20 to both sides of equation

$2X^2 + 3X - 20 = 0$

$(2X - 5)(X + 4) = 0$

$X = 5/2;\ X = -4$ Reject -4

X = 5/2 cm. height of triangle

2X+3 = 8 cm. base of triangle

4. **The base of a triangle is 5 times the height. If the area of the triangle is 90 square feet, find the dimensions of the triangle.**

SOLUTION: **Let X =** height of triangle

_____ = base of triangle

Equation: **½bh = Area**

5. **The base of a triangle is 3 less than twice the height. If the area of the triangle is 10 square centimeters, find the dimensions of the triangle.**

6. The height of a parallelogram is 3 less than the base. If the area of the parallelogram is 40 square meters, find the dimensions of the parallelogram.

SOLUTION: Let X = _____

_____ = _____

Equation: $b \cdot h = \text{Area}$

7. The base of a parallelogram is 3 more than twice the height. If the area is 35 square centimeters, find the dimensions of the parallelogram.

8. The base of a parallelogram is 3 less than twice the height. If the area is 35 square centimeters, find the dimensions of the parallelogram.

9. A radius of a circle is 500 m. Find the area using
a) $\pi = 3.14$ b) $\pi = 22/7$ c) $\pi = 3.14159$ d) calculator value

a) $A = \pi r^2$
$= 3.14(500^2)$
$=$ _____

b) $A = \pi r^2$
$= (22/7)(500^2)$
$=$ _____

c) $A = \pi r^2$
$=$ _____
$=$ _____

d) $A = \pi r^2$
$=$ _____
$=$ _____

Notice the differences in the above answers. Which is best? Why?

EXAMPLE 3: The area of a circle is 36π square centimeters. Find the radius and circumference of the circle. Give circumference in terms of π.

SOLUTION: Let X = radius of circle

Equation: $\pi r^2 = Area$

$\pi X^2 = 36\pi$ Divide both sides by π

$X^2 = 36$

X = ± 6, so X = 6 cm. = radius

C = $2\pi r = 2\pi 6 = 12\pi$ cm.

EXAMPLE 4: The area of a circle is 500 square centimeters. Find the radius of the circle. Use π=3.14 (or calculator value), and round to nearest hundredth.

SOLUTION: Let X = radius of circle

Equation: $\pi r^2 = Area$

$\pi X^2 = 500$ Divide both sides by π.

$X^2 = \frac{500}{\pi} = 159.24$ (*or* 159.15) Use calculator!

$X = \pm\sqrt{\frac{500}{\pi}} = 12.62$, so X = 12.62 cm.

10. The area of a circle is 49π square centimeters. Find the radius and circumference of the circle. Give circumference in terms of π.

11. The area of a circle is 529π square centimeters. Find the radius and circumference of the circle. Give circumference in terms of π.

12. The circumference of a circle is 24π centimeters. Find the radius and area of the circle. Give area in terms of π.

Equation: $2\pi r = \textit{Circumference}$

$2\pi r = 24\ \pi$ Divide both sides by 2π

$r =$ _____cm.

$A = \pi r^2 =$ _____ sq. cm. or _____ cm^2

13. The circumference of a circle is 26π centimeters. Find the radius and area of the circle. Give area in terms of π.

Equation: $2\pi r = \textit{Circumference}$

"What good is math?"

14. The circumference of a tree is measured and found to be 20 feet around. Find the diameter of the tree (without cutting it down to measure it!). Round to nearest hundredth.

15. A circular track has a radius of 100 feet. How many times must you run around the track to have run a mile, if there are 5280 feet in a mile?

EXAMPLE 5: A rectangular box with height 3 centimeters has a volume of 24 cubic centimeters. The length of the box is two more than the width. Find the dimensions of the box.

SOLUTION: Let X = width of box
$X+2$ = length of box
3 = height of box

Equation: $L\ W\ H$ = Volume of box

$$3X(X+2) = 24$$
$$3X^2 + 6X - 24 = 0$$
$$3(X^2 + 2X - 8) = 0$$
$$3(X + 4)(X - 2) = 0$$

$X = -4$; Reject

$X = 2$ cm. Width of box
$X+2 = 4$ cm. Length of box
3 cm. Height of box

Check: 24 cu.cm. Volume of box

16. A rectangular box with height 4 meters has a volume of 576 cubic meters and a square base. Find the dimensions of the box.

SOLUTION: Let X = width of box
X = length of box
4 = height of box

Equation: L W H = Volume of box

17. A rectangular box with height 2 centimeters has a volume of 50 cubic centimeters. The length of the box is 5 more than twice the width. Find the dimensions of the box.

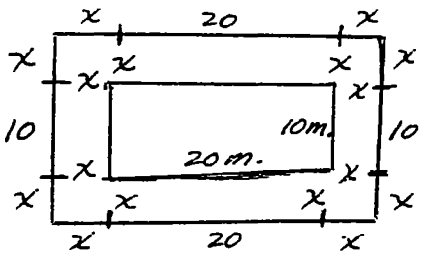
SOLUTION: Let X = _____ of box
_____ = _____ of box
_____ = _____ of box

18. A rectangular box with height 2 centimeters has a volume of 50 cubic centimeters. The length of the box is 5 less than twice the width. Find the dimensions of the box.

"What good is math?"

19. A man wants to build a rectangular speaker box whose volume is 3 cubic feet for his 15 inch (diameter) speakers. If the square base is 18 inches on each side, how deep are the speaker boxes?

EXAMPLE 6: A rectangular garden is 10 m. by 20 m. The owner, who has enough pine bark to cover 136 square meters, wants a pine bark path around the outside of the garden. Find the width of the path and the outside dimensions.



SOLUTION: Let **X** = width of the path
2X+10 = total outside width
2X+20 = total outside length
$10 \cdot 20 = 200\ m^2$ = Inner area
$136\ m^2$ = Border area (Pine bark)

Equation:

Total area	=	**Inner area**	+	**Border area**
(2X+10)(2X+20)	=	**200**	+	**136**
$4X^2 + 60X + 200$	=	336		
$4X^2 + 60X - 136$	=	0		
$4(X^2 + 15X - 34)$	=	0		
$4(X + 17)(X - 2)$	=	0		

$X = -17$; **X = 2 m. Width of path**
Reject **2X+10 = 14 m. Outside width**
2X+20 = 24 m. Outside length

20. A rectangular garden is 10 meters by 20 meters. The owner, who has enough pine bark to cover an area of 216 square meters, wants a pine bark path around the outside of the garden. Find the width of the path and the outside dimensions.

21. The combined area of an 8" by 10" picture and its frame (border around the outside) is 288 square inches. Find the width and the dimensions of the picture's frame.

22. The an 8" by 10" picture is surrounded by a matte (border) whose area is 40 square inches. (That is, the area of the matte is 40 square inches.) Find the width of the matte and the outside dimensions of the matte.

EXAMPLE 7: **The longer leg of a right triangle is 2 more than the shorter leg, and the hypotenuse is 2 less than twice the shorter leg. Find the sides of the triangle.**

SOLUTION:

Let X = shorter leg
$X+2$ = longer leg
$2(X) - 2$ = hypotenuse

Equation:

$$\mathbf{X^2 + (X+2)^2 = (2X-2)^2}$$
$$X^2 + X^2 + 4X + 4 = 4X^2 - 8X + 4$$
$$2X^2 + 4X + 4 = 4X^2 - 8X + 4$$
$$0 = 2X^2 - 12X$$
$$0 = 2X(X - 6)$$
$$2X=0 \quad \text{or} \quad X-6=0$$
$$X=0 \qquad \mathbf{X= 6} \quad \textbf{shorter leg}$$
$$\textbf{Reject} \qquad \mathbf{X+2= 8} \quad \textbf{longer leg}$$
$$\mathbf{2X-2=12} \quad \textbf{hypotenuse}$$

EXAMPLE 8: **The diagonal of a rectangle is 3 less than 4 times the width of the rectangle. The length of the rectangle is 1 less than the diagonal. Find the length of the diagonal of the rectangle.**

SOLUTION:

Let X = width of rectangle
$4X-3$ = diagonal of rectangle
$(4X-3) - 1$ = length of rectangle
$4X-4$ = length of rectangle

Equation:

$$\mathbf{X^2 + (4X-4)^2 = (4X-3)^2}$$
$$X^2 + 16X^2 - 32X + 16 = 16X^2 - 24X + 9$$
$$17X^2 - 32X + 16 = 16X^2 - 24X + 9$$
$$X^2 - 8X + 7 = 0$$
$$(X - 1)(X - 7) = 0$$
$$\mathbf{X = 1 \text{ or } X = 7} \quad \textbf{width of rectangle}$$
$$\mathbf{4X-4 = 0} \qquad \mathbf{4X-4 =24} \quad \textbf{length of rectangle}$$
$$\textbf{No Way!!} \qquad \mathbf{4X-3 =25} \quad \textbf{diagonal of rectangle}$$

23. The longer leg of a right triangle is 1 less than twice the shorter leg, and the hypotenuse is 1 more than twice the shorter leg. Find the sides of the triangle.

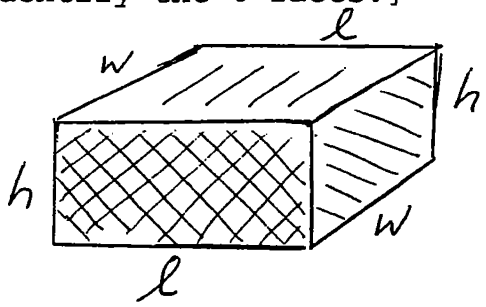
24. The shorter leg of a right triangle is 1 less than the longer leg, and the hypotenuse is 1 less than twice the shorter leg. Find the sides of the triangle.

25. The longer leg of a right triangle is 4 less than the hypotenuse. The longer leg is 2 less than twice the shortest leg. Find the sides of the triangle.
[Hint: Let X = shortest leg, and paraphrase first sentence.]

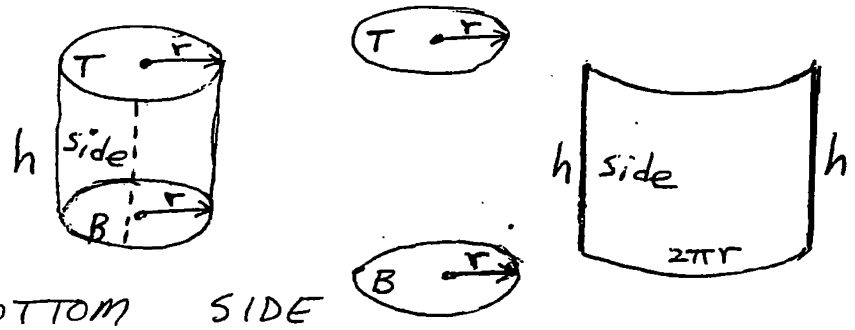
26. The hypotenuse of a right triangle is 4 less than 3 times the shortest side, and the longer leg is 4 more than twice the shortest side. Find the sides of the triangle.

"FLORIDA CLAST SKILLS"

27. Find the surface area of a rectangular box of width **w**, length **l**, and height **h**. [Hint: Identify the 6 faces.]



28. Find the surface area of a cylinder whose height is **h** and whose base radius is **r**. [Hint: Identify the top, the bottom, and "side" areas. Notice that the "side" area is actually a rectangle whose width is **h** and whose length is the circumference of the circle **2πr**.]



TOP BOTTOM SIDE

$A =$ ____ + _____ + _____

$=$ _____.

ANSWERS 4.06

p.351-362:

1. 7cm, 35cm; **2.** 2cm, 5cm; **3.** 5m, 10m; **4.** h=6ft, b=30ft; **5.** h=4cm, b=5cm; **6.** b=8m, h=5m; **7.** h=7/2 cm, b=10cm; **8.** h=5 cm, b=7cm; **9a)** 785000 m^2, **9b)** 785714.29 m^2, **9c)** 785397.5 m^2, **9d)** 785398.16 m^2 (varies with calc.) Important to use an accurate value of π); **10.** r=7cm, C=14π cm; **11.** r=23cm, C=46π cm; **12.** r=12 cm, A=144π cm^2; **13.** r=13 cm, A=169π cm^2; **14.** 6.37 ft; **15.** 8.4 laps; **16.** 12m, 12m, 4m; **17.** 5/2 cm, 10 cm, 2 cm; **18.** 5cm, 5cm, 2cm; **19.** 16"; **20.** 3m, 16m by 26m; **21.** 4", 16" by 18"; **22.** 1", 10" by 12"; **23.** 8, 15, 17; **24.** 3, 4, 5; **25.** 16, 30, 34; **26.** 10, 24, 26; **27.** 2LH + 2WL + 2WH; **28.** $2\pi r^2 + 2\pi rh$.

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