4.05 *Theorem of Pythagoras*

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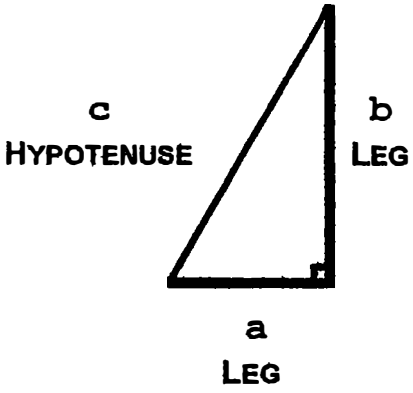
If the quadratic formula is one of the most important formulas in all of mathematics, then certainly the **Theorem of Pythagoras** is the other one. Although this theorem was known to the Babylonians 1000 years earlier, the first proof was given by the Greek mathematician **Pythagoras**, 6th century B.C.

The **Theorem of Pythagoras** deals specifically with **right triangles.** In a right triangle, the two sides that are mutually perpendicular are called **legs**, and the third side, always opposite the right angle, and always the longest side, is called the **hypotenuse** of the triangle. According to the **Theorem of Pythagoras**, if "**a**" and "**b**" are **legs**, and "**c**" is the **hypotenuse**, then $a^2 + b^2 = c^2$.

Given any two sides of a right triangle, the **Theorem of Pythagoras** can be used to find the third side. The first step is to identify which side is the hypotenuse.

THEOREM OF PYTHAGORAS
In any right triangle, where "**a**" and "**b**" are **legs**, and "**c**" is the **hypotenuse**,
$a^2 + b^2 = c^2$.

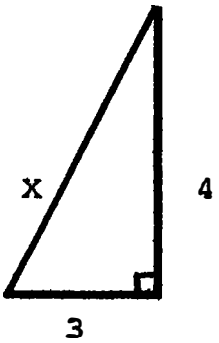


EXAMPLE 1: **Find the hypotenuse of a right triangle whose legs are 3 cm. and 4 cm.**

SOLUTION: **Let X = hypotenuse.**

Equation: $3^2 + 4^2 = X^2$
$9 + 16 = X^2$
$X^2 = 25$
$X = \pm 5$ cm

Answers: X = -5 is meaningless
X = 5 cm. is the hypotenuse



EXAMPLE 2: If the hypotenuse of a right triangle is 13 ft., and one of the legs is 5 ft., find the other leg.

SOLUTION: Let X = other leg.

Equation: $5^2 + X^2 = 13^2$

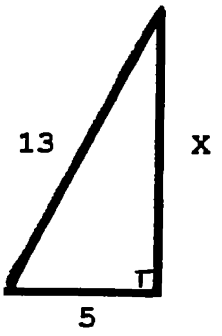
$25 + X^2 = 169$

$X^2 = 144$

$X = \pm 12$ ft

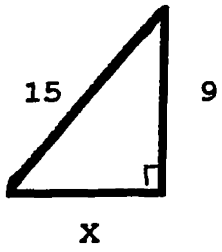
Answers: X = -12 is meaningless

X = 12 ft. is the other side



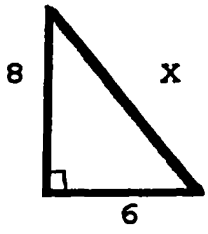
EXERCISES. Find the missing side of each triangle. (Solve for X.)

1.



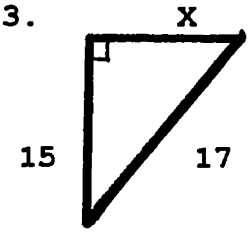
$X^2 + 9^2 = 15^2$

2.



$6^2 + 8^2 = X^2$

3.



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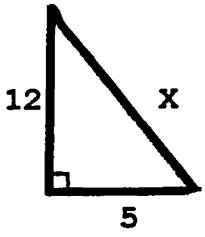

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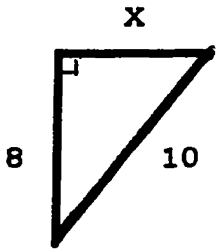

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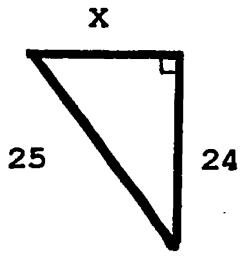


Did you notice that in the first two examples and the first 6 exercises all of the sides came out even? Do you think in all such problems, in which you are given two sides of a triangle and asked to find the third side, that the answers come out whole numbers as these did? The truth is that they do not always come out even, and in fact there are certain "special" triangles that are like this. Of course, those who make up the exercises (and test questions!) are well aware of these "special" triangles that come out even, and consequently exercises are frequently (usually?)"rigged" to work out.

Perhaps it would be helpful to let you in on these special numbers. They are called **"Pythagorean Triples"**. Although there are infinitely many such special triangles, only a few have numbers that are small enough to be "reasonable". The two most commonly used are the two from the first two examples: **3,4,5** and **5,12,13.** Two triples that are not as frequent are **8,15,17** (see #3) and **7,24,25** (see #6). In addition to these, any multiple of these numbers is also a "triple." As examples, **6,8,10** or **9,12,15** are multiples of **3,4,5.** Multiples of **5,12,13** are **10,24,26** or **15,36,39.**

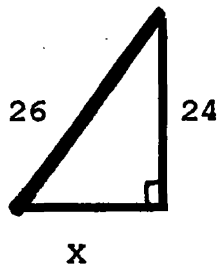
PYTHAGOREAN TRIPLES

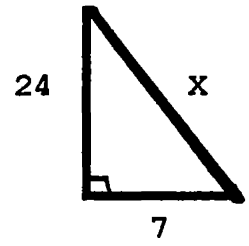
When three integers a, b, and c, are such that $a^2 + b^2 = c^2$, it is called a Pythagorean Triple. The most common are:

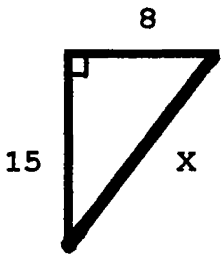
(3, 4, 5)
(5, 12, 13)
(8, 15, 17)
(7, 24, 25)
or any multiple of the above.

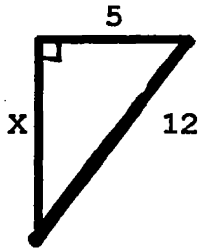
[In general: **$(2xy, x^2 - y^2, x^2 + y^2)$**
for X>Y, X>0, and Y>0
where x and y have no common factors.]

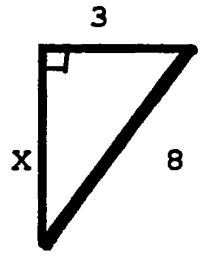
EXERCISES. Find the missing side of each triangle. (Solve for X.) For answers that do not come out even, use a calculator and round to nearest hundredth. Watch for "special" triangles.

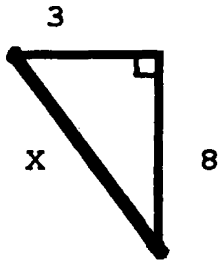
7. 

8. 

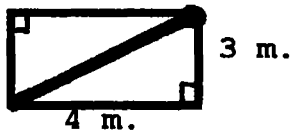
9. 

10. 

11. 

12. 

13. Notice in the figure at the right that the diagonal of a rectangle divides the rectangle into two triangles. Use this to find the diagonal if the width is 3 m. and the length is 4 m.



14. Find the diagonal of a rectangle whose width is 6 ft. and length is 8 ft.

15. Find the diagonal of a rectangle whose width is 12 cm. and length is 16 cm.

16. Find the length of a rectangle whose width is 8 ft. and whose diagonal is 17 ft.

17. Find the width of a rectangle whose diagonal is 25 cm. and length is 24 cm.

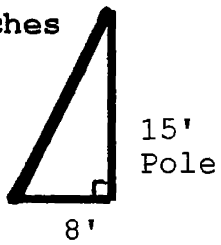
18. Find the width of a rectangle whose diagonal is 25 cm. and length is 20 cm.

19. Find the length of a rectangle whose diagonal is 29 cm. and width is 20 cm.

20. Find the diagonal of a rectangle whose width is 40 ft. and length is 42 ft.

21. Find the diagonal of a rectangle whose width is 13 cm. and length is 84 cm.

22. A guy wire to the top of a 15 foot pole reaches the ground 8 feet from the base of the pole. How long is the wire?



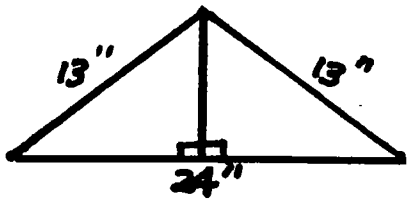
23. A guy wire to the top of a 35 foot pole reaches the ground 18 feet from the base of the pole. How long is the wire?

24. A guy wire to the top of a pole is 35 feet. It reaches the ground 18 feet from the base of the pole. How tall is the pole?

25. A guy wire to the top of a pole is 73 feet long and reaches the ground 48 feet from the base of the pole. How tall is the pole?

In the next exercises, it will be helpful to know that an **isosceles triangle** is a triangle with exactly **two** equal sides. Also, the height of the triangle is always perpendicular to the base and it cuts the base in half to form two equal triangles as shown in the illustration for **#26.**

26. Find the height of an isosceles triangle whose base is 24 inches and whose equal sides are each 13 inches.



27. Find the height of an isosceles triangle whose base is 140 inches and whose equal sides are each 74 inches.

28. An isosceles triangle has a base of 10 cm. and a height of 12 cm. How long are the equal sides?

29. An isosceles triangle has a base of 48 cm. and a height of 70 cm. How long are the equal sides?

30. An isosceles triangle has a base of 64 cm. and a height of 126 cm. How long are the equal sides?

ANSWERS 4.05

p.343-348:

1. 12; **2.** 10; **3.** 8; **4.** 13; **5.** 6; **6.** 7; **7.** 10; **8.** 25; **9.** 17;

10. $\sqrt{119}$ or 10.91; **11.** $\sqrt{55}$ or 7.42; **12.** $\sqrt{73}$ or 8.54; **13.** 5 m;

14. 10 ft; **15.** 20 cm; **16.** 15 ft; **17.** 7cm; **18.** 15 cm;
19. 21 cm; **20.** 58 ft; **21.** 85 cm; **22.** 17 ft;

23. $\sqrt{1549}$ or 39.36 ft; **24.** $\sqrt{901}$ or 30.02 ft; **25.** 55 ft;

26. 5 in; **27.** 24 in; **28.** 13 cm; **29.** 74 cm; **30.** 130 cm.

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE