5.01 *Linear Graphing, Slope of a Line*

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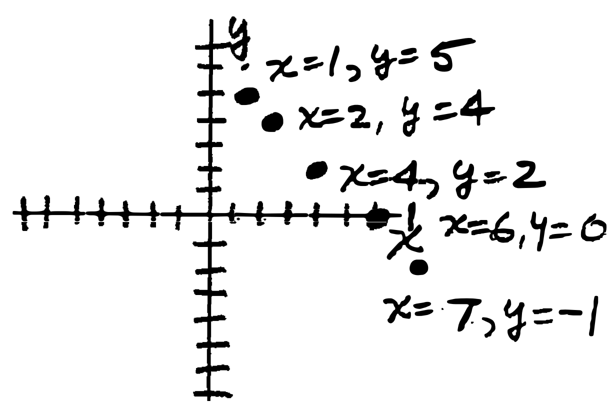
ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

Up to this point the equations you have graphed have been one dimensional. You have solved <u>linear equations</u>--that is, equations with the variable raised to the first power like $X + 4 = 6$, and you have solved <u>quadratic equations</u>--that is, equations with the variable raised to the second power like $X^2 - 3X + 2 = 0$. It was easy to express the solutions to these types of equations, since linear equations usually have only one solution (quadratic equations up to two solutions!).

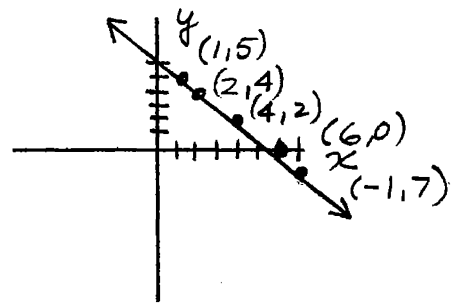
Now consider an equation with **two variables**, like $X + Y = 6$. Since there are two variables, each solution will have two parts, an "X" part and a "Y" part. One solution would be X=1 and Y=5. Another solution would be X=2 and Y=4. Other solutions would be X=4 and Y=2, X=6 and Y=0, X=7 and Y=-1, X=1.5 and Y=4.5, etc. As you can see, the number of solutions is infinite--considering the negative and fractional values, there is no way to list all the solutions. It is helpful to find a few of the solution values of X and Y, and put them in a table for convenience, as shown at the right.

x	y
1	5
2	4
4	2
6	0
7	-1
1.5	4.5

When you were solving a linear "X" type equation, you could graph the solution on a horizontal numberline. Now that you have equations in "X" and "Y" it will be helpful to graph the "X" values along a horizontal numberline, as before, and then construct a vertical line to measure the "Y" values. In this way each pair of X and Y values represents a point and may be graphed as shown at the right. The X-axis is always horizontal, and the Y-axis is always vertical. The intersection of the X and Y axes (where X=0 and Y=0) is called the <u>**origin**</u>. To



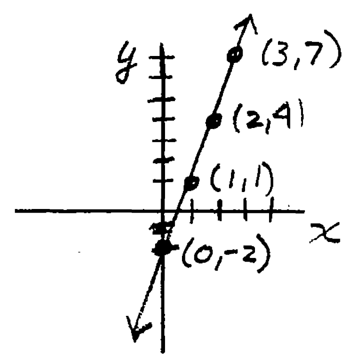
graph each point, always begin at the origin, and count the X value to the right (or the left if X is negative); from there count the Y value up (down if Y is negative), and <u>then</u> put the point.



Instead of having to label the points for example with "X=4 and Y=2", we shorten the notation using parentheses (4,2), where the first number is always the X value, and the second number is always the Y value (alphabetical order!)

Did you notice that all of the solution points you graphed for X + Y = 6 lie on a straight line? Does that tell you about other points that you did not actually graph? Notice that the graph does not stop with the last point graphed, but continues in either direction, as signified by the arrows on the ends of the line.

As another example, use the equation **Y = 3X - 2.** Let X equal any numbers you select, and use the equation to find the corresponding values of Y.
For example, if you let X=0, then Y=-2.
Next let X=1, then Y=3·1-2, so Y=1.
Next, let X=2, then Y=3·2-2, so Y=4.
Next, let X=3, then Y=3·3-2, so Y=7.
The graph of these points also forms a straight line.



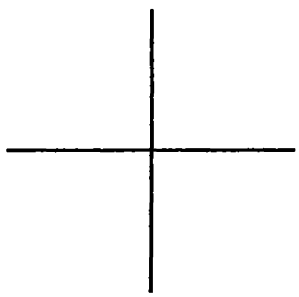
While not every equation in X and Y will be a straight line graph, it is true that every graph in which the X and Y terms are raised to the first power (linear) will be a straight line. The simplest approach to graphing a line (even simpler than graphing calculators!) is point plotting. Just select a value of X (any value of X you choose!) and from the equation find the corresponding value of Y. And how many points does it take? For straight lines, two points will be enough (perhaps a third point to check). For graphs that are not straight lines, more points will be needed, depending upon your own insight and experience.

After graphing straight lines by point plotting, you will pursue additional graphing techniques, including **slope-intercept method, X and Y-intercept method, non-linear methods,** and of course, the use of the **graphing calculator.**

EXERCISES. **Complete the following table of values and graph the line. If table values are not given, make up a few values of your own.**

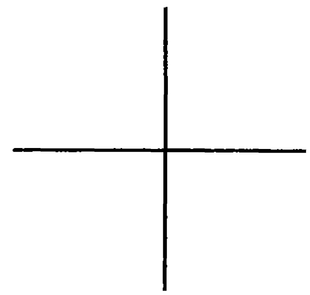
1. $Y = 1X + 2$

X	Y
0	2
1	3
2	
-1	



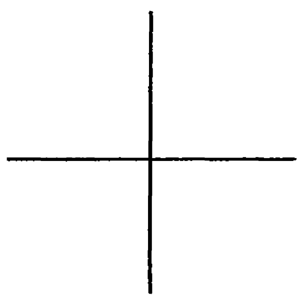
2. $Y = 2X + 2$

X	Y
0	2
1	
2	
-1	



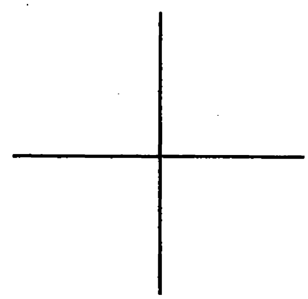
3. $Y = 3X + 2$

X	Y
0	
1	
-1	
-2	



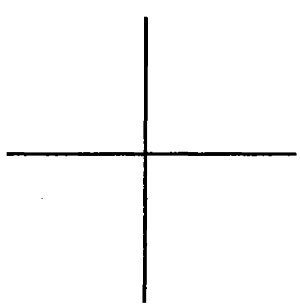
4. $Y = -1X + 2$

X	Y
0	
1	
2	
-1	



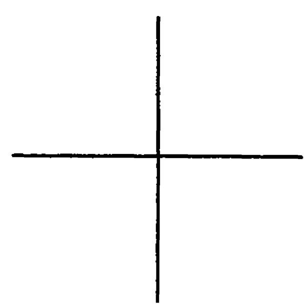
5. $Y = -2X + 2$

X	Y



6. $Y = -3X + 2$

X	Y



7. $Y = X + 4$

X	Y

8. $Y = 2X + 4$

X	Y

9. $Y = -X + 4$

X	Y

10. $Y = -2X + 4$

X	Y

11. $Y = X - 2$

X	Y

12. $Y = 2X - 2$

X	Y

13. $Y = -2X - 2$

X	Y

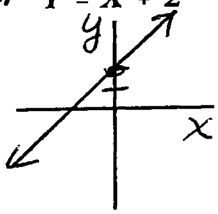
14. $Y = -X - 2$

X	Y

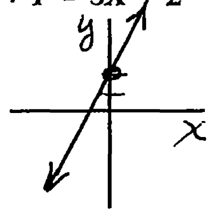
DEFINITIONS		
X-intercept:	where the graph crosses the X-axis	(Y=0)
Y-intercept:	where the graph crosses the Y-axis	(X=0)

Now, look at each of the following equations and their graphs.

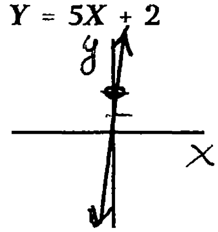
A) $Y = X + 2$



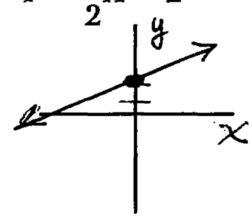
B) $Y = 3X + 2$



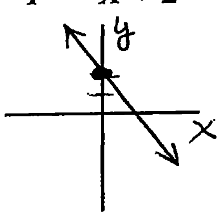
C) $Y = 5X + 2$



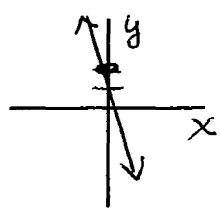
D) $Y = \frac{1}{2}X + 2$



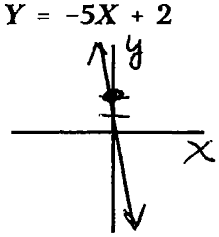
E) $Y = -X + 2$



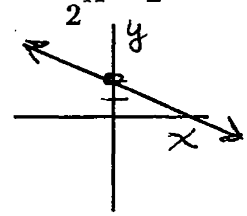
F) $Y = -3X + 2$



G) $Y = -5X + 2$



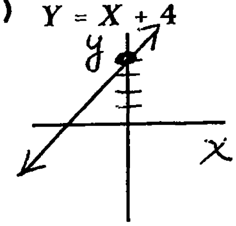
H) $Y = -\frac{1}{2}X + 2$



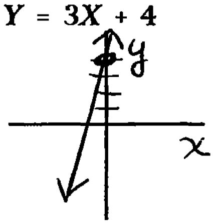
1a) What do the above equations have in common? ________________

b) What do the above graphs have in common? ________________

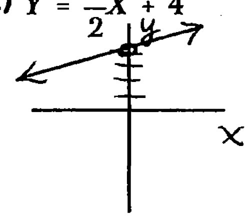
I) $Y = X + 4$



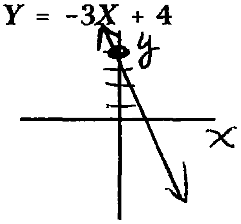
J) $Y = 3X + 4$



K) $Y = \frac{1}{2}X + 4$



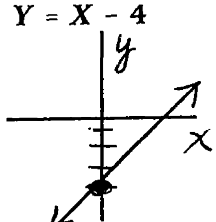
L) $Y = -3X + 4$



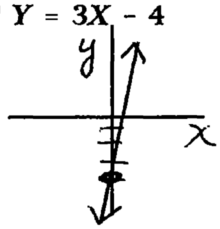
2a) What do the above equations have in common? ________________

b) What do the above graphs have in common? ________________

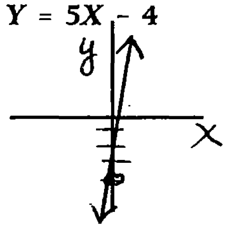
M) $Y = X - 4$



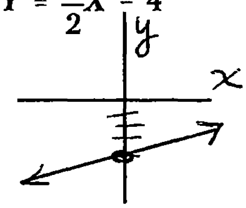
N) $Y = 3X - 4$



O) $Y = 5X - 4$



P) $Y = \frac{1}{2}X - 4$



3a) What do the above equations have in common? ________________

b) What do the above graphs have in common? ________________

4. Find the the Y-intercept:

A) $Y = 7X + 2$	______	G) $Y = -X - 12$	______
B) $Y =-7X + 2$	______	H) $Y = -3X + 3/2$	______
C) $Y =-5X + 6$	______	I) $Y = 3/2X - 5/2$	______
D) $Y = 8X - 7$	______	J) $Y = 7 + 3X$	______
E) $Y = 8X + 1$	______	K) $Y = -7 + 3X$	______
F) $Y =-8X + 156$	______	L) $Y = 3X$	______

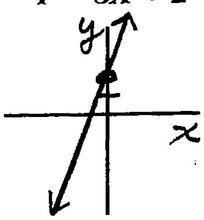
M) GENERALIZATION: For $Y = mX + b$, the Y-intercept is _____

5. VERBALIZATION:

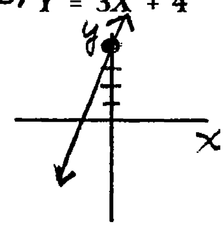
Graphically, the Y-intercept is where the graph crosses the __________. It can be found by letting the value of ______ equal ________. In a linear equation it is the ________ term.

Look at each of the following equations and their graphs:

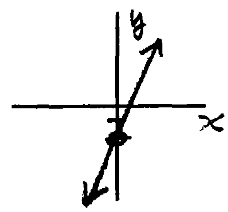
A) $Y = 3X + 2$



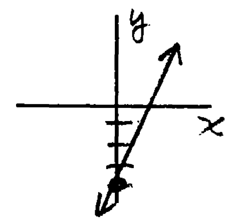
B) $Y = 3X + 4$



C) $Y = 3X - 2$



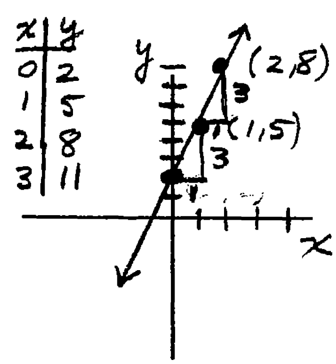
D) $Y = 3X - 4$



6. What do the above equations have in common? ______________

What do the above graphs have in common? ______________

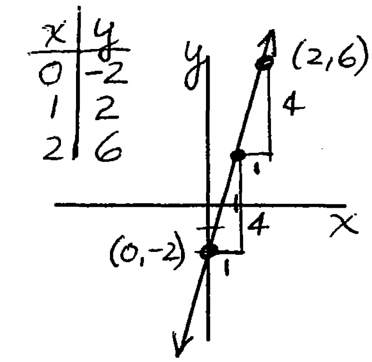
You should have noticed that in the four equations above, in each equation, the coefficient of X was 3. As for the graphs, you should have noticed that they are parallel. Lines that are parallel have the same "steepness" or **slope**.



There is a way of describing the "steepness" or "slope" of a line. Consider the graph of **Y = 3X + 2.** First, notice that the Y-intercept is 2. Beginning at the Y-intercept (0,2), move one unit right, then up 3 units. You are now at the point (1,5). Now, from (1,5) move another unit right and up 3 more units. You are now at (2,8).

Since for each horizontal unit, you moved vertically 3 units, we say this line has a slope of 3/1 or 3. Slope then is defined:

$$\textit{SLOPE} = \frac{\textit{VERTICAL}}{\textit{HORIZONTAL}} \text{ or } \frac{\textit{RISE}}{\textit{RUN}} \text{ or } \frac{\textit{Y distance}}{\textit{X distance}}$$



Returning to the previous equation **Y = 3X + 2,** notice that the slope of the graph was 3. Also, the coefficient of X is 3.

Consider **Y = 4X - 2.** Beginning at the **Y-intercept** at **(0,-2),** move 1 unit right and up 4 to (1,2). Next move from (1,2) another unit to the right, then move 4 more units to (2,6). Do you see that the slope is $\frac{\textit{rise}}{\textit{run}} = \frac{4}{1}$ ***or* 4 ?** Do you see that the equation**Y= 4X - 2** has **X-coefficient 4?** When the equation is in the form **Y = _?_X + _?_,** the **coefficient of X is the slope.**

7. Give the slope of each of the following lines;

A)	**Y = 5X + 2**	________	**I)**	**Y = 3 - X**	________
B)	**Y = 2 + 5X**	________	**J)**	**Y = 3/2 X + 5**	________
C)	**Y = -5X**	________	**K)**	**Y = -2/3 X**	________
D)	**Y = -3X + 5**	________	**L)**	**Y = 4/5 X - 7**	________
E)	**Y = 3 - 2X**	________	**M)**	**Y = 1/2 - 10/3 X**	________
F)	**Y = X + 5/2**	________	**N)**	**Y = 0·X + 3**	________
G)	**Y = 5 + X**	________	**O)**	**Y = 3**	________
H)	**Y = -X - 3/2**	________	**P)**	**Y = mX + b**	________

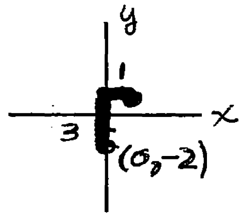
If the equation is written **Y = mX + b, m** represents the **slope,** and **b** represents the **y-intercept.** This gives a fast way to graph a line if it is given in this form. The exercises on the next pages will help you understand.

Graph each of the following by locating the Y-intercept. From this point measure the slope (rise over run) and plot the next point.

8. $Y = 3X - 2$

Y-int = -2

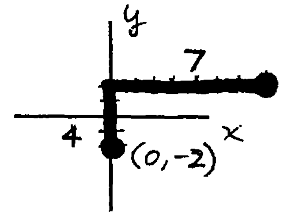
Slope $m = \frac{3}{1} = \frac{rise}{run}$



9. $Y = \frac{4}{7}X - 2$

Y-int = -2

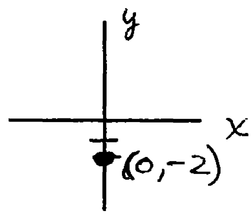
Slope $m = \frac{4}{7} = \frac{rise}{run}$



10. $Y = \frac{2}{3}X - 2$

Y-int = ____

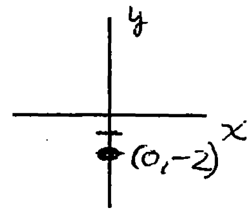
Slope = ____



11. $Y = \frac{4}{1}X - 2$

Y-int = ____

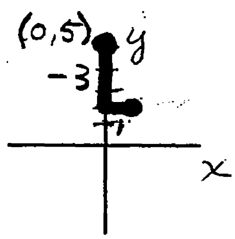
Slope = ____



12. $Y = \frac{-3}{1}X + 5$

Y-int = ____

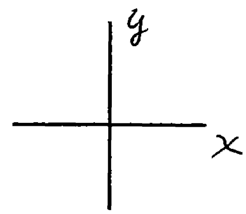
Slope = ____



13. $Y = \frac{-3}{5}X + 4$

Y-int = ____

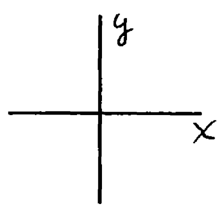
Slope = ____



14. $Y = 1X + 5$

Y-int = ____

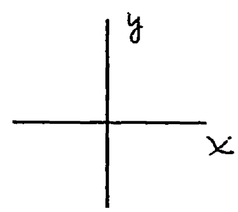
Slope = ____



15. $Y = -X + 3$

Y-int = ____

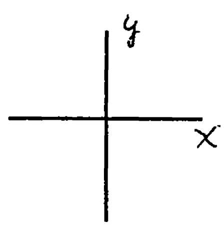
Slope = ____



16. $Y = 5 - \frac{2}{3}X$

Y-int = ____

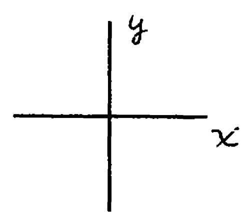
Slope = ____



17. $Y = 4 + 3X$

Y-int = ____

Slope = ____



18. $Y = -2X + 3$

19. $Y = 5X - 2$

20. $Y = \frac{2}{3}X - 4$

21. $Y = -\frac{2}{3}X + 6$

22. $Y = X - 4$

23. $Y = 4 - X$

24. $Y = 3 - \frac{1}{2}X$

25. $Y = \frac{5}{2}X - 4$

26. $Y = \frac{3}{4}X + 2$

27. $Y = -\frac{3}{4}X - 2$

28. $Y = 5 - \frac{2}{3}X$

29. $Y = 4 + 3X$

Notice that each of the equations on the previous pages was given in the form **$Y = mX + b$ (slope-intercept form).** However, equations are frequently given in another form called **standard form** in which the **X** and **Y terms** are on the same side of the equation: **$AX + BY = C$.** The tradition is to make A, B, and C integers, and the value of A positive. If the equation is given in **standard form,** the way to find the **slope** is to **solve for Y in terms of X.**

EXAMPLE. Find the slope and Y-intercept of $3X + Y = 6$.

SOLUTION: Add $-3X$ to both sides: $\underline{-3X \quad\quad -3X}$

$$Y = -3X + 6$$

Y-intercept is 6 and slope is -3.

In 30-38, solve for Y in terms of X. Give Y-intercept and slope.

30. $4X + Y = 8$ **31.** $-4X + Y = 8$ **32.** $Y - 3X = -4$

33. $2X + 3Y = 12$

$\underline{-2X \quad\quad -2X}$

$3Y = -2X+12$

$Y =$

Y-Int=____ m=____

34. $-2X + 3Y = 12$ **35.** $3X + 2Y = 8$

36. $X - 2Y = 6$

$-2Y = -X + 6$

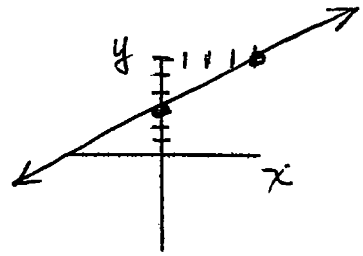
$Y =$

Y-Int=____ m=____

37. $3X - 2Y = 6$ **38.** $5X - 3Y = -6$

EXAMPLE FOR DISCUSSION:

Graph $3X - 4Y = -12$ by slope-intercept method



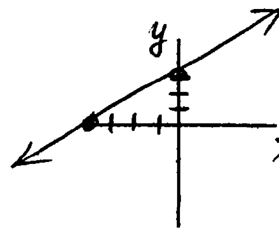
$$-4Y = -3X - 12$$

$$Y = \frac{3}{4}X + 3$$

Y-int = 3; $m = \frac{3}{4} = \frac{rise}{run}$

When the equation is given in **standard form,** it seems a lot of work to solve for Y in order to find the Y-intercept and slope. It may be easier to find the **X and Y-intercepts** and graph in this way:

Same example by "TWO INTERCEPT" method:



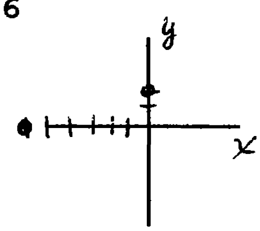
$3X - 4Y = -12$

X	Y
0	3
-4	0

In 39 - 52, find the X and Y intercepts, and sketch the graph.

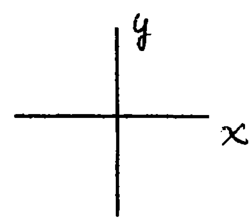
39. $-X + 3Y = 6$

X	Y
0	2
-6	0



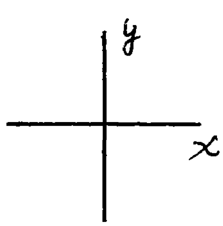
40. $4X - 2Y = -8$

X	Y
0	4
-2	0



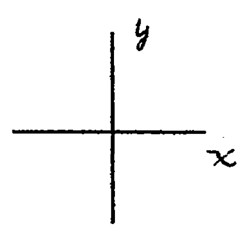
41. $-2X + 3Y = 12$

X	Y
0	
	0



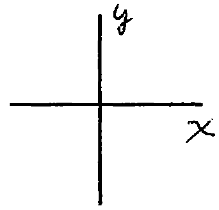
42. $4X + 2Y = -8$

X	Y
0	
	0



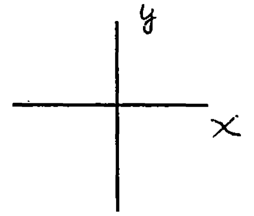
43. X - 2Y = -8

X	Y
0	
	0



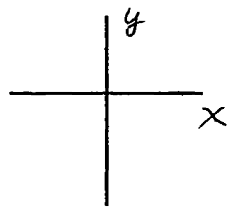
44. 2X - 3Y = 12

X	Y
0	
	0



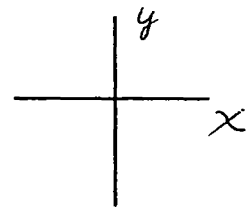
45. 3X - Y = -3

X	Y



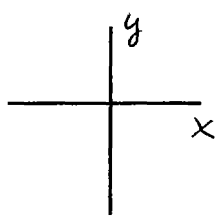
46. X - 2Y = 6

X	Y



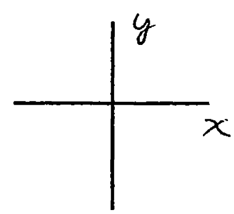
47. X + Y = -3

X	Y



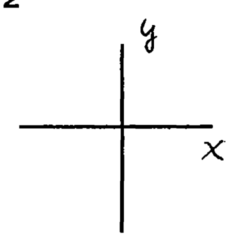
48. X + 2Y = 8

X	Y



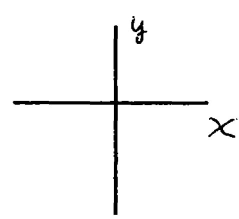
49. -3X + 2Y = -12

X	Y



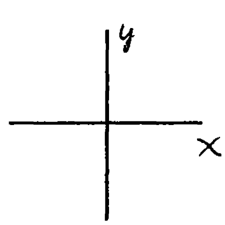
50. 4X - 2Y = -12

X	Y



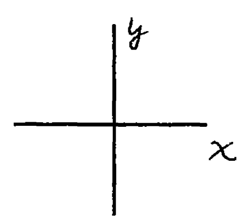
51. X + 2Y = -6

X	Y



52. -2X - 3Y = 6

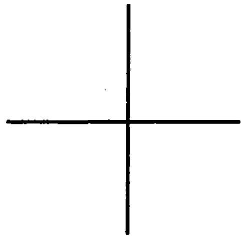
X	Y



53. Sometimes an equation has no X term. For example, Y=4. This could also be written as $0 \cdot X + Y = 4$. In this case, X could be any value, as long as the value of Y is 4. Complete the following table, and draw the graph.

Y = 4

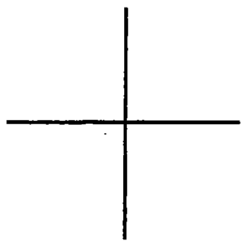
X	Y
0	
1	
2	
4	
-2	



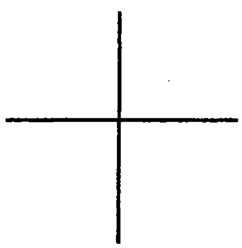
54. Sometimes an equation has no Y term. For example, X=4. This could also be written as $X + 0 \cdot Y = 4$. In this case, Y could be any value, as long as the value of X is 4. Complete the following table, and draw the graph.

X = 4

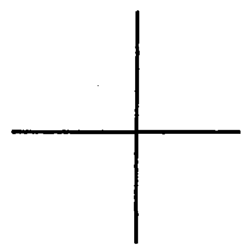
X	Y
	0
	1
	2
	4
	-2



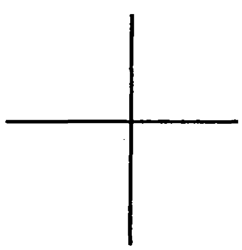
55. Y = -2



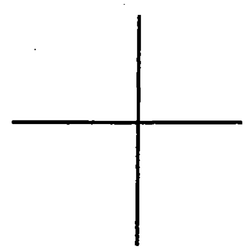
56. X = -2



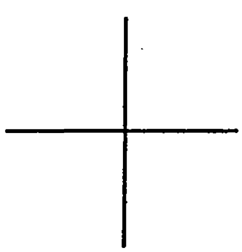
57. X = 2



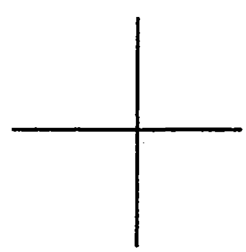
58. Y = 2



59. Y = 0



60. X = 0



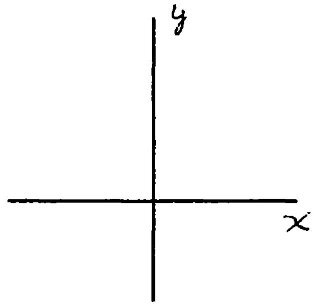
61. The graph of "Y = (a number)" is always a ____________ line. There is no X-intercept.

62. The graph of "X = (a number)" is always a ____________ line. There is no ____________.

The following equations are not straight lines. A few more points and some "dot-to-dot" ingenuity will be needed.

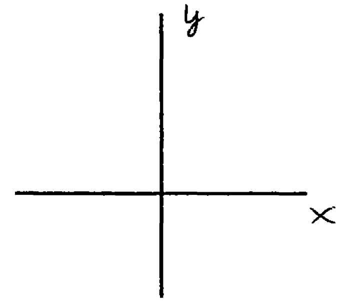
63. $Y = X^2$

X	Y
0	
1	
2	
3	
-1	1
-2	
-3	



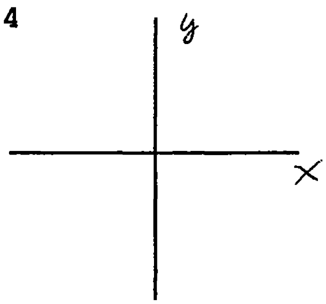
64. $Y = X^2 + 2$

X	Y
0	
1	
2	
3	
-1	3
-2	
-3	



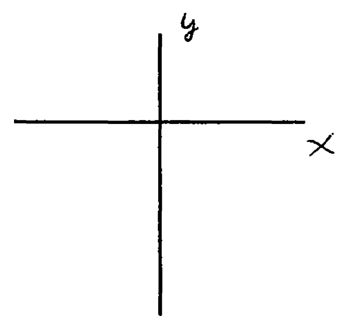
65. $Y = X^2 - 4$

X	Y
0	
1	
2	
3	
-1	-3
-2	
-3	



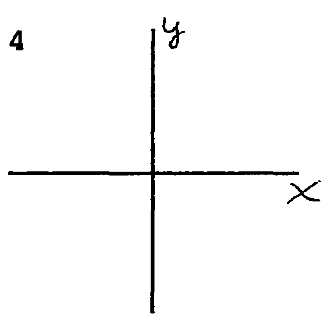
66. $Y = -X^2$

X	Y
0	
1	-1
2	
3	
-1	-1
-2	
-3	



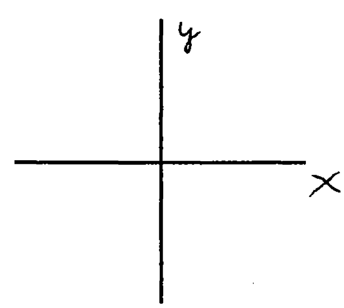
67. $Y = -X^2 + 4$

X	Y
0	
1	3
2	
3	
-1	3
-2	
-3	



68. $Y = X^3$

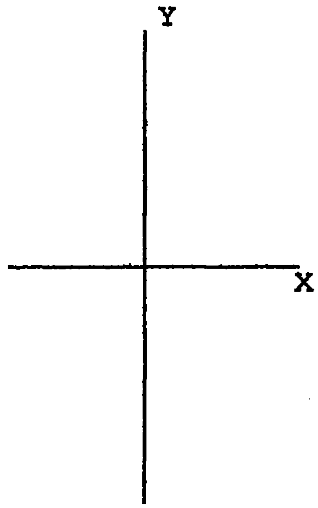
X	Y
0	
1	
2	
-1	
-2	



69. $Y = \frac{12}{X}$

(Notice that $X \neq 0$!)

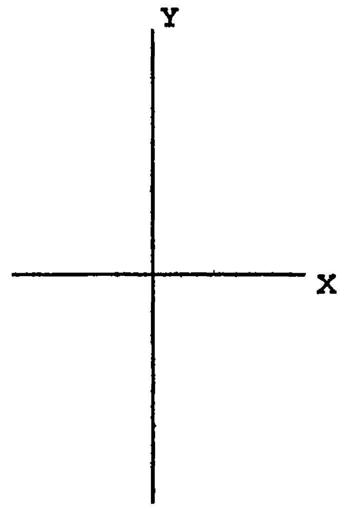
X	Y
1	
2	
3	
4	
6	
-1	
-2	
-3	
-4	
-6	



70. $Y = \frac{-12}{X}$

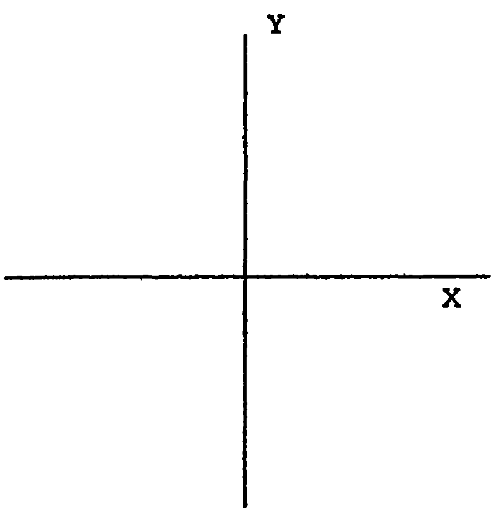
(Again, $X \neq 0$!)

X	Y
1	
2	
3	
4	
6	
-1	
-2	
-3	
-4	
-6	



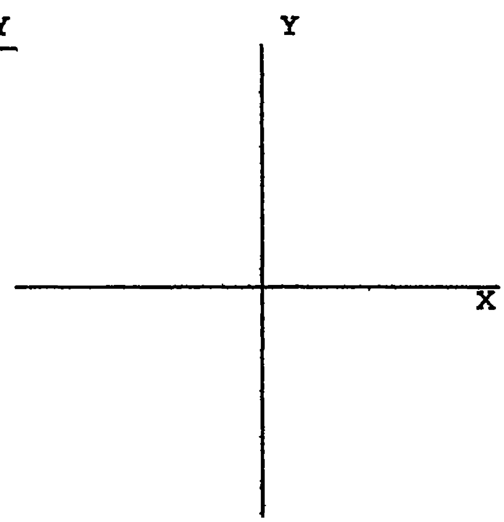
71. $Y = \frac{36}{X^2}$

X	Y
2	
3	
6	
-2	
-3	
-6	
0	??



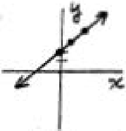
72. $Y = \sqrt{25 - X^2}$

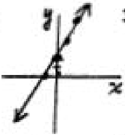
X	Y
0	
3	
4	
5	
-3	
-4	
-5	



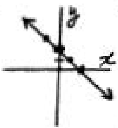
ANSWERS 5.01

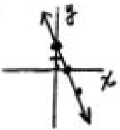
p.373-374:

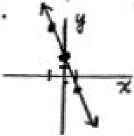
1. 

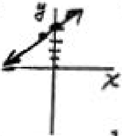
2. 

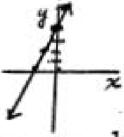
3. 

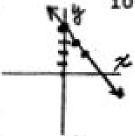
4. 

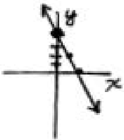
5. 

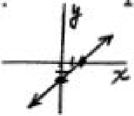
6. 

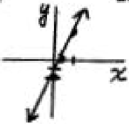
7. 

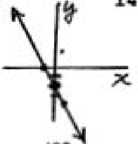
8. 

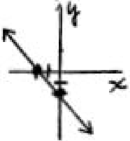
9. 

10. 

11. 

12. 

13. 

14. 

p.375-385:

1a) "Y= +2", **b)** Y-int=2; **2a)** "Y= +4", **b)** Y-int=4;

3a) "Y= -4", **b)** Y-int=-4; **4A)** 2, **B)** 2, **C)** 6, **D)** -7, **E)** 1,

4F) 156, **G)** -12, **H)** 3/2, **I)** -5/2, **J)** 7, **K)** -7, **L)** 0, **M)** b;

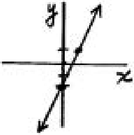
5A) Y-axis, **B)** X, **C)** 0, **D)** constant or number term;

6A) "Y=3X ..." **B)** Same slope (steepness); **7A)** 5, **B)** 5, **C)** -5,

7D) -3, **E)** -2, **F)** 1, **G)** 1, **H)** -1, **I)** -1, **J)** 3/2, **K)** -2/3,

7L) 4/5, **M)** -10/3, **N)** 0, **O)** 0, **P)** m;

8.



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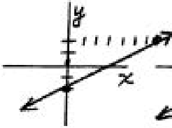

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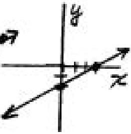

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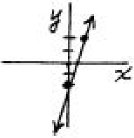

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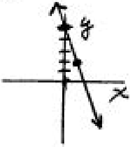

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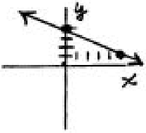

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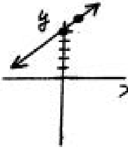

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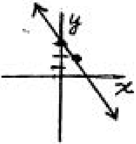

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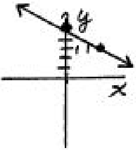

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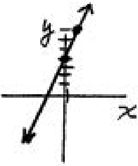

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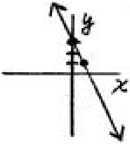

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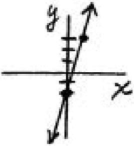

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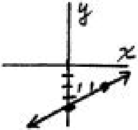

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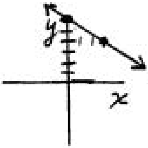
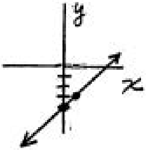


22.



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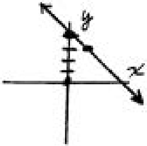

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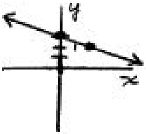

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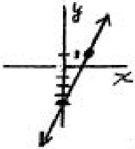

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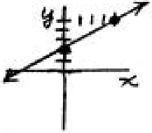

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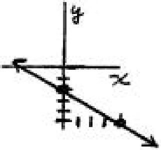

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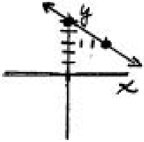

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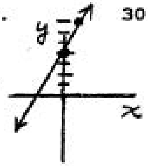


30. $y_{int} = 8$, $m = -4$

31. $y_{int} = 8$, $m = 4$

32. $y_{int} = {}^{-}4$, $m = 3$

33. $y_{int} = 4$, $m = -2/3$

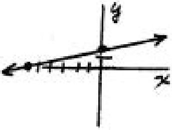
34. $y_{int} = 4$, $m = 2/3$

35. $y_{int} = 4$, $m = -3/2$

36. $y_{int} = -3$, $m = \frac{1}{2}$

37. $y_{int} = -3$, $m = 3/2$

38. $y_{int} = 2$, $m = 5/3$

39.


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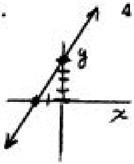

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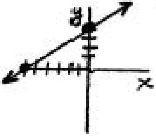

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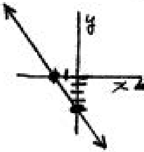

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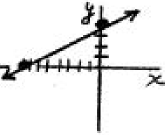

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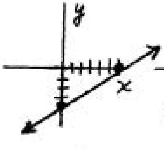

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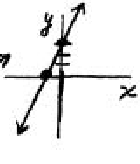

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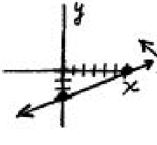

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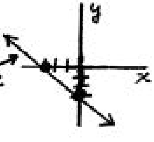

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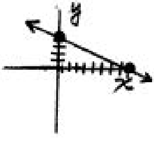

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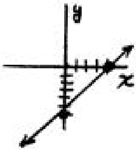

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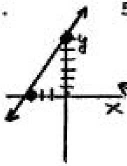

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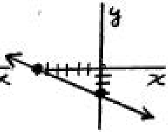

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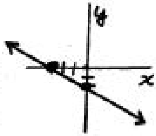

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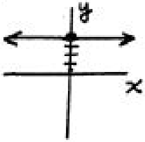

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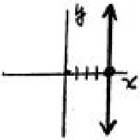

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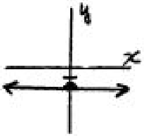

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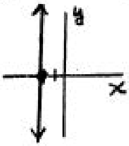

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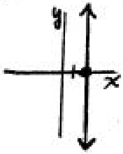

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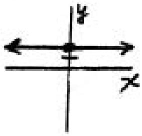

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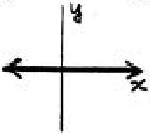

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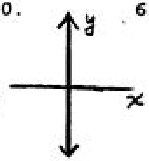
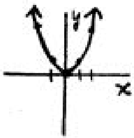


61. horizontal

62. vertical, y intercept.

63.



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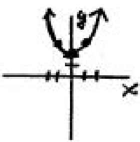

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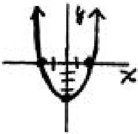

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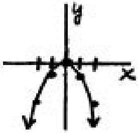

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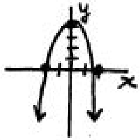

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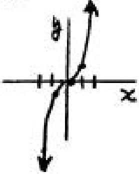

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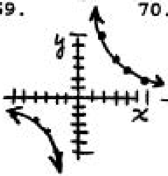

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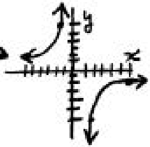

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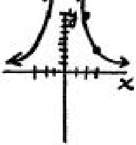

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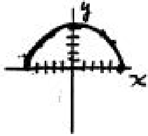

Dr. Robert J. Rapalje

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE