5.03 *Equation of a Line*

In the last two sections, an equation in x and y was generally given to you. Given an equation, the problem was usually to find the slope, to find the x and y-intercepts, or perhaps to graph the line whose equation was given. Now in this lesson, we turn things around. As in the television game show ***Jeopardy***, where they are given answers and the contestants are required to give the question, now you will be given the slope and a point on the line, and you will be required to find the **equation of a line**. (However, you must always remember to answer in the form of an equation!)

When giving the equation of a line, it is traditional to express the equation in either **standard form, $Ax + By = C$**, or in **slope-intercept form, $y = mx + b$.** With the increase in use of graphing that the **slope-intercept form, $y = mx + b$** should be the equation form of choice. **In order to find the equation of a line, you must know the slope and a point on the line.**

The easiest way to accomplish this is to begin with the familiar equation **$y = mx + b$.**

If you know the slope and a point on the line,
you can find the equation of the line using

$$y = mx + b$$

EXAMPLE 1. Find the equation of the line with slope m=4 that passes through the point (-3,2).

Step 1: Write the equation $y = mx + b$

Step 2: Substitute the numbers. You are given the value of m=4, and you know that x=-3 when y=2. Substitute these numbers into the equation

$y = mx + b$
$2 = 4(-3) + b$

Step 3: Solve for b.
$2 = -12 + b$
Add +12 to each side: $+12 \quad +12$
$14 = b$

Step 4: Write the equation:
$y = mx + b$
$y = 4x + 14$ **(Final Answer!)**

Step 5: Check by substitution:
$2 = 4(-3) + 14$
$2 = -12 + 14$

EXERCISES: In each of the following exercises, express the equation of the line in slope-intercept form.

1. m = 3, through (-2,4)

Step 1: Write the equation: $y = mx + b$

Step 2: Substitute m=3, x=-2, y=4: $4 = 3(-2) + b$

Step 3: Solve for b: ___ = ____ + b

___ = b

Step 4: Write the equation: y = ___x + ___ **Final Answer!**

Step 5: Check by substitution:

2. m = -3 through (4,2)

3. m = -3 through (2,-4)

4. **m = -2 through (-3,4)**

5. **m = 2 through (-3,-4)**

6. **m = 4 through (5,-3)**

7. **m = -4 through (3,-5)**

8. **m = 5/2, through (3,4)**

Step 1: Write the equation: $y = mx + b$

Step 2: Substitute $m=\frac{5}{2}$, $x=3$, $y=4$: $4 = \frac{5}{2}(3) + b$

Step 3: Solve for b. $4 = \frac{15}{2} + b$

Multiply both sides by 2: $2(4) = 2(\frac{15}{2}) + 2(b)$

$8 = 15 + 2b$

$\_\_\_\_ = 2b$

$b = \_\_\_\_$

Step 4: Write the equation:

Step 5: Check:

9. **m = 3/4 through (-5,-2)**

10. **m = 3/2 through (-4,3)**

11. **m = -3/2 through (4,-3)**

12. **m = -5/3 through (6,-3)**

13. **m = 5/3 through (-6,3)**

14. **m = -7/2 through (4,1)**

In order to find the equation of a line containing two points, think, what ***two*** elements are required? Answer: a **point** and a **slope.** Now, if you are given two points and not the slope**,** it shouldn't take a rocket scientist to see what is needed here! If you are given two points, your first step must be to **find the slope m between the two points**. Use the formula as follows:

15. **Find the equation of the line through (2, -4) and (-5, 6).**

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{6 - (-4)}{-5 - 2} = \frac{10}{-7} = -\frac{10}{7}$

y = mx + b

Now use **either** point, first (2, -4): $-4 = -\frac{10}{7}(2) + b$

Multiply both sides by 7: $7(-4) = 7(-\frac{20}{7}) + 7(b)$

____ = _____ + 7b

Solve for b:

b = ___

Write the equation: y = _____

16. As a check, complete the previous problem, find the equation of the line through (2, -4) and (-5, 6), using the other point (-5,6). Recall that the slope of the line was $-\frac{10}{7}$.

$$\mathbf{y = mx + b}$$

Use **the other** point (-5, 6): $6 = -\frac{10}{7}(-5) + b$

Multiply both sides by 7: $7(6) = 7(\frac{50}{7}) + 7(b)$

$\_\_\_\_ = \_\_\_\_ + 7b$

Solve for b:

$b = \_\_\_\_\_$

$y = \_\_\_\_\_$

Does this agree with answer on previous page? _____

17. **Find the equation of the line through (-4, 2) and (2, 14).**

18. **Find the equation of the line through (-4, 2) and (-2, -6).**

19. **Find the equation of the line through (-2, -3) and (2, 4).**

20. **Find the equation of the line through (2, 6) and (-2, 3).**

21. **Find the equation of the line through (-3, 6) and (-1, 3).**

22. **Find the equation of the line through (4, -3) and (-1, 3).**

23. **Find the equation of the line through (-4, 5) and (1, 8).**

24. **Find the equation of the line through (3, -2) and (-4, 4).**

25. **Find the equation of the line with *x*-int 4 and y-int -3.**

[Hint: (4, 0) and (0, -3)]

26. **Find the equation of the line with *x*-int -2 and y-int -5.**

27. **Find the equation of the line with *x*-int -4 and y-int 6.**

28. **Find the equation of the line with *x*-int 6 and y-int 3.**

In the next exercises, remember that if two lines are parallel, they have the same slope. If two lines are perpendicular, the slope of one is the negative reciprocal of the other.

29a) **Find the equation of the line through (4, -2) and parallel to y = -3*x* + 5.**

m = _____ $m_{parallel}$= _____

b) **Find the equation of the line through (4,-2) and perpendicular to y = -3*x* + 5.**

m = _____ m⊥ = _____

30a) **Find the equation of the line through (-3, 5) and parallel to y = -7*x* - 1.**

m = _____ $m_{parallel}$= _____

b) **Find the equation of the line through (-3,5) and perpendicular to y = -7*x* - 1.**

m = _____ m⊥ = _____

31a) **Find the equation of the line through (4, -5) and parallel to** $y = \frac{3}{4}x + 2$.

m = _____ $m_{parallel}$ **= _____**

b) **Find the equation of the line through (4,-5) and perpendicular to** $Y = \frac{3}{4}x + 2$.

m = _____ $m_{\perp}$ **= _____**

32a) **Find the equation of the line through (4, -5) and parallel to** $Y = -\frac{3}{4}x + 2$.

m = _____ $m_{parallel}$ **= _____**

b) **Find the equation of the line through (4,-5) and perpendicular to** $Y = -\frac{3}{4}x + 2$.

m = _____ $m_{\perp}$ **= _____**

33a) **Find the equation of the line through (4, 2) and parallel to** $4x + 3y = 12$.

m = _____ $m_{parallel}$ **= _____**

b) **Find the equation of the line through (4, 2) and perpendicular to** $4x + 3y = 12$.

m = _____ $m_{\perp}$ **= _____**

34a) **Find the equation of the line through (4, 2) and parallel to $4x - 3y = 12$.**

b) **Find the equation of the line through (4, 2) and perpendicular to $4x - 3y = 12$.**

35a) **Find the equation of the line through (-3, 5) and parallel to $7x - 4y = 1$.**

b) **Find the equation of the line through (-3, 5) and perpendicular to $7x - 4y = 1$.**

36a) **Find the equation of the line through (4, -5) and parallel to $3x + 5y = -15$.**

b) **Find the equation of the line through (4,-5) and perpendicular to $3x + 5y = -15$.**