

# 5.03 Equation of a Line

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**ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE**

In the last two sections, an equation in  $x$  and  $y$  was generally given to you. Given an equation, the problem was usually to find the slope, to find the  $x$  and  $y$ -intercepts, or perhaps to graph the line whose equation was given. Now in this lesson, we turn things around. As in the television game show *Jeopardy*, where they are given answers and the contestants are required to give the question, now you will be given the slope and a point on the line, and you will be required to find the **equation of a line**. (However, you must always remember to answer in the form of an equation!)

When giving the equation of a line, it is traditional to express the equation in either **standard form**,  $Ax + By = C$ , or in **slope-intercept form**,  $y = mx + b$ . With the increase in use of graphing calculators, which require the equation of a graph to be entered in the form of  $y = \underline{\hspace{2cm}}$ , it seems that the **slope-intercept form**,  $y = mx + b$  should be the equation form of choice. In order to find the equation of a line, you must know the slope and a point on the line.

The easiest way to accomplish this is to begin with the familiar equation  $y = mx + b$ .

*If you know the slope and a point on the line,  
you can find the equation of the line using*

$$y = mx + b$$

**Example 1.** Find the equation of the line with slope  $m=4$  that passes through the point  $(-3,2)$ .

**Step 1:** Write the equation  $y = mx + b$

**Step 2:** **Substitute the numbers.** You are given the value of  $m=4$ , and you know that  $x=-3$  when  $y=2$ . Substitute these numbers into the equation  $y = mx + b$ .

$$2 = 4(-3) + b$$

**Step 3:** **Solve for b.**

$$2 = -12 + b$$

Add +12 to each side:

$$\begin{array}{r} +12 \quad +12 \\ \hline 14 = b \end{array}$$

**Step 4:** **Write the equation.**

$$y = mx + b$$

$$y = 4x + 14 \text{ (Answer!)}$$

**Step 5:** **Check by substitution:**  $2 = 4(-3) + 14$

$$2 = -12 + 14$$

**Exercises:** In each of the following exercises, express the equation of the line in slope-intercept form.

1.  $m = 3$ , through  $(-2,4)$

**Step 1:** Write the equation:  $y = mx + b$

**Step 2:** Substitute  $m=3$ ,  $x=-2$ ,  $y=4$ :  $4 = 3(-2) + b$

**Step 3:** Solve for b:  $\underline{\quad} = \underline{\quad} + b$

$$\underline{\quad} = b$$

**Step 4:** Write the equation:  $y = \underline{\quad}x + \underline{\quad}$  Answer

**Step 5:** Check by substitution:

2.  $m = -3$  through  $(4,2)$

3.  $m = -3$  through  $(2,-4)$

4.  $m = -2$  through  $(-3, 4)$

5.  $m = 2$  through  $(-3, -4)$

6.  $m = 4$  through  $(5, -3)$

7.  $m = -4$  through  $(3, -5)$

8.  $m = 5/2$ , through  $(3, 4)$

Step 1: Write the equation:

$$y = mx + b$$

Step 2: Substitute  $m = \frac{5}{2}$ ,  $x=3$ ,  $y=4$ :

$$4 = \frac{5}{2}(3) + b$$

Step 3: Solve for b.

$$4 = \frac{15}{2} + b$$

Multiply both sides by 2:

$$2(4) = 2\left(\frac{15}{2}\right) + 2(b)$$

$$8 = 15 + 2b$$

$$\underline{\quad} = 2b$$

$$b = \underline{\quad}$$

Step 4: Write the equation:

Step 5: Check:

9.  $m = 3/4$  through  $(-5, -2)$

10.  $m = 3/2$  through  $(-4, 3)$

11.  $m = -3/2$  through  $(4, -3)$       12.  $m = -5/3$  through  $(6, -3)$

13.  $m = 5/3$  through  $(-6, 3)$       14.  $m = -7/2$  through  $(4, 1)$

In order to find the equation of a line containing two points, think, what **two** elements are required? Answer: a **point** and a **slope**. Now, if you are given two points and not the slope, it shouldn't take a rocket scientist to see what is needed here! If you are given two points, your first step must be to **find the slope  $m$  between the two points**. Use the formula as follows:

15. Find the equation of the line through  $(2, -4)$  and  $(-5, 6)$ .

$$m = \frac{6 - (-4)}{-5 - 2} = \frac{10}{-7} = -\frac{10}{7}$$

$$y = mx + b$$

Now use **either** point, first  $(2, -4)$ :  $-4 = -\frac{10}{7}(2) + b$

Multiply both sides by 7:  $7(-4) = 7(-\frac{20}{7}) + 7(b)$

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}} + 7b$$

$$b = \underline{\hspace{2cm}}$$

Write the equation:  $y = \underline{\hspace{4cm}}$

16. As a check,

$$y = mx + b$$

use the other point  $(-5, 6)$ :

$$6 = -\frac{10}{7}(-5) + b$$

Multiply both sides by 7:

$$7(6) = 7\left(\frac{50}{7}\right) + 7(b)$$

$$\underline{\hspace{2cm}} = \underline{\hspace{2cm}} + 7b$$

$$b = \underline{\hspace{2cm}}$$

$$y = \underline{\hspace{2cm}}$$

Does this agree with answer on previous page?                     

17. Find the equation of the line through  $(-4, 2)$  and  $(2, 14)$ .

18. Find the equation of the line through  $(-4, 2)$  and  $(-2, -6)$ .

19. Find the equation of the line through  $(-2, -3)$  and  $(2, 4)$ .

20. Find the equation of the line through  $(2, 6)$  and  $(-2, 3)$ .

21. Find the equation of the line through  $(-3, 6)$  and  $(-1, 3)$ .      22. Find the equation of the line through  $(4, -3)$  and  $(-1, 3)$ .

23. Find the equation of the line through  $(-4, 5)$  and  $(1, 8)$ .      24. Find the equation of the line through  $(3, -2)$  and  $(-4, 4)$ .

25. Find the equation of the line with x-int  $4$  and y-int  $-3$ .      26. Find the equation of the line with x-int  $-2$  and y-int  $-5$ .

[Hint:  $(4, 0)$  and  $(0, -3)$ ]

27. Find the equation of the line with x-int -4 and y-int 6.

28. Find the equation of the line with x-int 6 and y-int 3.

In the next exercises, remember that if two lines are **parallel**, they have the same slope. If two lines are **perpendicular**, the slope of one is the **negative reciprocal** of the other.

29a) Find the equation of the line through (4, -2) and parallel to  $y = -3x + 5$ .  
 $m =$   $m_{\text{parallel}} =$

b) Find the equation of the line through (4, -2) and perpendicular to  $y = -3x + 5$ .  
 $m =$   $m_{\perp} =$

30a) Find the equation of the line through (-3, 5) and parallel to  $y = -7x - 1$ .  
 $m =$   $m_{\text{parallel}} =$

b) Find the equation of the line through (-3, 5) and perpendicular to  $y = -7x - 1$ .  
 $m =$   $m_{\perp} =$

31a) Find the equation of the line through (4, -5) and parallel to  $Y = \frac{3}{4}X + 2$ .

$m =$   $m_{\text{parallel}} =$

b) Find the equation of the line through (4, -5) and perpendicular to  $Y = \frac{3}{4}X + 2$ .

$m =$  \_\_\_\_\_  $m_{\perp} =$

32a) Find the equation of the line through (4, -5) and parallel to  $Y = -\frac{3}{4}X + 2$ .

$m =$   $m_{\text{parallel}} =$

b) Find the equation of the line through (4, -5) and perpendicular to  $Y = -\frac{3}{4}X + 2$ .

$m =$  \_\_\_\_\_  $m_{\perp} =$

33a) Find the equation of the line through (4, 2) and parallel to  $4X + 3Y = 12$ .

$m =$   $m_{\text{parallel}} =$

b) Find the equation of the line through (4, 2) and perpendicular to  $4X + 3Y = 12$ .

$m =$  \_\_\_\_\_  $m_{\perp} =$



- 34a) Find the equation of the line through  $(4, 2)$  and parallel to  $4X - 3Y = 12$ .
- b) Find the equation of the line through  $(4, 2)$  and perpendicular to  $4X - 3Y = 12$ .

- 35a) Find the equation of the line through  $(-3, 5)$  and parallel to  $7X - 4Y = 1$ .
- b) Find the equation of the line through  $(-3, 5)$  and perpendicular to  $7X - 4Y = 1$ .

- 36a) Find the equation of the line through  $(4, -5)$  and parallel to  $3X + 5Y = -15$ .
- b) Find the equation of the line through  $(4, -5)$  and perpendicular to  $3X + 5Y = -15$ .

## ANSWERS 5.03

p.397-404:

1.  $Y=3X+10$  ; 2.  $Y=-3X+14$  ; 3.  $Y=-3X+2$  ; 4.  $Y=-2X-2$  ; 5.  $Y=2X+2$  ;  
 6.  $Y=4X-23$  ; 7.  $Y=-4X+7$  ; 8.  $Y=\frac{5}{2}X-\frac{7}{2}$  ; 9.  $Y=\frac{3}{4}X+\frac{7}{4}$  ; 10.  $Y=\frac{3}{2}X+9$  ;  
 11.  $Y=-\frac{3}{2}X+3$  ; 12.  $Y=-\frac{5}{3}X+7$  ; 13.  $Y=\frac{5}{3}X+13$  ; 14.  $Y=-\frac{7}{2}X+15$  ;  
 15.  $Y=-\frac{10}{7}X-\frac{8}{7}$  ; 16.  $Y=-\frac{10}{7}X-\frac{8}{7}$  ; 17.  $Y=2X+10$  ; 18.  $Y=-4X-14$  ;  
 19.  $Y=\frac{7}{4}X+\frac{1}{2}$  ; 20.  $Y=\frac{3}{4}X+\frac{9}{2}$  ; 21.  $Y=-\frac{3}{2}X+\frac{3}{2}$  ; 22.  $Y=-\frac{6}{5}X+\frac{9}{5}$  ;  
 23.  $Y=\frac{3}{5}X+\frac{37}{5}$  ; 24.  $Y=-\frac{6}{7}X+\frac{4}{7}$  ; 25.  $Y=\frac{3}{4}X-3$  ; 26.  $Y=-\frac{5}{2}X-5$  ;  
 27.  $Y=\frac{3}{2}X+6$  ; 28.  $Y=-\frac{1}{2}X+3$  ; 29a)  $Y=-3X+10$  , b)  $Y=\frac{1}{3}X-\frac{10}{3}$  ;  
 30a)  $Y=-7X-16$  , b)  $Y=\frac{1}{7}X+\frac{38}{7}$  ; 31a)  $Y=\frac{3}{4}X-8$  , b)  $Y=-\frac{4}{3}X+\frac{1}{3}$  ;  
 32a)  $Y=-\frac{3}{4}X-2$  , b)  $Y=\frac{4}{3}X-\frac{31}{3}$  ; 33a)  $Y=-\frac{4}{3}X+\frac{22}{3}$  , b)  $Y=\frac{3}{4}X-1$  ;  
 34a)  $Y=\frac{4}{3}X-\frac{10}{3}$  , b)  $Y=-\frac{3}{4}X+5$  ; 35a)  $Y=\frac{7}{4}X+\frac{41}{4}$  , b)  $Y=-\frac{4}{7}X+\frac{23}{7}$  ;  
 36a)  $Y=-\frac{3}{5}X-\frac{13}{5}$  , b)  $Y=\frac{5}{3}X-\frac{35}{3}$  .

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