5.05 *Inequalities in Two Variables/ Systems of Inequalities*

Dr. Robert J. Rapalje

Solving a linear equation in one variable, like **3X + 6 = 12**, usually resulted in a single solution, like **X = 2.** The graph was represented by a **single point** on a number**line.** A corresponding inequality, like **3X + 6 < 12**, required shading the numberline to the left of the solution, not including the endpoint. The inequality **3X + 6 > 12** required shading to the right, not inclucing the endpoint. The inequalities **3X + 6 ≤ 12** and **3X + 6 ≥ 12** also require shading, but the solution to these include the endpoints.

In two dimensions, the equation **3X + Y = 12** represents a **line** that can be graphed in the **XY plane.** As the **point X = 2** divides the number**line** into two regions, the **line 3X + Y = 12** divides the **XY plane** into two regions,above and below (or right and left) of the line. It should be clear that **3X + Y < 12** represents the shading on one side of the line, while **3X + Y > 12** represents the shading on the other side. The inequalities **3X + Y < 12** and **3X + Y > 12 do not include the line** itself, and therefore are represented by **dotted lines.** The inequalities **3X + Y ≤ 12** and **3X + Y ≥ 12 do include the line** and are represented by **solid lines.** The only question that remains is, "Which side of the the line should be shaded?" Probably the easiest way to decide is to solve the inequality for **Y.** For example, **3X + Y < 12** can be written as **Y < -3X + 12.** The values of Y are measured up and down the Y-axis, with values of **Y = -3X + 12** representing values <u>on</u> the line. Therefore, it seems reasonable that **Y < -3X + 12** represents values that are ***below the line*** **Y = -3X + 12**, and **Y > -3X + 12** represents values that are ***above the line*.**

Remember that when graphing a line whose equation is in the form **Y = mX + b**, it is usually easiest to use the **Y-intercept** and **slope** to draw the line. If the equation is in **standard form, AX + BY = C**, then it is usually easiest to graph by finding the **X** and **Y-intercepts.**

When graphing a **linear inequality** there are three steps:

I. Change the inequality to an equation and graph the line.
- **A.** If **Y = mX + b**, then use **Y-intercept/slope** method.
- **B.** If **AX + BY = C** form, then use **intercepts** method.

II. Decide whether the line is included or not included.
- **A.** If "<" or ">", then use a **dotted line.**
- **B.** If "≤" or "≥", then use a **solid line.**

III. Decide whether to **shade above or below** the line.
- **A.** If the equation has a **positive Y-coefficient** and "<" or "≤", then **shade <u>below</u> the graph** of the line.
- **B.** If the equation has a **positive Y-coefficient** and ">" or "≥", then **shade <u>above</u> the graph** of the line.
- **C.** If the equation has a **negative Y-coefficient,** then multiply both sides of the inequality by **-1,** which reverses the direction of the inequality sign. Then shade above or below as indicated.

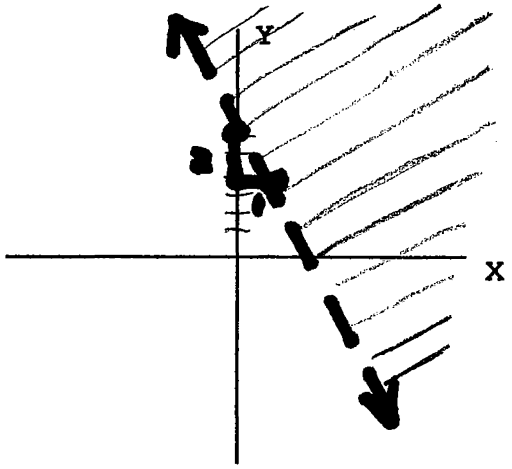
EXAMPLE 1: Graph Y > -2X + 6

Solution: I. Graph Y = -2X + 6
Easiest to use slope-intercept method
Y-int = (0,6); m = -2

(See below!)

II. Use a dotted line

III. Since +Y > , you must shade above the line.



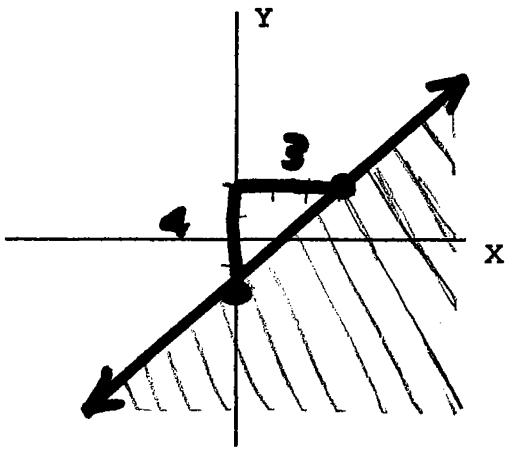
EXAMPLE 2: Graph $Y \leq \frac{4}{3}X - 2$

Solution: I. Graph $Y = \frac{4}{3}X - 2$
Easiest to use slope-int method
Y-int = (0, -2)

$m = \frac{4}{3}$

II. Use a solid line

III. Since +Y ≤, shade below the line



EXERCISES. **Graph each of the following inequalities. Shade the appropriate areas.**

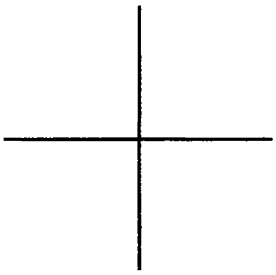
1. $Y < 3X + 2$

Y-int_____

m = _____

Type line ____

Shade_____



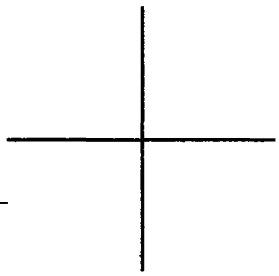
2. $Y > -2X + 4$

Y-int_____

m = _____

Type line_____

Shade_____



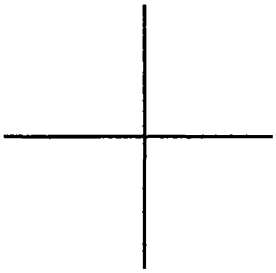
3. $Y \geq -X - 4$

Y-int_____

m = _____

Type line ____

Shade_____



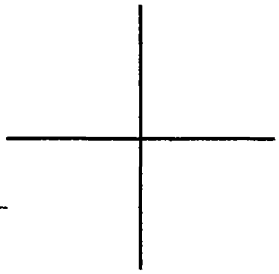
4. $Y \leq 2X - 4$

Y-int_____

m = _____

Type line_____

Shade_____



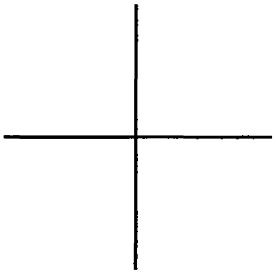
5. $Y < -\frac{4}{3}X + 2$

Y-int_____

m = _____

Type line ____

Shade_____



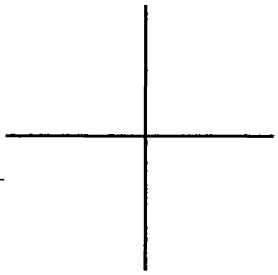
6. $Y \geq \frac{3}{2}X - 2$

Y-int_____

m = _____

Type line_____

Shade_____

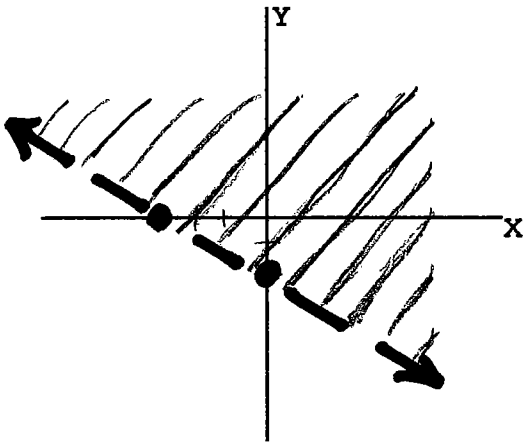


EXAMPLE 3: Graph $2X + 3Y > -6$

Solution: I. Graph $2X + 3Y = -6$
Easiest to use two intercepts method
X-int = (-3, 0)
Y-int = (0, -2)

II. Use a dotted line

III. Since $+Y >$, you must shade above the line.

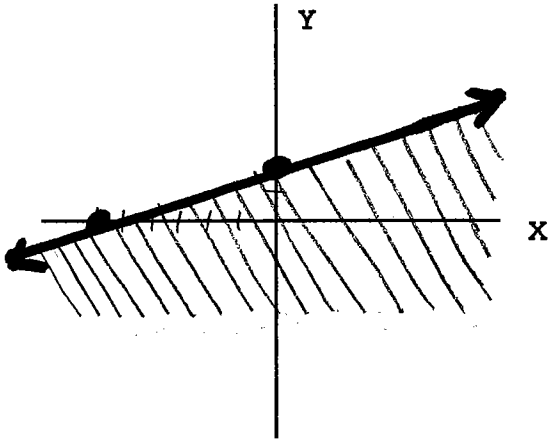


EXAMPLE 4: Graph $-X + 3Y \leq 6$

Solution: I. Graph $-X + 3Y = 6$
Easiest to use two intercepts method
X-int = (-6, 0)
Y-int = (0, 2)

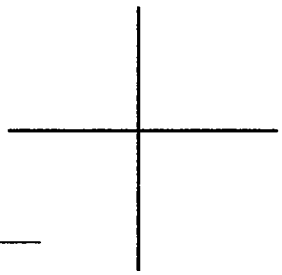
II. Use a solid line

III. Since $+Y \leq$, shade below the line.



7. $-X + 3Y \geq 6$

X	Y
0	2
-6	0

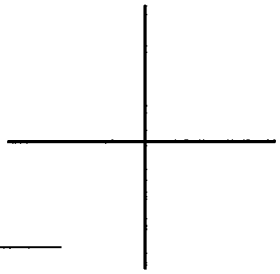


Type line: _____

Shade: _____

8. $3X + 2Y > -12$

X	Y
0	
	0

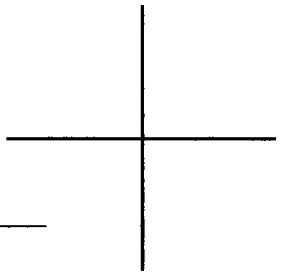


Type line: _____

Shade: _____

9. $3X + Y < -6$

X	Y
0	
	0

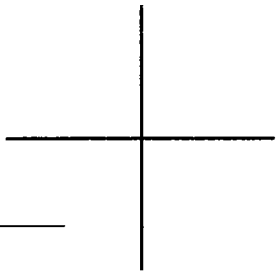


Type line: _____

Shade: _____

10. $-2X + Y \leq 8$

X	Y
0	
	0

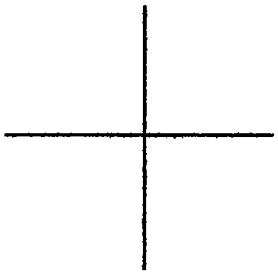


Type line: _____

Shade: _____

11. $-3X + 4Y > -12$

X	Y

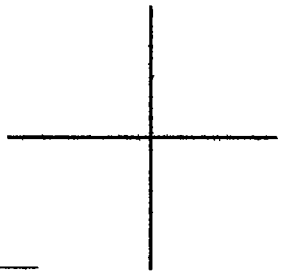


Type line: _____

Shade: _____

12. $-3X + 2Y \geq 12$

X	Y



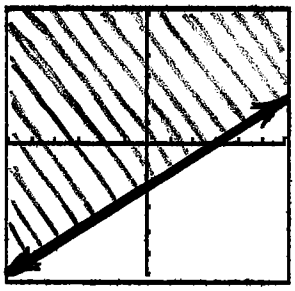
Type line: _____

Shade: _____

EXAMPLE 5: Graph $2X - 3Y \leq 6$

Solution:

$2X - 3Y \leq 6$

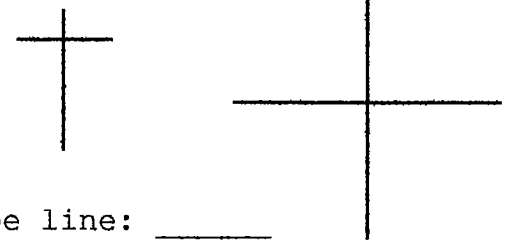


I. Graph $2X - 3Y = 6$
Easiest to use two int. method
X-int = (3, 0); Y-int = (0,-2)

II. Use a solid line

III. Since $-Y \leq$, divide both sides by -1.
This means $+Y \geq$, so shade above line.

13. $3X - Y < -6$



Type line: _____

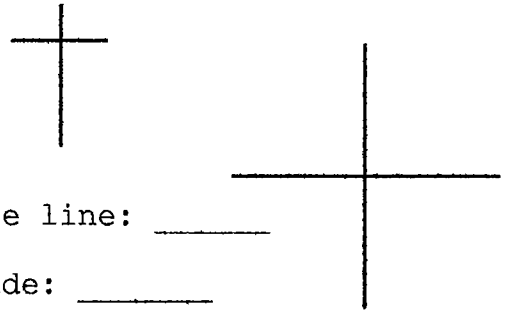
Shade: _____

14 . $-2X - Y \leq 8$

Type line: _____

Shade: _____

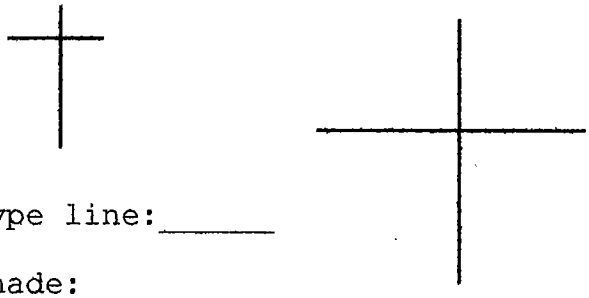
15. $3X - 4Y \geq 12$



Type line: _____

Shade: _____

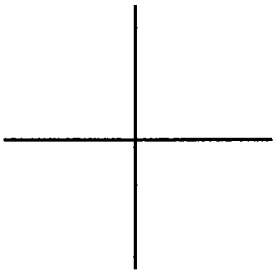
16. $3X - 2Y > -12$



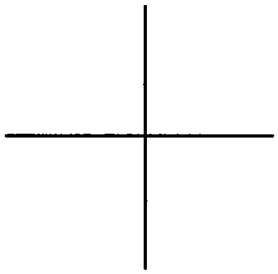
Type line: _____

Shade: _____

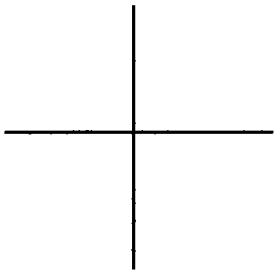
17. $Y > -3X - 4$



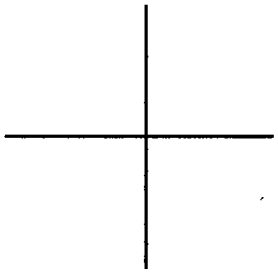
18. $Y \geq X$



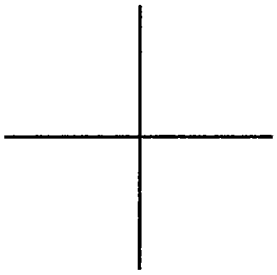
19. $X - 3Y < -6$



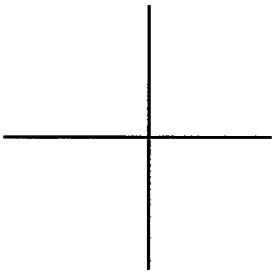
20. $Y \geq -\frac{1}{4}X - 2$



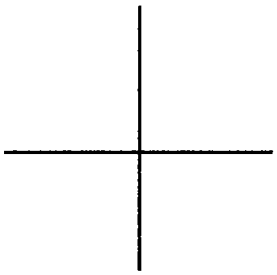
21. $Y \geq \frac{3}{2}X + 5$



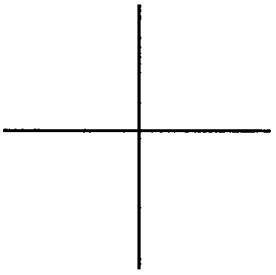
22. $Y < -3$



23. $X \leq 4$



24. $X > -4$



It is frequently necessary to graph a **system of inequalities**--that is, two or more inequalities with two unknowns. In such problems, there will be two or more regions to graph, with instructions to shade the **union** or the **intersection** of the regions. As before, remember that the **union** includes all shaded regions, while the **intersection** of the regions includes only the regions common to both (or all) of the shadings. It will be very helpful to use colored pencils, using a different color for each equation/shaded region.

EXAMPLE 6a) Graph the region represented by the union of

$3X - 2Y > 6$

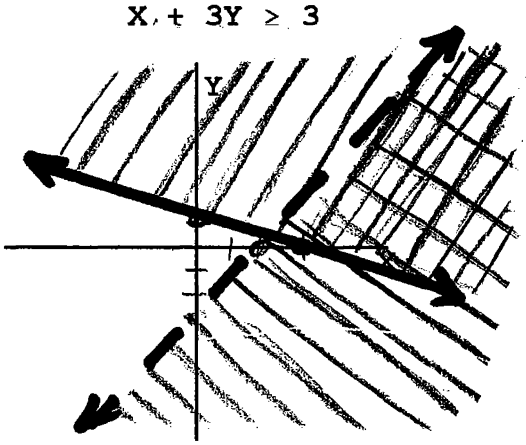
$X + 3Y \geq 3$

$3X - 2Y > 6$

X	Y
0	-3
2	0

Dotted Line

Shade below



$X + 3Y \geq 3$

X	Y
0	1
3	0

Solid Line

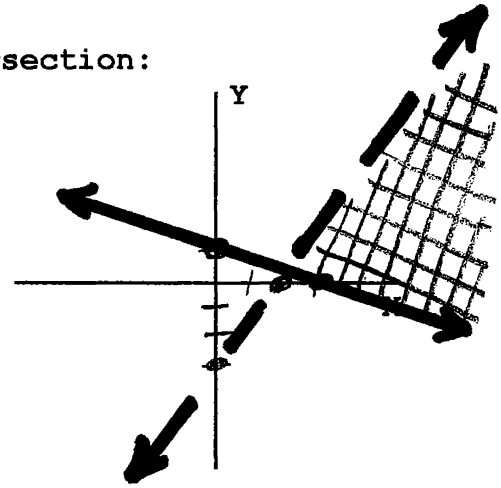
Shade above

<u>UNION</u> is <u>everything</u> that is shaded.

EXAMPLE 6b) Graph the intersection:

$3X - 2Y > 6$

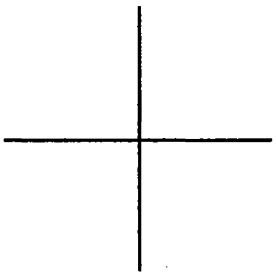
$X + 3Y \geq 3$



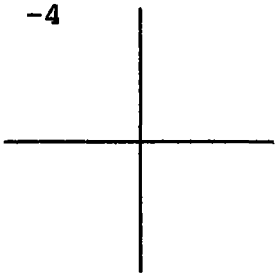
<u>INTERSECTION</u> is only what is common to BOTH.

In 25 - 28, find the region represented by the unions of the following inequalities.

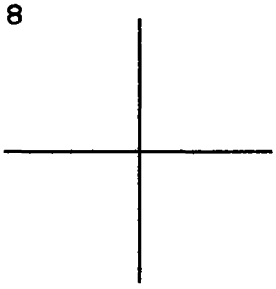
25. $Y > X + 2$
$Y < -X$



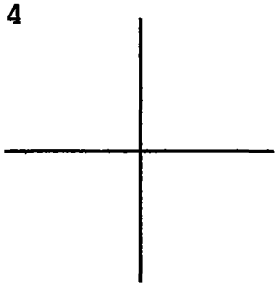
26. $4X + Y \leq -8$
$X - 2Y \leq -4$



27. $2X - 3Y \geq -12$
$-Y \leq 4X - 8$

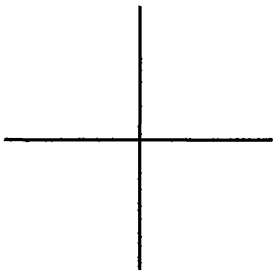


28. $Y < 4X - 4$
$Y > -2X + 4$

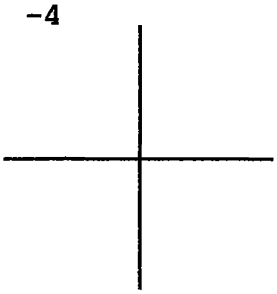


In 29 - 32, find the region represented by the intersections of each of the following.

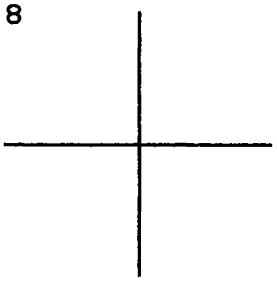
29. $Y \leq -X + 2$
$Y \geq X$



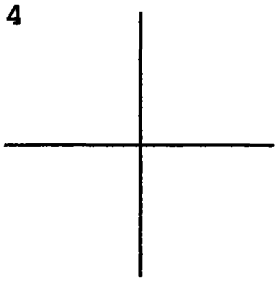
30. $4X - Y > -8$
$X + 2Y < -4$



31. $2X - 3Y < -12$
$-Y > 4X - 8$



32. $Y \geq -4X + 4$
$Y \leq 2X - 4$



EXAMPLE 7: Graph the region represented by the intersection of

$$3X + 2Y < 12$$
$$X \geq 0$$
$$Y \geq -3$$

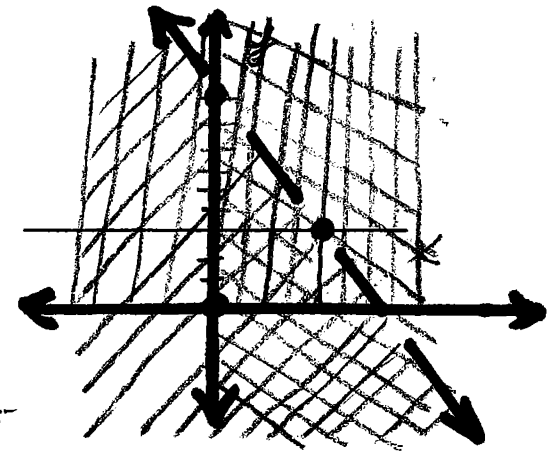
$3x+2y=12$

x	y
0	6
4	0

$x=0$ Yaxis

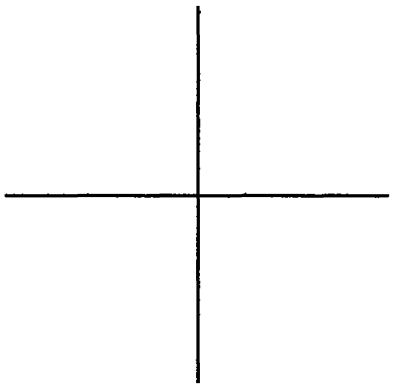
$y=-3$ Horizontal Line

Region = TRIANGLE

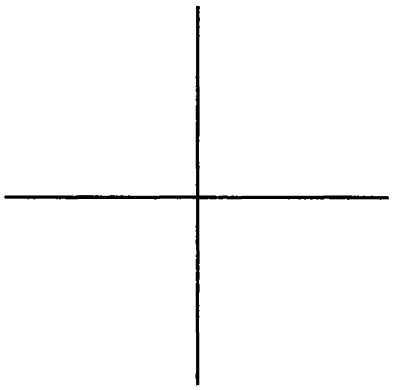


In 33 - 34, graph the region represented by the intersections of each of the following inequalities.

33. $3X - 2Y \leq 12$
$X \geq 0$
$Y \leq 0$

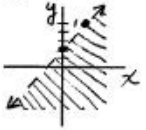


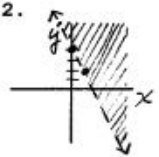
34. $X + 3Y > -6$
$X < 2$
$Y < 3$

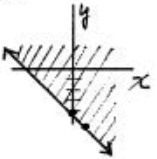


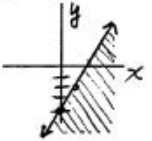
ANSWERS 5.05

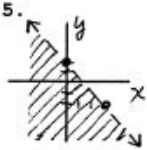
p.420-426:

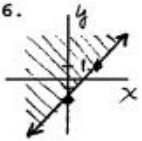
1. 

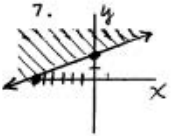
2. 

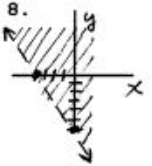
3. 

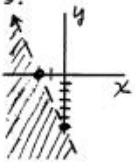
4. 

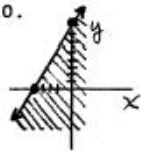
5. 

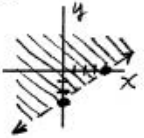
6. 

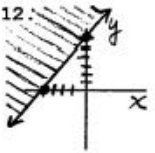
7. 

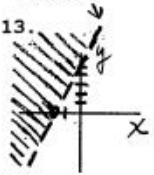
8. 

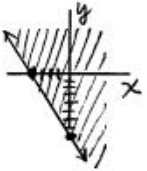
9. 

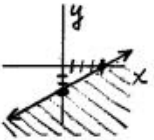
10. 

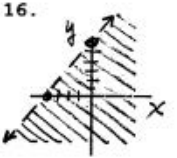
11. 

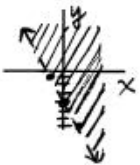
12. 

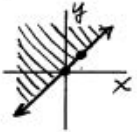
13. 

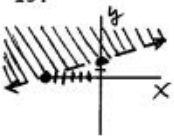
14. 

15. 

16. 

17. 

18. 

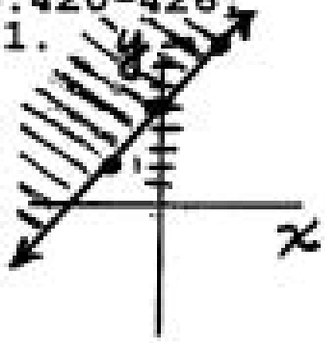
19. 

20. 

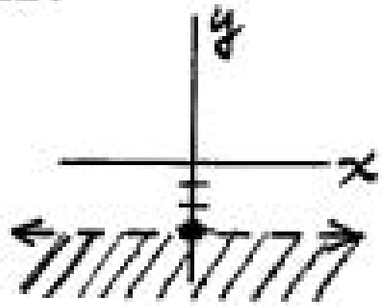
ANSWERS 5.05 (Continued)

p.420-426:

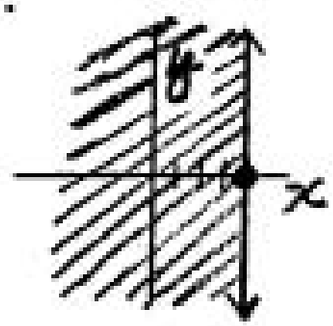
21.



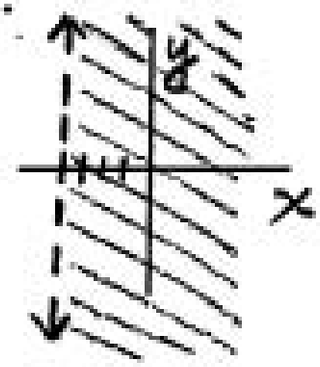
22.



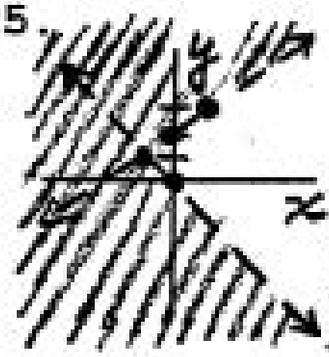
23.



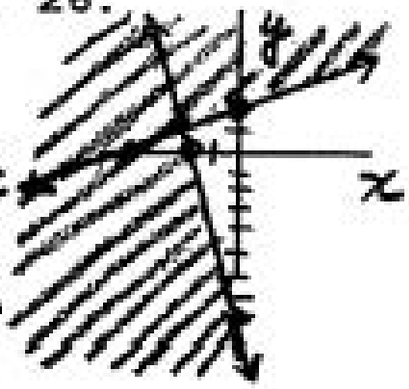
24.



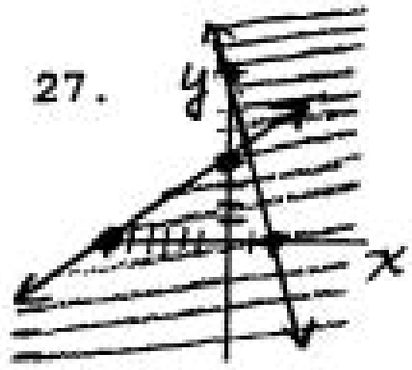
25.



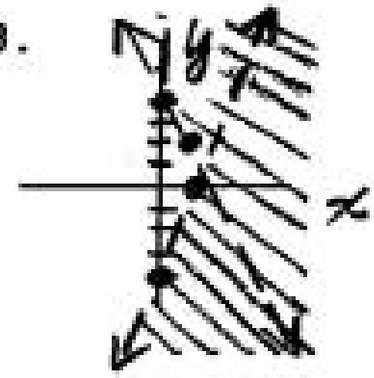
26.



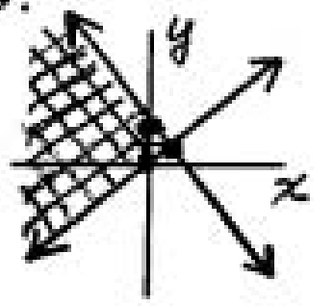
27.



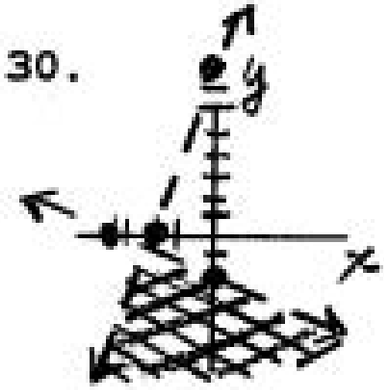
28.



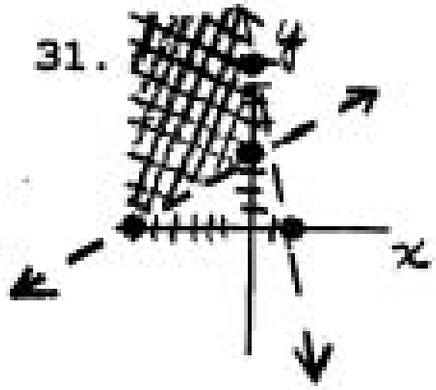
29.



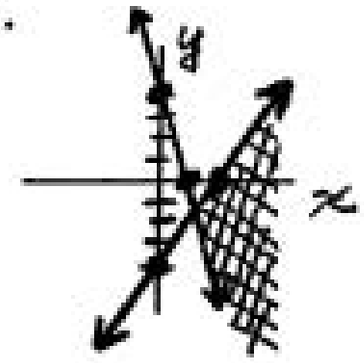
30.



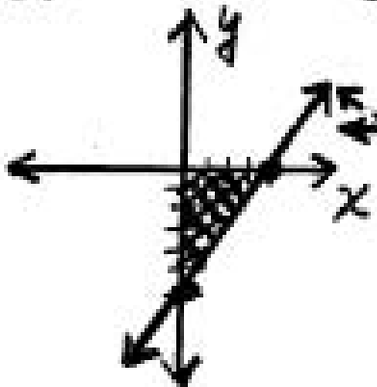
31.



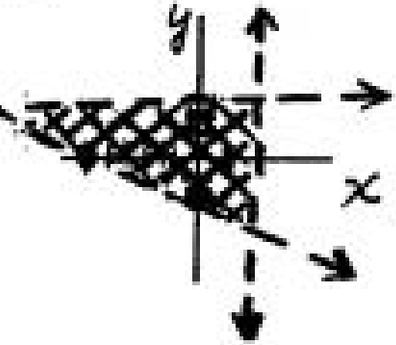
32.



33.



34.



Dr. Robert J. Rapalje

ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE