

1.05 Fractions

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

For many students, the greatest challenges in mathematics are "fractions" (or "**rational expressions**") and "word problems." In both of these topics, the basic concepts have been taught since the very first algebra course. Now, in this third level of algebra or "college algebra," you have the same basic concepts applied with somewhat greater complexity and abstraction. With fractions especially, the mechanics are more difficult. In this section there will be five types of problems: **reducing fractions, multiplying and dividing fractions, adding and subtracting fractions, complex fractions, and "continued fractions."** All of these topics, except "continued fractions," are probably familiar to you from your previous mathematics background. As you might expect, in higher levels of math, the factoring becomes more difficult, the common denominators become more complicated, and the complex fractions become more complex (of course!).

$$\text{PRINCIPLE: } \frac{ac}{bc} = \frac{a}{b}, \quad b \neq 0, c \neq 0$$

When **reducing fractions**, remember that the first step is to express the numerator and denominator in **factored form**. Then, you may reduce a fraction, only when you are able to divide numerator and denominator by the same non-zero factor. Remember these?

EXERCISES:

1. $\frac{X^2 - 16}{X^2 - 8X + 16}$

2. $\frac{X^2 - 25}{X^2 - 4X - 5}$

3. $\frac{X^3 - 27}{X^2 - 6X + 9}$

4.
$$\frac{X^3 - 27}{X^2 + 3X + 9}$$

5.
$$\frac{X^2 - 5X + 25}{X^3 + 125}$$

6.
$$\frac{X^2 - 25}{X^3 - 125}$$

7.
$$\frac{X^2 - 2X - XY + 2Y}{X^2 + 2X - XY - 2Y}$$

8.
$$\frac{X^2 + 4X - 2XY - 8Y}{X^2 - 2X - 2XY + 4Y}$$

PRINCIPLE:
$$\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}, \quad b \neq 0, d \neq 0$$

PRINCIPLE:
$$\frac{a}{b} \div \frac{c}{d} = \frac{a \cdot d}{b \cdot c}, \quad b \neq 0, c \neq 0, d \neq 0$$

When multiplying or dividing fractions, the first step is to factor each numerator and denominator if possible. Divide any factor of any numerator with any factor of any denominator. When multiplying fractions, multiply numerator times numerator and denominator times denominator. When dividing, invert the second fraction and multiply.

$$9. \frac{X^3 - 27}{X^2 - 6X + 9} \cdot \frac{X^2 - 9}{X^2 + 3X + 9} \quad 10. \frac{X^3 - 27}{X^3 + 3X^2 - 9X - 27} \cdot \frac{(X - 3)^3}{X^2 + 3X + 9}$$

$$11. \frac{X^4 - 81}{X^4 + 5X^2 - 36} + \frac{X^4 - 3X^3 - 27X + 81}{X^3 - 2X^2 - 9X + 18} \quad 12. \frac{X^2 - 25}{X^3 - 125} + \frac{X^2 - 10X + 25}{(X - 5)^3}$$

$$13. \frac{X^8 - 16}{X^8 - 8X^4 + 16} + \frac{X^8 - 1}{X^8 - 5X^4 + 4} \quad 14. \frac{X^6 - 64}{X^6 - 16X^3 + 64} + \frac{X^4 - 16}{X^4 - 8X^2 + 16}$$

$$= \frac{(X^3 -) (X^3 +)}{(X^3 -) (X^3 -)} \cdot \frac{(X^2 -) (X^2 -)}{(X^2 -) (X^2 -)}$$

=

=

$$\text{PRINCIPLE: } \frac{a}{d} + \frac{b}{d} - \frac{c}{d} = \frac{a + b - c}{d}, \quad d \neq 0$$

When adding or subtracting fractions, you must have a **common denominator**. The denominator of the resulting fraction is the common denominator. To find the numerator of the resulting fraction, add (or subtract as necessary) the numerators of the given fractions.

If the fractions do not have a common denominator, then factor each denominator completely. If there are no common factors among the denominators, then the least common denominator is the product of all the denominators. If there are common factors involved, then determine the highest power of each factor. **The least common denominator is the product of the factors of the denominators, each factor being raised to the highest power in any given denominator.** After finding the least common denominator, determine what factors are missing from each denominator, and multiply the numerator and denominator of each fraction by the missing factors. Finally, add (or subtract) the numerators of the resulting fractions, divided by the common denominator. Combine like terms and reduce the fraction if possible. Please see the summary on the next page.

ADDITION AND SUBTRACTION OF FRACTIONS

Summary

I. FIND THE LEAST COMMON DENOMINATOR (LCD).

- A. Factor each denominator to determine what factors are needed for the common denominator.
- B. For each of the denominator factors, determine the highest power of each factor. The **LCD** is the product of each factor raised to its highest power.
- C. The LCD becomes the denominator of the fraction.

II. PLAY "WHAT'S MISSING."

- A. Compare each denominator to the LCD, and determine the **missing factors** for each denominator.
- B. Multiply each **numerator and denominator** by "**What's missing!**"

III. ADD OR SUBTRACT NUMERATORS.

- A. Add (or subtract) numerators, and place over the common denominator.
- B. Combine like terms and reduce the resulting fraction, if possible.

In 15 - 26, find the LCD and perform the indicated operations:

15. $\frac{X - 2}{X - 4} - \frac{X}{X + 4}$

16. $\frac{X - 4}{X - 2} - \frac{X}{X + 4}$

= $\frac{\quad}{(x-4)(x+4)}$

=

$$17. \quad \frac{X+2}{X+4} - \frac{X}{X-4}$$

$$18. \quad \frac{X+4}{X-2} - \frac{X}{X-4}$$

$$19. \quad \frac{X-2}{X-4} - \frac{X}{X+4} + 4$$

$$20. \quad \frac{X-4}{X-2} - \frac{X}{X+4} + 4$$

$$21. \quad \frac{X+2}{X+4} - \frac{X}{X-4} - 4$$

$$22. \quad \frac{X+4}{X-2} - \frac{X}{X-4} - 4$$

$$23. \quad \frac{X}{X^2 - 4} - \frac{2}{X^2 - 6X + 8} + \frac{1}{X^2 - 2X - 8}$$

$$24. \quad \frac{X}{X^2 - 4} + \frac{2}{X^2 - 6X + 8} - \frac{1}{X^2 - 2X - 8}$$

$$25. \quad \frac{X}{X^2 - 5X + 6} - \frac{X}{X^2 - 6X + 8} - \frac{12 - X^2}{(X - 2)(X - 3)(X - 4)}$$

$$26. \quad \frac{X}{X + 4} + \frac{2X}{X^2 - 4} - \frac{2}{(X + 2)(X + 4)}$$

Complex fractions are fractions whose numerator and/or denominator contain fractions. Compare and contrast the complex

fractions: $\frac{\frac{a}{b}}{c}$ and $\frac{a}{\frac{b}{c}}$. In the first case, the fraction $\frac{\frac{a}{b}}{c}$ means the numerator $\frac{a}{b}$ is divided by the denominator "c". This

$$\begin{aligned} \text{may be written } \frac{\frac{a}{b}}{c} &= \frac{a}{b} \div \frac{c}{1} \\ &= \frac{a}{b} \cdot \frac{1}{c} = \frac{a}{bc} \end{aligned}$$

Now compare this to the complex fraction $\frac{a}{\frac{b}{c}}$. In this case, the numerator "a" is divided by the fraction $\frac{b}{c}$.

$$\begin{aligned} \text{This means } \frac{a}{\frac{b}{c}} &= \frac{a}{1} \div \frac{b}{c} \\ &= \frac{a}{1} \cdot \frac{c}{b} = \frac{ac}{b} \end{aligned}$$

The answer is not the same. If the fraction lines are equal length, then the fraction is undefined.

In general, $\frac{\frac{a}{b}}{\frac{c}{d}}$ means $\frac{a}{b} \div \frac{c}{d}$ or $\frac{a}{b} \cdot \frac{d}{c}$.

EXERCISES. Simplify each of the following:

$$\begin{array}{lll} 1. \quad \frac{\frac{3}{2}}{\frac{12}{7}} = \frac{3}{2} \div \frac{12}{7} & 2. \quad \frac{\frac{3}{25}}{\frac{9}{20}} = & 3. \quad \frac{\frac{7}{8}}{\frac{21}{10}} = \\ & & \\ & & \\ & & \end{array}$$

$$4. \frac{\frac{X}{4}}{\frac{8}{Y}} =$$

$$5. \frac{\frac{X}{12}}{\frac{Y}{4}} =$$

$$6. \frac{\frac{3X^3}{14Y^2}}{\frac{21X^5}{8Y}} =$$

$$7. \frac{\frac{4X}{X-4}}{\frac{8Y}{X^2-16}} = \frac{4X}{X-4} \div \frac{8Y}{X^2-16}$$

$$8. \frac{\frac{12X}{X^2-9}}{\frac{4X}{X+3}} =$$

$$= \frac{4X}{(X-4)} \cdot \frac{(X-4)(X+4)}{8Y}$$

=

$$9. \frac{\frac{X^3-8}{X+2}}{\frac{X^2+2X+4}{X^2-4}} =$$

$$10. \frac{\frac{X^2-2X-3}{X^3+1}}{\frac{X^2-3X}{X^2-X+1}} =$$

When the numerator and/or denominator of a complex fraction consists of two or more fractions, there are two methods of simplifying. Consider the problem: $\frac{\frac{1}{X} - \frac{1}{Y}}{\frac{1}{X} + \frac{1}{Y}}$.

METHOD I. Rewrite the problem in the form of a division problem.

$$\begin{aligned} \frac{(\frac{1}{X} - \frac{1}{Y})}{(\frac{1}{X} + \frac{1}{Y})} &= \text{(NUMERATOR)} \div \text{(DENOMINATOR)} \\ &= (\frac{1}{X} - \frac{1}{Y}) \div (\frac{1}{X} + \frac{1}{Y}) \quad \text{Work as two separate} \\ &= \frac{Y - X}{XY} \div \frac{Y + X}{XY} \quad \text{problems. Find LCD} \\ &= \frac{Y - X}{XY} \cdot \frac{XY}{Y + X} = \frac{Y - X}{Y + X} \quad \text{for each.} \end{aligned}$$

METHOD II. Find a common denominator for the entire problem. Multiply the numerator and denominator by this LCD. (LCD = XY!)

$$\begin{aligned} \frac{XY (\frac{1}{X} - \frac{1}{Y})}{XY (\frac{1}{X} + \frac{1}{Y})} & \quad \text{(USE DISTRIBUTIVE PROPERTY!)} \\ &= \frac{XY \cdot \frac{1}{X} - XY \cdot \frac{1}{Y}}{XY \cdot \frac{1}{X} + XY \cdot \frac{1}{Y}} \quad \text{(You may wish to do this step mentally!)} \\ &= \frac{Y - X}{Y + X} \end{aligned}$$

Which method is easier? Sometimes one method is easier; sometimes the other is easier. Sometimes it is a toss up! There will be more explanation on this later.

In 11-13, use method I.

11.

$$\frac{\frac{5}{X} - \frac{3}{Y}}{\frac{2}{X} + \frac{3}{Y}} = (\quad) \div (\quad)$$

=

=

12.

$$\frac{\frac{X}{5} - \frac{5}{X}}{1 + \frac{5}{X}} = (\quad) \div (\quad)$$

=

=

13.

$$\frac{1 - \frac{3}{X}}{\frac{X}{3} - \frac{3}{X}} = (\quad) \div (\quad)$$

=

=

In 14 - 16, use method II.

14.

$$\frac{\left(\quad\right) \left(\frac{5}{X} - \frac{3}{Y}\right)}{\left(\quad\right) \left(\frac{2}{X} + \frac{3}{Y}\right)} =$$

15.

$$\frac{\left(\quad\right) \left(\frac{X}{5} - \frac{5}{X}\right)}{\left(\quad\right) \left(1 + \frac{5}{X}\right)} =$$

$$=$$

16.

$$\frac{\left(\quad\right) \left(1 - \frac{3}{X}\right)}{\left(\quad\right) \left(\frac{X}{3} - \frac{3}{X}\right)} =$$

$$=$$

For exercises in which the common denominator for the numerator is the same as the common denominator for the denominator, Method II will be easier. For more complex exercises, especially when the LCD for the numerator differs greatly from the LCD for the denominator, Method I will be easier. The great advantage for Method I is that you can separate the problem into two smaller problems. In this way, you can ignore half of the problem, while you work on the other half.

In 17 - 24, use either method:

$$17. \frac{\frac{1}{X} + \frac{1}{2}}{\frac{4}{X^2} - 1}$$

$$18. \frac{\frac{1}{X^2} - 1}{\frac{1}{X} - 1}$$

(METHOD II works nicely in #19 - 24. Why??)

$$19. \frac{3 - \frac{6}{X-2}}{8 - \frac{16}{X-2}}$$

$$20. \frac{1 - \frac{5}{X} + \frac{6}{X^2}}{1 - \frac{3}{X} + \frac{2}{X^2}}$$

$$21. \frac{4 + \frac{4}{X} - \frac{3}{X^2}}{4 - \frac{8}{X} + \frac{3}{X^2}}$$

$$22. \frac{1 + \frac{1}{X-1}}{1 - \frac{1}{X-1}}$$

$$23. \frac{3 + \frac{9}{X-2}}{3 - \frac{3}{X-2}}$$

$$24. \frac{\frac{1}{X} + \frac{1}{X+2}}{\frac{1}{X} - \frac{1}{X+2}}$$

In 25 - 28, where common denominators are different, Method I is probably easier.

$$25. \frac{1 - \frac{2}{X+2}}{1 + \frac{2}{X-2}}$$

$$26. \frac{\frac{X}{X+1} + 1}{\frac{2X+1}{X-1}}$$

() ÷ ()

$$27. \frac{\frac{X}{X-1} + 2}{\frac{6X-4}{X+2}}$$

$$28. \frac{\frac{6x}{X-1} - 3}{4 - \frac{12}{X+4}}$$

$$29. \frac{\frac{X+2}{X} + \frac{X}{X+2}}{\frac{X+2}{X} - \frac{X}{X+2}}$$

$$30. \frac{\frac{X+2}{X} + \frac{X}{X-2}}{\frac{X+2}{X} - \frac{X}{X-2}}$$

(not I?)
why?

$$31. \frac{\frac{X+2}{X} + \frac{X}{X+2}}{\frac{X+2}{X} - \frac{X}{X-2}}$$

$$32. \frac{\frac{4}{X} - \frac{1}{X-2}}{\frac{4}{X+2} - \frac{4}{X}}$$

$$33. \frac{\frac{4}{X-4} - \frac{2}{X}}{\frac{-6}{X+1} + \frac{8}{X}}$$

$$34. \frac{\frac{4}{X+Y} - \frac{2}{X}}{\frac{4}{X-Y} - \frac{2}{X}}$$

In 35 - 54, eliminate negative exponents and simplify:

$$35. \frac{X^{-2} + Y^{-2}}{X^{-2} - Y^{-2}} = \frac{\frac{1}{X^2} + \frac{1}{Y^2}}{\frac{1}{X^2} - \frac{1}{Y^2}}$$

$$= \frac{(X^2 Y^2) \left(\frac{1}{X^2} + \frac{1}{Y^2} \right)}{(X^2 Y^2) \left(\frac{1}{X^2} - \frac{1}{Y^2} \right)}$$

$$= \underline{\hspace{4cm}}$$

$$36. \frac{2X^{-1} + (2Y)^{-1}}{2XY^{-1}} = \frac{2 \cdot \frac{1}{X} + \frac{1}{2Y}}{2X \cdot \frac{1}{Y}}$$

$$= \frac{(2XY) \left(2 \cdot \frac{1}{X} + \frac{1}{2Y} \right)}{(2XY) \left(2X \cdot \frac{1}{Y} \right)}$$

$$= \underline{\hspace{4cm}}$$

$$37. \frac{X^{-1} - Y^{-1}}{X^{-1} + Y^{-1}} =$$

$$38. \frac{(3X)^{-1} + 3X^{-1}}{X^{-1} + 3^{-1}} =$$

$$39. \frac{(4X)^{-2} + 4^{-2}}{4X^{-2} - 4^{-2}} =$$

$$40. \frac{X^{-1} - Y^{-1}}{X^{-2} - Y^{-2}} =$$

$$41. \frac{2X - (2X)^{-1}}{1 + (2X)^{-1}} =$$

$$42. \frac{1 + (2X)^{-1}}{2X + (2X)^{-2}} =$$

$$43. \frac{2^{-1}X - 2X^{-1}}{4^{-1}X^2 - 2X^{-1}} =$$

$$44. \frac{2X^{-2} + (2X)^{-2}}{3X^{-1} + 3X^{-2}} =$$

$$45. (X^{-1} + Y^{-1})^{-1}$$

$$46. (X^{-1} - Y^{-1})^{-1}$$

$$47. (X^{-2} - Y^{-2})^{-1}$$

$$= \left(\frac{1}{X} + \frac{1}{Y} \right)^{-1}$$

$$= \left(\frac{Y + X}{XY} \right)^{-1}$$

$$= \underline{\hspace{2cm}}$$

$$48. (X^{-1} + Y^{-1})^{-2}$$

$$49. (X^{-1} - Y^{-1})^{-2}$$

$$50. (3X^{-1} + 3Y^{-1})^{-1}$$

$$51. (3X^{-1} - 3Y^{-1})^{-1}$$

$$52. [(3X)^{-1} + (3Y)^{-1}]^{-1}$$

$$53. [(3X)^{-1} - (3Y)^{-1}]^{-1}$$

$$54. [(3X)^{-2} - 3Y^{-2}]^{-1}$$

A **continued fraction** is any fraction whose numerator and/or denominator is a complex fraction. When simplifying a continued fraction, it is usually best to start at the bottom and work up..

For example, $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + 1}}}}$. It might be helpful to

begin with a smaller example: $1 + \frac{1}{1 + \frac{1}{1 + 1}} = 1 + \frac{1}{1 + \frac{1}{2}}$

$$= 1 + \frac{1}{\frac{3}{2}}$$

$$= 1 + \frac{2}{3} = \frac{5}{3}$$

Calculate each of the following. Do as much possible in your head.

1. $1 + \frac{1}{1 + 1}$

2. $\frac{1}{1 + \frac{1}{1 + 1}}$

3. $1 + \frac{1}{1 + \frac{1}{1 + 1}}$

4. $1 + \frac{1}{1 - \frac{1}{1 + 1}}$

5. $1 - \frac{1}{1 - \frac{1}{1 + 1}}$

6. $1 - \frac{1}{1 + \frac{1}{1 - \frac{1}{1 + 1}}}$

$$7. \ 1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1+1}}} \quad 8. \ 1 + \frac{1}{1 + \frac{1}{1 - \frac{1}{1+1}}} \quad 9. \ 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1+1}}}$$

$$10. \ 6 - \frac{5}{4 - \frac{3}{2-1}} \quad 11. \ 6 + \frac{5}{4 + \frac{3}{2+1}} \quad 12. \ 6 + \frac{5}{4 - \frac{3}{2+1}}$$

$$(13.) \ 10 - \frac{9}{8 - \frac{7}{6 - \frac{5}{4 - \frac{3}{2-1}}}}$$

$$(14.) \ 10 + \frac{9}{8 + \frac{7}{6 + \frac{5}{4 + \frac{3}{2+1}}}}$$

$$15. \ 1 + \frac{1}{1 - \frac{1}{1 + \frac{1}{1 - \frac{1}{1+1}}}}$$

$$16. \ 1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1+1}}}}$$

$$17. \quad 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + 1}}}}$$

$$18. \quad 2 + \frac{2}{2 + \frac{2}{2 + \frac{2}{2 + \frac{2}{2 + 2}}}}$$

$$19. \quad 3 + \frac{3}{3 + \frac{3}{3 + \frac{3}{3 + \frac{3}{3 + 3}}}}$$

$$20. \quad 1 - \frac{1}{1 + \frac{1}{1 - \frac{1}{1 + \frac{1}{1 - \frac{1}{1 + 1}}}}}$$

$$21. \quad 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + 1}}}}}$$

$$22. \quad 1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 - \frac{1}{1 + 1}}}}}$$

1.05 p 45-51: 1. $\frac{x+4}{x-4}$ 2. $\frac{x+5}{x+1}$ 3. $\frac{x^2+3x+9}{x-3}$ 4. $x-3$ 5. $\frac{1}{x+5}$

6. $\frac{x+5}{x^2+5x+25}$ 7. $\frac{x-2}{x+2}$ 8. $\frac{x+4}{x-2}$ 9. $x+3$

10. $\frac{(x-3)^3}{(x+3)^2}$ 11. $\frac{(x+3)^2}{(x+2)(x^2+3x+9)}$ 12. $\frac{(x-5)(x+5)}{x^2+5x+25}$ 13. $\frac{x^4+4}{x^4+1}$

14. $\frac{(x+2)^2(x-2)(x+4)}{(x^2+4)(x^2+2x+4)}$ 15. $\frac{6x-8}{(x-4)(x+4)}$ 16. $\frac{2x-16}{(x-2)(x+4)}$ 17. $\frac{-6x-8}{(x-4)(x+4)}$

18. $\frac{2x-16}{(x-2)(x-4)}$ 19. $\frac{4x^2+6x-72}{(x-4)(x+4)}$ 20. $\frac{4x^2+10x-48}{(x-2)(x+4)}$

21. $\frac{-4x^2-6x+56}{(x-4)(x+4)}$ 22. $\frac{-4x^2+26x-48}{(x-2)(x-4)}$ 23. $\frac{x^2-5x-6}{(x-2)(x+2)(x-4)}$

ANSWERS 1.05 (Continued)

p. 45-51: 24. $\frac{x^2-3x+6}{(x-2)(x+2)(x-6)}$ 25. $\frac{x+3}{(x-7)(x-2)}$ 26. $\frac{x^2+2}{(x+4)(x-2)}$

p. 52-62: 1. $\frac{7}{8}$ 2. $\frac{4}{15}$ 3. $\frac{5}{12}$ 4. $\frac{xy}{32}$ 5. $\frac{x}{18}$ 6. $\frac{4}{49x^2y}$

7. $\frac{x(x+4)}{24}$ 8. $\frac{2}{x-3}$ 9. $(x-2)^2$ 10. $\frac{1}{x}$ 11. $\frac{54-3x}{2y+3x}$

12. $\frac{x-5}{5}$ 13. $\frac{2}{x+3}$ 14. $\frac{54-3x}{2y+3x}$ 15. $\frac{x-5}{5}$

16. $\frac{3}{x+3}$ 17. $\frac{x}{2(x-x)}$ 18. $\frac{1+x}{x}$ 19. $\frac{2}{8}$

20. $\frac{x-3}{x-1}$ 21. $\frac{2x+3}{2x-3}$ 22. $\frac{x}{x-2}$ 23. $\frac{x+1}{x-3}$

24. $x+1$ 25. $\frac{x-2}{x+2}$ 26. $\frac{x-1}{x+1}$ 27. $\frac{x+2}{2(x-1)}$

28. $\frac{3(x+4)}{4(x-1)}$ 29. $\frac{x^2+2x+2}{2(x+1)}$ 30. $\frac{2-x^2}{2}$ 31. $\frac{(x-2)(x^2+2x+2)}{-2(x+2)}$

32. $\frac{(3x-8)(x+2)}{-8(x-2)}$ 33. $\frac{x+1}{x-4}$ 34. $\frac{(x-y)^2}{(x+y)^2}$ 35. $\frac{y^2+x^2}{y^2-x^2}$

36. $\frac{4x-x}{4x^2}$ 37. $\frac{y-x}{y+x}$ 38. $\frac{10}{3+x}$ 39. $\frac{1+x^2}{64-x^2}$ 40. $\frac{xy}{y+x}$

41. $2x-1$ 42. $\frac{2x}{4x^2-2x+1}$ 43. $\frac{2(x+2)}{x^2+2x+4}$ 44. $\frac{3}{4(x+1)}$ 45. $\frac{xy}{x+y}$

46. $\frac{xy}{y-x}$ 47. $\frac{x^2y^2}{y^2x^2}$ 48. $\frac{x^2y^2}{(y+x)^2}$ 49. $\frac{x^2y^2}{(y-x)^2}$ 50. $\frac{xy}{3(y+x)}$

51. $\frac{xy}{3(y-x)}$ 52. $\frac{2xy}{y+x}$ 53. $\frac{3xy}{y-x}$ 54. $\frac{9x^2}{y^2-27x^2}$

- ¹⁰⁵
p 63-65:
1. $\frac{3}{2}$
 2. $\frac{4}{5}$
 3. $\frac{5}{3}$
 4. 3
 5. -1
 6. $\frac{2}{3}$
 7. 2
 8. $\frac{9}{3}$
 9. $\frac{8}{5}$
 10. 1
 11. 7
 12. $\frac{5}{3}$
 13. 1
 14. 11
 15. $\frac{5}{2}$
 16. $\frac{1}{2}$
 17. $1\frac{3}{8}$
 18. $\frac{52}{19}$
 19. $\frac{129}{34}$
 20. $\frac{3}{5}$
 21. $\frac{21}{73}$
 22. -1

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