1.10 *Equations in Quadratic Form*

There are many equations that are not really quadratic equations, but when a particular substitution is made, the resulting equation is quadratic. Although they are not technically "quadratic" equations, they are **quadratic in form** (or "shape"), and the solutions are obtained using the methods of solving quadratic equations by factoring, if possible, completing the square, or quadratic formula. This is essentially a section involving the technique of **substitution**: select a different variable (if the equation is given in "X", then let **"u"** be the variable, and also find the value of **"u²"**. The following examples and exercises will show the way. The trick is to know what to use for the variable **"u"**. This will be easier with practice. You will probably quickly recognize that the problems here have been "rigged" or "contrived" to work out as they do. (If you think it is a challenge to work out the answers, you should try making up the problems!!)

For fairly simple problems in "quadratic form," the **straight-out factoring method** is an excellent alternative to the **substitution method**. Consider the following example each way:

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SUBSTITUTION METHOD:

$(X^2+7X)^2 - 2(X^2+7X) - 48 = 0$

Let $u = X^2+7X$, and so $u^2 = (X^2+7X)^2$

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The equation now reads: $u^2 - 2u - 48 = 0$

which factors: $(u - 8)(u + 6) = 0$

Solve for u: $u = 8; \quad u = -6$

Substitute/solve for X: $X^2+7X=8; \quad X^2+7X = -6$

$X^2+7X-8 = 0; \quad X^2+7X+6 = 0$

$(X+8)(X-1)=0; \quad (X+6)(X+1)=0$

$\boxed{X=-8; \ X=1; \quad X=-6; \ X=-1}$

"STRAIGHT-OUT FACTORING METHOD":

$(X^2+7X)^2 - 2(X^2+7X) - 48 = 0$

$[(X^2+7X) - 8] [(X^2+7X) + 6] = 0$

$(X^2 + 7X - 8)(X^2 + 7X + 6) = 0$

$(X + 8)(X - 1)(X + 6)(X + 1) = 0$

$X=-8;$	$X=1;$	$X=-6;$	$X=-1$

In exercises 1 - 6, find all solutions (use complex numbers if necessary. For some use the factoring method; in others, try substitution in order to familiarize yourself with this technique.

1. $X^4 - 5X^2 - 36 = 0$

$(x^2 - 9)(x^2 + 4) = 0$

$x^2 = \_\_\_\_ \quad x^2 = \_\_\_\_$

$x = \_\_\_\_ \quad x = \_\_\_\_$

2. $X^4 + 5X^2 - 36 = 0$

3. $X^4 - 2X^2 - 48 = 0$

4. $X^4 + 2X^2 - 48 = 0$

5. $(X^2+5X)^2 - 2(X^2+5X) - 24 = 0$

$[(x^2+5x) - 6][(x^2+5x) + 4] = 0$

$(x^2+5x \quad)(x^2+5x \quad) = 0$

$(\quad)(\quad)(\quad)(\quad) = 0$

6. $(X^2-6X)^2 + 13(X^2-6X) + 40 = 0$

7. $(X^2-3X)^2 - 14(X^2-3X) + 40 = 0$ 8. $(X^2-5X)^2 - 2(X^2-5X) - 24 = 0$

9. $(X^2-4X)^2 + 8(X^2-4X) + 15 = 0$ 10. $(X^2+6X)^2 + 3(X^2+6X) - 70 = 0$

11. $\underset{\scriptstyle u^2}{\underline{\left(X+\frac{12}{X}\right)^2}}$ + 15 $\underset{\scriptstyle u}{\underline{\left(X+\frac{12}{X}\right)}}$ + 56 = 0   12. $\left(X+\frac{12}{X}\right)^2 - 15\left(X+\frac{12}{X}\right) + 56 = 0$

Let u = $\left(X+\frac{12}{X}\right)$; u^2 = $\left(X+\frac{12}{X}\right)^2$

$$u^2 + 15u + 56 = 0$$

13. $\left(X+\frac{6}{X}\right)^2 + 2\left(X+\frac{6}{X}\right) = 35$ 14. $\left(X-\frac{18}{X}\right)^2 + 4\left(X-\frac{18}{X}\right) = 21$

15. $\left(\dfrac{X-7}{X}\right) + \dfrac{16X}{X-7} = 10$

Let $u = \dfrac{X-7}{X}$; $\quad \dfrac{1}{u} = \dfrac{X}{X-7}$

$$u + 16 \cdot \dfrac{1}{u} = 10$$

$$u^2 + 16 = 10u$$

$$u^2 - 10u + 16 = 0$$

$$(u-8)(u-2) = 0$$

$$u=8 \qquad u=2$$

$$\dfrac{X-7}{X} = 8; \qquad \dfrac{X-7}{X} = 2$$

$X=\underline{\qquad}$; $\quad X=\underline{\qquad}$

16. $\left(\dfrac{X+7}{X}\right) + \dfrac{16X}{X+7} = 10$

17. $\dfrac{X^2-6}{X} - \dfrac{5X}{X^2-6} = 4$

18. $\dfrac{X^2+12}{X} + \dfrac{48X}{X^2+12} = 14$

19. $\dfrac{X^2+12}{X} - \dfrac{32X}{X^2+12} = 4$

20. $\dfrac{X^2-12}{X} - \dfrac{20X}{X^2-12} = 1$

21. $(X^2-6X)^{\frac{2}{3}} - (X^2-6X)^{\frac{1}{3}} - 6 = 0$ 22. $(X^2+30X)^{\frac{2}{3}} - (X^2+30X)^{\frac{1}{3}} - 12 = 0$

Let $u = (X^2-6X)^{\frac{1}{3}}$; $u^2 = (X^2-6X)^{\frac{2}{3}}$

$$u^2 - u - 6 = 0$$

23. $\overset{u^2}{\overbrace{(X^2-3X)}} - \overset{u}{\underbrace{5\sqrt{X^2-3X}}} + 6 = 0$

Let $u = (X^2-3X)^{\frac{1}{2}};\quad u^2 = X^2-3X$

24. $X^2+6X - 7\sqrt{X^2+6X} + 12 = 0$

25. $X^2 + 8X + 12 = 7\sqrt{X^2+8X}$

p. 148-154:

1. ±3, $\pm2i$; 2. ±2, $\pm3i$; 3. $\pm2\sqrt{2}$, $\pm i\sqrt{6}$; 4. $\pm2i\sqrt{2}$, $\pm\sqrt{6}$;

5. $-6,1,-4,-1$; 6. $5,1,4,2$; 7. $4,-1,5,-2$; 8. $6,-1,1,4$;

9. $3,1,2\pm i$; 10. $-7,1,-3\pm i$; 11. $-3,-4,-6,-2$; 12. $2,3,4,6$;

13. $-6,-1,3,2$; 14. $-9,2,6,-3$; 15. -1, -7; 16. 1, 7;

17. $6,-1,-3,2$; 18. $6,2$, $3\pm1\sqrt{3}$; 19. $6,2,-2\pm2i\sqrt{2}$;

20. $-6,2$, $\dfrac{5\pm\sqrt{73}}{2}$; 21. $9,-3,4,2$; 22. $-32,2,-15\pm3\sqrt{22}$;

23. $4,-1$, 24. -8, 2, $-3\pm3\sqrt{2}$; 25. $-9,1$, $-4\pm4\sqrt{2}$.