1.09 *Radical Equations*

Dr. Robert J. Rapalje

When solving equations involving radicals, it will be necessary to square or cube both sides or to raise both sides of an equation to some power. However, it must be noted that the process of raising both sides of an equation to an **even** power, especially squaring both sides, does not guarantee equivalent equations. This means that the solutions of the resulting equation may not work in the original equation. Consider, for example, the very simple equation $X = 3$. Now, square both sides to get $X^2 = 9$. The resulting equation, $X^2=9$, has two solutions: $X=3$ and $X=-3$. However, $X=-3$ does not work in the original equation. Solutions like $X=-3$ that satisfy the resulting equation but do not work in the original equation are called **extraneous roots**, or **extraneous** (extra) **solutions**, and they must be rejected.

> **RULE**
>
> **Whenever solving an equation by squaring both sides, you must check each solution in the original equation and reject all extraneous roots.**

Another word of caution, if you square both sides of an equation, be sure to square the entire side. Never square term by term. Consider: $5 = 2 + 3$

Square both sides: $5^2 = (2 + 3)^2$, not $5^2 = 2^2 + 3^2$

$25 = 4 + 9$ **WRONG!!**

To "undo" a **square root**, you must **square** both sides of the equation. To "undo" a **cube root**, you must **cube** both sides of the equation. To "undo" a **fourth root**, you must **raise** both sides **to the fourth power**, and so on. If there are other terms involved in the equation, before squaring both sides, it is an important first step to **isolate the radical term on one side of the equation.** This is demonstrated in the examples that follow.

EXAMPLE 1. Solve for X:

$$\sqrt{2X + 3} = 5$$
$$\left(\sqrt{2x+3}\right)^2 = 5^2$$
$$2x+3 = 25$$
$$2x = 22$$
$$x = 11$$
$$\text{Ch: } \sqrt{22+3} = 5$$
$$\sqrt{25} = 5 \checkmark$$

EXAMPLE 2. Solve for X:

$$\sqrt[3]{2X + 3} = 5$$
$$\left(\sqrt[3]{2x+3}\right)^3 = 5^3$$
$$2x + 3 = 125$$
$$2x = 122$$
$$x = 61$$
$$\text{Ch: } \sqrt[3]{122+3} = 5$$
$$\sqrt[3]{125} = 5 \checkmark$$

EXERCISES. Solve for X:

1. $\sqrt{2X - 3} = 5$

2. $\sqrt[4]{2X - 3} = 5$

3. $\sqrt{2X + 3} = -5$

4. $\sqrt[3]{2X + 3} = -5$

5. $\sqrt[5]{3X + 4} = -2$

6. $\sqrt[4]{2X + 4} = -2$

7. $\sqrt{5X + 1} - 9 = 0$

Get radical term isolated
on left side of equation.

$$\sqrt{5X + 1} = 9$$

Now square both sides & solve:

8. $\sqrt{3X + 1} - 8 = 2$

EXAMPLE 3.

$2\sqrt{X + 3} - 5 = 2$

$2\sqrt{x+3} = 7$

$\left(2\sqrt{x+3}\right)^2 = 7^2$

$4(x+3) = 49$

$4x + 12 = 49$

$4x = 37$

$x = \frac{37}{4}$

Ch: $2\sqrt{\frac{37}{4}+3} - 5 = 2$

$2\sqrt{\frac{49}{4}} - 5 = 2$

$2 \cdot \frac{7}{2} - 5 = 2$ ✓

9. $3\sqrt{X - 2} + 2 = 7$

EXAMPLE 4

$2\sqrt[3]{X-2}+3=7$

$\frac{2\sqrt[3]{x-2}}{2}=\frac{4}{2}$

$\left(\sqrt[3]{x-2}\right)^3=2^3$

$x-2=8$

$x=10$

Ch: $2\sqrt[3]{8}+3=7$

$2\cdot 2+3=7$ ✓

10. $2\sqrt[3]{X-2}+7=3$

EXAMPLE 5

$\left(\sqrt{X^2-3X+18}\right)^2=(X+2)^2$

$x^2-3x+18=x^2+4x+4$

$-7x=-14$

$x=2$

Ch: $\sqrt{4-6+18}=2+2$

$\sqrt{16}=4$ ✓

11. $\sqrt{X^2-3X+7}=2-X$

EXAMPLE 6

$\sqrt{X+3}=X-3$

$\left(\sqrt{x+3}\right)^2=(x-3)^2$

$x+3=x^2-6x+9$

$0=x^2-7x+6$

$0=(x-6)(x-1)$

$x=6$ ~~$x=1$~~ Reject!

Ch: $x=6$ Ch: $x=1$

$\sqrt{6+3}=6-3$ $\sqrt{1+3}=1-3$

$\sqrt{9}=3$ ✓ $\sqrt{4}=-2$ No!

12. $\sqrt{X+6}=X+4$

13. $\sqrt{2X + 15} = 2X + 3$

14. $\sqrt{2X + 3} = 2X - 3$

15. $\sqrt{3X + 1} = \frac{1}{2}X + 1$

16. $\sqrt{X + 3} = \frac{1}{3}X + 1$

EXAMPLE 7.

$\sqrt{X + 264} = \sqrt{X} + 12$

$\left(\sqrt{X+264}\right)^2 = \left(\sqrt{X} + 12\right)^2$

$x + 264 = x + 24\sqrt{x} + 144$

$\frac{120}{24} = \frac{24\sqrt{x}}{24}$

$5 = \sqrt{x}$

$5^2 = \left(\sqrt{x}\right)^2$

$25 = x$

Ch: $\sqrt{25+264} = \sqrt{25} + 12$

$\sqrt{289} = 5 + 12$

$17 = 5 + 12$ ✓

17. $\sqrt{X + 33} = \sqrt{X} + 3$

18. $\sqrt{X+7} + \sqrt{X} = 7$ **19.** $\sqrt{X+40} - \sqrt{X} = 4$

(You must first isolate one of the radical terms!)

$\sqrt{X+7} = 7 - \sqrt{X}$

EXAMPLE 8

$\sqrt{3X+1} = \sqrt{X} + 3$

$\left(\sqrt{3x+1}\right)^2 = \left(\sqrt{x}+3\right)^2$

$3x+1 = x + 6\sqrt{x} + 9$

$2x - 8 = 6\sqrt{x}$

$x - 4 = 3\sqrt{x}$

$(x-4)^2 = (3\sqrt{x})^2$

$x^2 - 8x + 16 = 9x$

$x^2 - 17x + 16 = 0$

$(x-16)(x-1) = 0$

$x = 16$ ~~$x = 1$~~

Ch: $x = 16$

$\sqrt{49} = \sqrt{16} + 3$ ✓

$x = 1$

$\sqrt{4} = \sqrt{1} + 3$

No!

20. $\sqrt{5X+1} = \sqrt{3X} + 1$

21. $\sqrt{5X-4} - \sqrt{X} = 2$ **22.** $\sqrt{2X} - \sqrt{3X+1} = 1$

23. $\sqrt{3X + 1} + \sqrt{2X} = 1$

24. $\sqrt{X + 4} = \sqrt{3X} - 2$

25. $\sqrt{5X + 4} - \sqrt{X} = 4$

26. $\sqrt{5 - X} = \sqrt{X + 20} - 1$

27. $\sqrt{2X + 20} = \sqrt{1 - 6X} - 5$

28. $\sqrt{2X + 20} = 5 - \sqrt{1 - 6X}$

$$16X^2 + 126X - 16 = 0$$

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29. $\sqrt{4X + 7} - \sqrt{2X + 3} = 1$

30. $\sqrt{2X + 10} = \sqrt{12 - 8X} - 1$

$$100X^2 - 28X - 39 = 0$$

31. $\sqrt{5X + 1} + \sqrt{X + 2} = 3$

32. $\sqrt{4X + 5} - \sqrt{3X - 6} = 2$

$$4X^2 - 29X + 7 = 0$$

$$X^2 - 34X + 145 = 0$$

33. $2\sqrt{3X + 4} - 3\sqrt{X} = 2$

34. $2\sqrt{X} - \sqrt{2X - 7} = 3$

35. $\sqrt{7X + 4} = \sqrt{2X + 3} + 2$

36. $\sqrt{7X + 4} = 2 - \sqrt{2X + 3}$

$25X^2 - 62X - 39 = 0$

$(25X \quad)(X \quad) = 0$

E.C.

37. $3\sqrt{2X + 5} - 2\sqrt{7 - X} = 3$

38. **Does** $\sqrt{5 + 2\sqrt{6}} = \sqrt{2} + \sqrt{3}$ **?**

[To verify this equation, square both sides, which is legitimate for positive numbers.]

$121X^2 + 124X - 236 = 0$

ANSWERS 1.09

p. 139-147:

1. 14; 2. 314; 3. No solution; 4. -64; 5. -12 6. No solution;
7. 16; 8. 33; 9. 43/9; 10. -6; 11. -3; 12. -2 (reject -5);
13. 1/2 (reject -3); 14. 3 (reject 1/2); 15. 0,8; 16. 6,-3;
17. 16; 18. 9; 19. 9; 20. 0,3; 21. 4 (reject 1); 22. No solution (reject 0,8); 23. 0 (reject 8); 24. 12 (reject 0);
25. 9 (reject 1); 26. -4 (reject -11); 27. -8 (reject 1/8);
28. 1/8 (reject -8); 29. 1/2, -3/2; 30. -1/2 (reject 39/50);
31. 1/4 (reject 7); 32. 5,29; 33. 4; 34. 16,4;
35. 3 (reject -13/25); 36. -13/25 (reject 3);
37. 118/121 (reject -2); 38. Yes.