1.12 *Applications*

After solving equations and inequalities, the question "What good is this?" inevitably arises! Answers to questions such as "What good is math?" usually involve applications--that is, "word problems!" Word problems range in difficulty from simple to absurd. While it is helpful to arrange them in "categories" for study, it will be even more helpful to see that the "different" categories or types of problems are actually more similar than they are different.

As in lower mathematics courses, it may also be helpful to identify five steps in setting up and solving word problems:

STEP 1: ***IDENTIFY THE VARIABLE.*** State exactly what it is that the variable represents. For example, "Let X = the number of dimes" or "Let X = rate of the first plane" or "Let X = amount of pure alcohol to be added." Then express all other quantities to be used in the problem in terms of X. This is the most important, often the most difficult, and usually the most overlooked step of the problem.

STEP 2: ***WRITE THE EQUATION.*** Having completed step 1, use this step in writing the equation. This is often no more than translating a sentence of the problem into an equation. Read the problem carefully.

STEP 3: ***SOLVE THE EQUATION.*** This is usually the easy part!

STEP 4: ***ANSWER THE QUESTION.*** After solving for X, there may be other quantities to be determined. Be sure you have answered the question before going on to the next exercise.

STEP 5: ***CHECK.*** Check the answers in the worded problem itself and make sure the solution actually works. Reject any extraneous or "inappropriate" answers.

In this section, the following "categories" of applications will be considered:

- I. **Number problems, consecutive number problems**
- II. **Perimeter problems**
- III. **Coin problems**
- IV. **Interest problems**
- V. **Mixture problems**
- VI. **Distance, rate, time problems**
- VII. **Area, volume problems (quadratic)**
- VIII. **Theorem of Pythagoras (quadratic)**

I. NUMBER PROBLEMS, CONSECUTIVE NUMBER PROBLEMS.

When finding consecutive numbers (integers), as 7, 8, 9 or 29, 30, 31, it is usually convenient to let X represent the first number. Then since the difference between the numbers is 1, it follows that X+1 represents the second number, X+2 represents the third number, etc.

When finding consecutive odd numbers (integers), as 7, 9, 11 or 29, 31, 33, notice that the difference between the numbers is 2. For this reason, after letting X represent the first number, the second number will be 2 more than the first (that is, X+2), the third number will be 2 more than the second (that is, X+4), and so on. Likewise for consecutive even numbers, such as 8, 10, 12 or 22, 24, 26, the difference between consecutive numbers is 2. Therefore, if X represents the first, then X+2 and X+4 represent the second and third numbers respectively.

In summary, for consecutive numbers:

Let X = first number
X+1 = second number
X+2 = third number.

For consecutive even numbers <u>or</u> consecutive odd numbers:

Let X = first number
X+2 = second number
X+4 = third number.

The numbers represented by X, X+2, and X+4 will all be even, or they will all be odd, depending upon whether the value of X turns out to be even or odd.

EXAMPLE 1:

Three numbers are such that the second is six less than twice the first, and the third is five more than the sum of the first two numbers. The sum of the numbers is 293. Find the numbers.

SOLUTION: Let X = first number
$2X-6$ = second number
$X + 2X-6 + 5$ = third number
or $3X-1$

Equation:
$$\begin{aligned} X + (2X-6) + (3X-1) &= 293 \\ 6X - 7 &= 293 \\ 6X &= 300 \end{aligned}$$

Answer the question:
$X = 50$ first number
$2X-6 = 94$ second number
$3X-1 = 149$ third number

Check: $50 + 94 + 149 = 293$

EXAMPLE 2:

Find three consecutive odd numbers such that eight times the third plus twice the first is equal to six times the second. Find the numbers.

SOLUTION: Let X = first odd number
$X+2$ = second odd number
$X+4$ = third odd number

Equation:
$$\begin{aligned} 8(X+4) + 2(X) &= 6(X+2) \\ 8X+32 + 2X &= 6X + 12 \\ 10X + 32 &= 6X + 12 \\ 4X &= -20 \end{aligned}$$

Answer the question:
$X = -5$ first number
$X+2 = -3$ second number
$X+4 = -1$ third number

Check:
$$\begin{aligned} 8(-1) + 2(-5) &= 6(-3) \\ -8 + -10 &= -18 \end{aligned}$$

EXERCISES:

1. Three numbers are such that the second number is 4 more than three times the first, and the third number is 12 less than the sum of the first two numbers. The sum of the three numbers is 44. Find the numbers.

2. Three numbers are such that the first number is 10 less than twice the second, and the second number is 4 more than three times the third. Twice the second number is equal to the sum of the first and third numbers. Find the numbers.

3. Find three consecutive even integers such that three times the first plus six times the second is equal to ten times the third.

4. Find three consecutive integers such that twice the first minus the third is 3 less than the second.

II. PERIMETER PROBLEMS.

EXAMPLE 3: A piece of wire 120 cm in length is to be cut into two pieces, one to form a rectangle and the other an equilateral triangle. The length of the rectangle is to be three less than twice the width, and each side of the triangle is to be twice the length of the rectangle. Find the length of each piece of wire.

SOLUTION: Let X = width of rectangle
2X-3 = length of rectangle
2(2X-3) = side of triangle
or 4X-6

Equation: **2W + 2L + 3S = Total Perimeter**
2(X) + 2(2X-3) + 3(4X-6) = 120
2X + 4X - 6 + 12X - 18 = 120
18X - 24 = 120
18X = 144
X = 8 cm width of rectangle

Answer question: **2X - 3 = 13 cm length of rectangle**
4X - 6 = 26 cm side of triangle

Check: Perimeter = 2W + 2L + 3S
= 2(8) + 2(13) + 3(26)
= 16 + 26 + 78
= 120 cm.

EXAMPLE 4: The perimeter of a rectangle is 500 meters. If five times the width is 4 more than twice the length, find the dimensions of the rectangle.

SOLUTION: Let X = width of the rectangle
5X = 2(length) + 4
5X - 4 = 2(length)

Equation: **2W + 2L = Perimeter**
2(X) + (5X-4) = 500
7X - 4 = 500
7X = 504
X = 72 m width
5X - 4 = 356 m 2 lengths
178 m length

Check: Perimeter = 2W + 2L
= 2(72) + 2(178)
= 144 + 356
= 500 m.

EXERCISES:

5. A wire 264 cm. in length is to be cut into two pieces, one to form a rectangle whose length is four less than twice the width, and the other to form a square whose side is equal to the sum of the width and the length of the rectangle. How long is the shorter piece of wire?

6. An envelope with an open flap is in the shape of a rectangle surmounted by an equilateral triangle. The length of the rectangle is 3 less than twice the width, and of course the side of the equilateral triangle is equal to the length of the rectangle. If the total distance around the open envelope is 71 cm., find the dimensions of the envelope.

7. The perimeter of a rectangle is 104 m. Three times the width is four less than twice the length. Find the dimensions.

8. A rectangle is such that the width is one tenth of the total perimeter. If the length is 8 more than three times the width, find the dimensions of the rectangle and the perimeter.

III. COIN PROBLEMS.

EXAMPLE 5: A box contains nickels, dimes, and quarters worth a total of \$16.75. The number of nickels is 5 less than four times the number of dimes, and the number of dimes is twice the number of quarters. How many of each coin are there?

SOLUTION:

	No. Coins	**Each**	**Values**
Q	X	25	25(X)
D	2X	10	10(2X)
N	4(2X) - 5	5	5(8X - 5)
			1675¢

Equation: 25X + 20X + 40X - 25 = 1675
85X - 25 = 1675
85X = 1700
X = 20 Quarters

Answer the question: **2X = 40 Dimes**
8X - 5 = 155 Nickels

Check:
20(.25) = \$ 5.00 Quarters
40(.10) = 4.00 Dimes
155(.05) = 7.75 Nickels
\$16.75 Total

EXERCISES:

9. A box contains nickels, dimes, and quarters worth \$12.60. The number of dimes is 2 less than three times the number of nickels, and the number of quarters is 4 less than twice the number of dimes. How many of each coin are there?

10. A box contains nickels, dimes, and quarters worth $69.50. The number of nickels is 10 more than twice the number of dimes. There are as many quarters as nickels and dimes combined. How many of each coin are there?

11. A large box contains nickels, dimes, and quarters worth $296. The number of quarters is three times the number of nickels. The number of quarters is 10 more than the number of dimes. How many of each coin are there? **[Hint: Let X = no. nickels, and rephrase the third sentence to read, "The number of dimes is _______ than the number of quarters."]**

12. A box contains pennies, nickels, and dimes worth $4.60. The number of nickels is 2 less than twice the number of pennies, and the number of pennies is 2 more than twice the number of dimes. How many of each coin are there?

IV. INTEREST PROBLEMS.

EXAMPLE 6: A woman invests a sum of money at 6% and $3000 more than this at 9%. If the total interest earned in one year is $4170, how much was invested at each rate?

SOLUTION:

	Principle X	**Rate =**	**Interest**
6%	X	0.06	0.06(X)
9%	X + $3000	0.09	0.09(X+3000)
			$4170

Equation:

$$\begin{aligned} 0.06X + 0.09(X + 3000) &= \$4170 \\ 0.06X + 0.09X + 270 &= \$4170 \\ 0.15X + 270 &= \$4170 \\ 0.15X &= \$3900 \\ X &= \$3900/0.15 \\ \mathbf{X} &= \mathbf{\$26000 \quad @ \quad 6\%} \end{aligned}$$

Answer the question: $\mathbf{X + 3000 = \$29000 \quad @ \quad 9\%}$

Check:

$$\begin{aligned} 26000(0.06) &= \$\ 1560.00 \\ 29000(0.09) &= \quad \underline{2610.00} \\ &\quad \$\ 4170.00 \text{ Total} \end{aligned}$$

EXERCISES:

13. A man has $10,000 to invest, some in a relatively safe account earning 5% interest per year, and the rest in more speculative investments earning 12% per year. If the total interest earned for the year was $955, how much was invested at each rate?

14. A sum of money was invested at 5% annual interest, and $500 less than twice this amount was invested at 12%. If the total interest earned for the year was $375, how much was invested at each rate?

15. A sum of money is invested in three accounts. The principle invested at 9% is $200 more than twice the principle invested at 7%. The principle invested at 12% is twice the principle invested at 9%. If the total interest earned for the year on all three accounts was $431, how much was invested at each rate?

16. A man has $20,000 to invest, some at 8% interest per year, and the rest at 12% per year. If the total interest earned for the year was $2160, how much was invested at each rate?

V. MIXTURE PROBLEMS.

EXAMPLE 7: Some 10% alcohol solution is to be mixed with some 30% alcohol solution to make 20 liters of 16% solution. How much of each must be used.

SOLUTION:	**Amt. Sol. X**	**Percent =**	**Pure Stuff**
10%	X	0.10	0.10(X)
30%	20 - X	0.30	0.30(20 - X)
16%	20	0.16	0.16(20)

Equation: $0.10(X) + 0.30(20 - X) = 0.16(20)$

$.10X + 6 - 0.30X = 3.20$

$-0.20X + 6 = 3.20$

$-0.20X = -2.80$

$X = -2.80/(-0.20)$

X = 14 liters of 10% solution

Answer the question: **20 - X = 6 liters of 30% solution**

Check:

14(0.10) = 1.4 liters alcohol
6(0.30) = 1.8 liters alcohol
20(0.16) = 3.2 liters Total

EXAMPLE 8: How much liquid must be drained from a 20 liter radiator at 20% antifreeze and replaced with pure antifreeze to bring the strength up to 50%?

SOLUTION:	**Amt. Sol. X**	**Percent =**	**Pure Stuff**
Begin	20	0.20	0.20(20)
Drain	**(-)** X	0.20	-0.20(X)
Add	**(+)** X	1.00	+1.00(X)
End Up	**(=)** 20	0.50	0.50(20)

Equation: $0.20(20) - 0.20(X) + 1.00(X) = 0.50(20)$

$4 + 0.80X = 10$

$0.80X = 6$

$X = 6/(0.80)$

X = 7.5 liters of antifreeze

EXAMPLE 9: How much water must be added to 60 liters of 20% acid solution in order to dilute the solution to 8%?

SOLUTION:

	Amt. Sol. X	**Percent =**	**Pure Stuff**
20%	60	0.20	0.20(60)
Water	X	0.00	0.00(X)
8%	X + 60	0.08	0.08(X + 60)

Equation: .20(60) + .00(X) = 0.08(X + 60)
12 + 0 = 0.08X + 4.8
7.2 = 0.08X
X = 7.2/(0.08)
X = 90 liters of water

Check: 60(0.20) = 12 liters acid
+ 90 Water = No acid
150(0.08) = 12 liters acid

EXAMPLE 10: Twenty kilograms of nuts consisting of cashews worth $6.00 per kg, pecans worth $2.50 per kg, and peanuts worth $1.50 per kg are mixed. If there are twice as many pecans as cashews, and the total value of the nuts is $56, how many of each are there? **[HINT: Let X = kg cashews; 2X = kg pecans**
3X = kg cashews and pecans combined
20-3X = kg peanuts]

SOLUTION:

	Amt. Sol. X	**Percent =**	**Pure Stuff**
Cashews	X	6.00	6.00(X)
Pecans	2X	2.50	2.50(2X)
Peanuts	20-3X	1.50	1.50(20-3X)
Total	20		56.00

Equation: 6.00(X) + 2.50(2X) + 1.50(20-3X) = 56.00
6.00X + 5.00X + 30.00 - 4.50X = 56.00
6.50X + 30.00 = 56.00
6.50 X = 26.00
X = 26.00/(6.50)
X = 4 kg cashews
2X = 8 kg pecans
20-3X = 8 kg peanuts

EXERCISES:

17. How much 10% alcohol solution must be added to 20 liters of 50% solution to make a 20% solution?

18. Some 80% acid solution is to be mixed with some 35% acid solution to make 300 liters of 50% solution. How much of each acid should be used?

19. How much water must be added to 50% alcohol solution to obtain 100 liters of 10% solution?

20. How much pure alcohol must be added to 100 liters of 10% alcohol solution to create an 80% solution?

21. How much liquid must be drained from a 24 liter radiator that is 25% antifreeze and replaced with pure antifreeze in order to bring the strength up to 50%?

22. How much liquid must be drained from an 18 liter radiator that is 10% antifreeze and replaced with pure antifreeze in order to bring the strength up to 50%?

23. Fifty tickets were sold to a chicken barbeque for a total of $219. Children's tickets sold for $2.50, youth tickets sold for $3.50, and adult's tickets sold for $5.00. There were 10 more youth tickets than children's tickets. How many of each ticket were sold?

24. A total of 180 tickets are sold, some at \$3, some at \$5, and some at \$10 each. The total value of the tickets was \$1100. The number of \$5 tickets was 20 more than the number of \$3 tickets. How many of each ticket were sold?

VI. DISTANCE, RATE, TIME PROBLEMS.

EXAMPLE 11: Two bicycles start at the same point traveling in the opposite direction. The speed of the second bike in miles per hour is 12 less than three times the first. At the end of 6 hours, the bicycles are 144 miles apart. Find the speed of each bicycle.

SOLUTION:

	Rate	**X**	**Time**	**=**	**Distance**
1st	X		6		6(X)
2nd	3X - 12		6		6(3X-12)
					144

Equation: $6X + 6(3X - 12) = 144$

$6X + 18X - 72 = 144$

$24X = 216$

$X = 9$ mph 1st bicycle

Answer the question: **$3X - 12 = 15$ mph 2nd bicycle**

EXAMPLE 12: A train leaves the Sanford terminal averaging 45 mph. A second train leaves the same terminal two hours later averaging 60 mph. How long will it take the second train to catch the first train? **[HINT: Let X = time of second train to catch first. Distances are equal.]**

SOLUTION:

	Rate	**X Time**	**= Distance**
1st	45	X + 2	45(X+2)
2nd	60	X	60(X)

Equation:

$$\begin{aligned} 45(X + 2) &= 60(X) \\ 45X + 90 &= 60X \\ 90 &= 15X \\ \mathbf{X} &= \textbf{6 hours for second train} \end{aligned}$$

EXERCISES:

25. Two boys are riding bicycles in the opposite direction. One travels 15 mph faster than the other. At the end of 3 hours they are 102 miles apart. How fast is each boy riding?

26. Two cars are driving in opposite directions, one at 55 mph and the other at 65 mph (on the interstate!). How long will it take before the cars are 300 miles apart?

27. At 8:00 a.m. a doctor leaves Sanford on a northbound train. Her husband, noticing that the doctor forgot her briefcase, boards another northbound train leaving at 10:00 a.m. and traveling 18 mph faster than the first train. The second train overtakes the first train at 4:00 p.m. Find the speeds of the trains, and the distance they traveled?

28. Two trains leave Winter Park terminal traveling in the opposite direction. The southbound train leaves at 8:00 a.m. The northbound train leaves at 10:00 a.m. traveling 20 mph faster than the southbound train. At 2:00 p.m. the trains are 600 miles apart. Find the speeds of the trains.

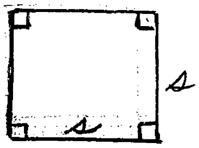
29. Two bicyclists begin at the same point on a circular path traveling in the same direction. The rate of one cyclist is 5 mph faster than the other. If it takes the faster cyclist 3 hours to "lap" the slower cyclist, then determine the distance around the path. **[Hint: The difference of the distances traveled equals one "lap".]** If the slower bicycle travels at 10 mph, how many laps did each bicycle make?

30. **EXTRA CHALLENGE.** Two bicylists begin at the same point on a circular path traveling in the same direction. One cyclist, who is traveling 3 mph faster than the other, "laps" the cyclist after 6 laps. Find the speeds of the cyclists. Is enough information given to determine the length of the path?

VII. AREA, VOLUME PROBLEMS (QUADRATIC)

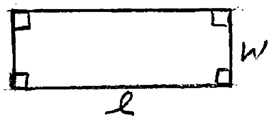
AREAS

SQUARE



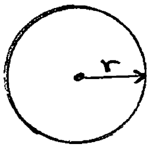
$A = s^2$

RECTANGLE



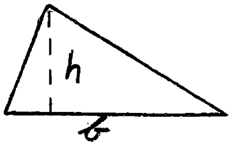
$A = l \cdot w$

CIRCLE



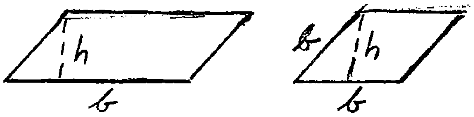
$A = \pi r^2$

TRIANGLE



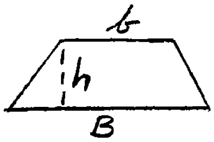
$A = \frac{bh}{2}$ or $\frac{1}{2} b \cdot h$

PARALLELOGRAM or **RHOMBUS**



$A = b \cdot h$

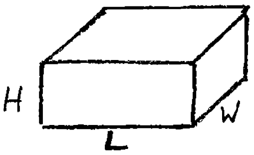
TRAPEZOID



$A = \frac{1}{2} (B + b) \cdot h$

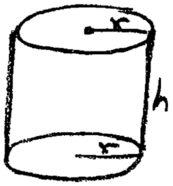
VOLUMES

RECTANGULAR BOX
(RECTANGULAR PARALLELOPIPED!)



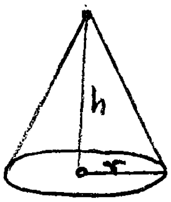
$V = L \cdot W \cdot H$

CYLINDER



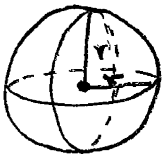
$V = \pi r^2 h$

CONE



$V = \frac{1}{3} \pi r^2 h$

SPHERE



$V = \frac{4}{3} \pi r^3$

EXAMPLE 13: The base of a triangle is 3 more than twice the height. If the area of the triangle is 10 square centimeters, find the dimensions of the triangle.

SOLUTION: Let X = height of triangle
2X+3 = base of triangle

Equation: ½·bh = Area
½·X(2X+3) = 10 Mult both sides of equation by 2
X(2X+3) = 20 Add -20 to both sides of equation
$2X^2 + 3X - 20 = 0$
(2X - 5) (X + 4) = 0
X = 5/2; X = -4 Reject -4
X = 5/2 cm. height of triangle
2X+3 = 8 cm. base of triangle

EXAMPLE 14: A rectangular box with height 3 centimeters has a volume of 24 cubic centimeters. The length of the box is two more than the width. Find the dimensions, and the volume of the box.

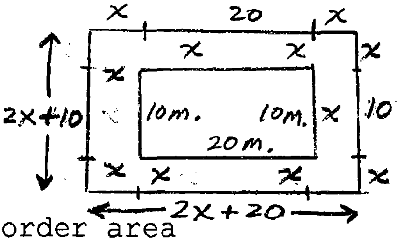
SOLUTION: Let X = width of box
X+2 = length of box
3 = height of box

Equation: L W H = Volume of box
3X(X+2) = 24
$3X^2 + 6X - 24 = 0$
$3(X^2 + 2X - 8) = 0$
3(X + 4)(X - 2) = 0
~~X = -4;~~ **X = 2 cm. Width of box**
~~Reject~~ **X+2 = 4 cm. Length of box**
3 cm. Height of box
24 cu.cm. Volume of box

EXAMPLE 15: A rectangular garden is 10 m. by 20 m. The owner, who has enough pine bark to cover 136 square meters, wants a pine bark path around the outside of the garden. Find the width of the path and the outside dimensions.

SOLUTION:

Let X = width of the path
$2X+10$ = total outside width
$2X+20$ = total outside length
$10 \cdot 20 = 200\ m^2$ = Inner area
$136\ m^2$ = Border area (Pine bark)



Equation: Total area = Inner area + Border area

$$(2X+10)(2X+20) = 200 + 136$$
$$4X^2 + 60X + 200 = 336$$
$$4X^2 + 60X - 136 = 0$$
$$4(X^2 + 15X - 34) = 0$$
$$4(X + 17)(X - 2) = 0$$

~~$X = -17$; Reject~~ **$X = 2$ cm. Width of path**

$2X+10 = 14$ cm. Outside width

$2X+20 = 24$ cm. Outside length

EXAMPLE 16: The volume of a cone is 36π cubic centimeters. The radius of the base is twice the height. Find the dimensions of the cone.

SOLUTION:

Let X = height of cone
$2X$ = radius of base of cone

Equation:

$$\frac{1}{3}\pi\, r^2 h = Volume$$

$$\frac{1}{3}\pi\, (2X)^2 X = 36\pi$$

$$\frac{1}{3}\pi\, 4X^2 \cdot X = 36\pi$$

$$\frac{4}{3}\pi\, X^3 = 36\pi$$

$$\frac{3}{4}\cdot\left(\frac{4}{3}\pi\, X^3\right) = \frac{3}{4}\cdot(36\pi)$$

$$\frac{\cancel{\pi} X^3}{\cancel{\pi}} = \frac{27\cancel{\pi}}{\cancel{\pi}}$$

$X = 3$ cm. height

$2X = 6$ cm. radius of base

EXERCISES:

31. The base of a triangle is 3 less than twice the height. If the area of the triangle is 10 square centimeters, find the dimensions of the triangle.

32. The height of a parallelogram is 3 less than the base. If the area of the parallelogram is 40 square meters, find the dimensions of the parallelogram.

33. A rectangular box with height 2 cm. has a volume of 50 cubic centimeters. The length of the box is 5 less than twice the width. Find the dimensions of the box.

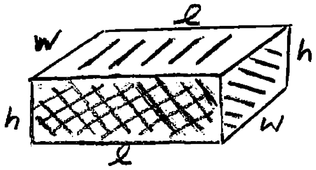
34. A rectangular box with height 4 meters has a volume of 576 cubic meters and a square base. Find the dimensions of the box.

35. A rectangular garden is 10 meters by 20 meters. The owner, who has enough pine bark to cover an area of 216 square meters, wants a pine bark path around the outside of the garden. Find the width of the path and the outside dimensions.

36. The combined area of an 8" by 10" picture and its frame (border around the outside) is 288 square inches. Find the width and the dimensions of the picture's frame.

37. The an 8" by 10" picture is surrounded by a matte (border) whose area is 40 square inches. (That is, the area of the matte is 40 square inches.) Find the width of the matte and the outside dimensions of the matte.

38. Find the surface area of a rectangular box of width **w**, length **l**, and height **h**. [Hint: Identify the 6 faces.]

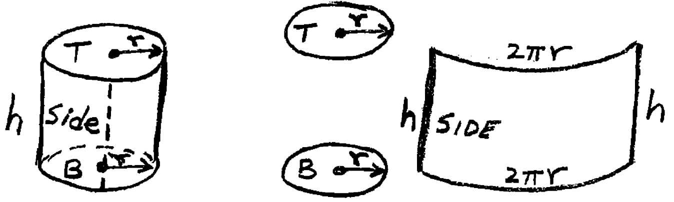


39. Find the surface area of a cylinder whose height is **h** and whose base radius is **r**. [Hint: Identify the top, the bottom, and "side" areas. Notice that the "side" area is actually a rectangle whose width is **h** and whose length is the circumference of the circle **2πr**.]

TOP BOTTOM SIDE

$A = \_\_\_\_ + \_\_\_\_ + \_\_\_\_$

$= \_\_\_\_\_\_\_\_\_\_\_\_.$



40. A cone has a base radius that is twice the height. If the volume of the cone is 36π cubic meters, find the dimensions of the cone.

41. A cylinder has a base diameter that is three times the height. If the volume of the cylinder is 144π cubic inches, find the dimensions of the cylinder.

42. A cone has a base radius that is three times the height. If the volume of the cone is 375π cubic centimeters, find the dimensions of the cone.

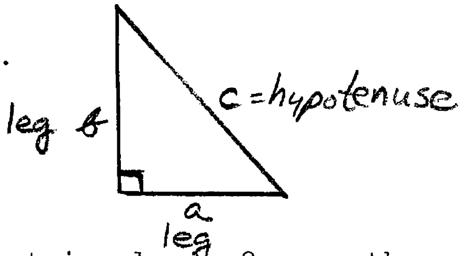
43. The height of a cone is twice the radius of the base. If the volume of the cone is 144π cubic feet, find the dimensions of the cone.

44. Find the radius of a sphere whose volume is 36π cubic centimeters.

45. Find the radius of a sphere whose volume is 4.5π cubic feet.

46. A man wants to build a rectangular speaker box whose volume is 3 cubic feet for his 15 inch (diameter) speakers. If the square base is 18 inches on each side, how deep are the speaker boxes?

VIII. THEOREM OF PYTHAGORAS (QUADRATIC).



In any right triangle,
where **"a"** and **"b"** are **legs**,
and **"c"** is the **hypotenuse**,
$a^2 + b^2 = c^2$.

EXAMPLE 17: The longer leg of a right triangle is 2 more than the shorter leg, and the hypotenuse is 2 less than twice the shorter leg. Find the sides of the triangle.

SOLUTION: Let

X = shorter leg
$X+2$ = longer leg
$2(X) - 2$ = hypotenuse

Equation:

$X^2 + (X+2)^2 = (2X-2)^2$
$X^2 + X^2 + 4X + 4 = 4X^2 - 8X + 4$
$2X^2 + 4X + 4 = 4X^2 - 8X + 4$
$0 = 2X^2 - 12X$
$0 = 2X(X - 6)$
$2X=0$ or $X-6=0$
$X=0$ **Reject**
$X= 6$ shorter leg
$X+2= 8$ longer leg
$2X-2=12$ hypotenuse

EXAMPLE 18: The diagonal of a rectangle is 3 less than 4 times the width of the rectangle. The length of the rectangle is 1 less than the diagonal. Find the length of the diagonal of the rectangle.

SOLUTION: Let

X = width of rectangle
$4X-3$ = diagonal of rectangle
$(4X-3) - 1$ = length of rectangle
$4X-4$ = length of rectangle

Equation:

$X^2 + (4X-4)^2 = (4X-3)^2$
$X^2 + 16X^2 - 32X + 16 = 16X^2 - 24X + 9$
$17X^2 - 32X + 16 = 16X^2 - 24X + 9$
$X^2 - 8X + 7 = 0$
$(X - 1)(X - 7) = 0$
~~**X = 1**~~ **or $X = 7$ width of rectangle**
~~4X-4 = 0~~ **$4X-4 = 24$ length of rectangle**
~~**No Way!!**~~ **$4X-3 = 25$ diagonal of rectangle**

EXERCISES:

47. The longer leg of a right triangle is 1 less than twice the shorter leg, and the hypotenuse is 1 more than twice the shorter leg. Find the sides of the triangle.

48. The shorter leg of a right triangle is 1 less than the longer leg, and the hypotenuse is 1 less than twice the shorter leg. Find the sides of the triangle.

49. The longer leg of a right triangle is 4 less than the hypotenuse. The longer leg is 2 less than twice the shortest leg. Find the sides of the triangle.
[Hint: Let X = shortest leg, and paraphrase first sentence.]

50. The hypotenuse of a right triangle is 4 less than 3 times the shortest side, and the longer leg is 4 more than twice the shortest side. Find the sides of the triangle.

ANSWERS 1.12

p.173-198:

1. 6,22,16; 2. 58,34,10; 3. -28,-26,-24; 4. All integers;
5. W=16cm, L=28cm; S=44cm, short wire=88cm; 6. W=10cm,L=17cm;
7. W=20cm, L=32cm; 8. N=8, L=32, P=80; 9. 8N, 22D, 40Q;
10. 70D, 150N, 220Q; 11. 270N, 810Q, 800D; 12. 14D, 30P, 58N;
13. $3500 @ 5%, $6500 @ 12%; 14. $1500 @ 5%, $2500 @12%;
15. $500 @7%; $1200 @9%, $2400 @12%; 16. $6000 @8%, $14000 @12%;
17. 60 l; 18. 100 l; 19. 80 l; 20. 350 l; 21. 8 l; 22. 8 l;
23. 4 children, 14 youth, 32 adult; 24. 50@$3, 70@$5, 60@$10;
25. 9.5mph, 24.5mph; 26. 2.5 hrs; 27. 54 mph, 72 mph, 432 mi;
28. 52 mph southbd, 72 mph northbd; 29. 15 mi, 2 & 3 laps;
30. 15mph, 18mph, no; 31. h=4cm, b=5cm; 32. b=8m, h=5m;
33. 5cm, 5cm, 2cm; 34. 12m, 12m, 4m; 35. 3m, 16m by 26m;
36. 4", 16" by 18"; 37. 1", 10" by 12"; 38. 2 LH + 2WL + 2WH;
39. $2\pi r^2 + 2\pi rh$; 40. h=3m, r=6m; 41. h=4", r=6";
42. h=5cm, r=15cm; 43. r=6', h=12'; 44. 3 cm; 45. 3/2';
46. 16"; 47. 8,15,17; 48. 3,4,5; 49. 16,30,34; 50. 10,24,26.