2.07 *Functional Notation, Functions, Domain, and Range*

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There is a special notation called **functional notation** that is frequently used in mathematics when one variable is described in terms of another. The notation **f(X)** [read "f of X"] is often used to name a second variable. Instead of writing "Y = 3X + 2" you sometimes write "f(X) = 3X + 2" or "g(X) = 3X + 2" or perhaps even "Y(X) = 3X + 2". Any letter may be used. This notation indicates that "f" or "g" or "Y" is a function of "X", or that it can be expressed in terms of "X". To find the value of f(2), just replace each X with the value 2. To find the value of f(4), replace each X with the value 4. To find the value of f(-3), replace each X with the value -3.

Complete the following exercises:

1. f(X) = 3X + 2

a) f(0) = 3( ) + 2
= __________

b) f(2) =
=

c) f(4) =
=

d) f(-3) =
=

e) f($) = 3( ) + 2

f) f(*) = __________

g) f(###) = __________

h) f(Junk) = __________

2. g(X) = -3X + 5

a) g(0) = -3( ) + 5
= __________

b) g(2) =
=

c) g(4) =
=

d) g(-3) =
=

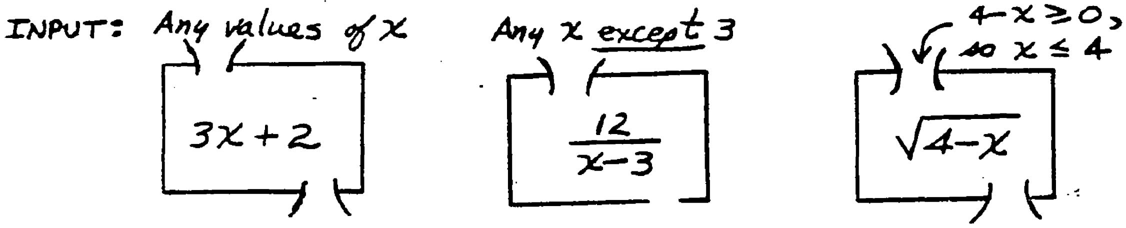
e) g($) = __________

f) g(*) = __________

g) g(###) = __________

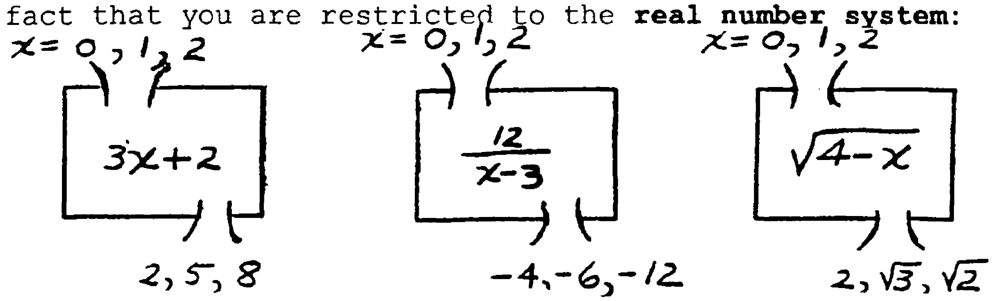
h) g(Junk) = __________

This concept can further be explained by use of **"function machines."** Consider the following examples (for this section, use only **real numbers**):



Each of these machines consists of an "input spout", a formula, and an "output spout". Whenever a number is dropped into the input spout the number is substituted in place of the X in the formula. The result becomes the output value. These machines, therefore, have three parts: input, calculation (formula), and output.

When operating any machine, it is nice to know what kinds of fuel will run the machine. Even more important, what kinds of fuel will the machine **not** operate the machine? Recall the machines and the fact that you are restricted to the **real number system**:



Notice that the first machine **"3X + 2"** will run on <u>anything</u>. The second machine $\frac{12}{X - 3}$ will run on <u>almost anything</u>--the value of X=3 is <u>not</u> allowed, since X=3 would require **division by zero**, which is not allowed. What values of X would be acceptable input for the third machine $\sqrt{4 - X}$ (assume you are dealing with **real numbers**)?

In this third machine, the values such as X=1, X=2, X=3, X=4 are all acceptable. However, if X>4, the result is a negative radicand, which is not acceptable in the real number system. These illustrate the **two requirements of the real number system:**

I. **Denominators cannot equal zero,**

II. **Radicands of <u>even</u> radicals must be greater than or equal to <u>zero</u>.**

Instead of drawing "function machines", it is usually more convenient to use $\mathbf{f(X) = 3X + 2}$, $\mathbf{g(X) = \frac{12}{X - 3}}$, or $\mathbf{h(X) = \sqrt{4 - X}}$.

Frequently the notation $\mathbf{Y = 3X + 2}$, $\mathbf{Y = \frac{12}{X - 3}}$, or $\mathbf{Y = \sqrt{4 - X}}$ is used. Any such equation that shows a **relationship** between two variables (such as X and Y) is defined to be a **relation.** When for each value of X (input) there is a unique value of Y (output), the equation is defined to be a **function.** In other words, a **function** is a **relation** that has a special **uniqueness property.** Furthermore, the set of all possible (permissible) **X values (input)** is defined to be the **domain.** The set of all possible **Y values (output)** is defined to be the **range.**

DOMAIN

To find the **domain,** you must begin with the function (or equation) in the form **"Y = _____."** If the equation is not in this form, then you must first solve for **Y** so as to have this form. Then, remember that there are two operations that are <u>not</u> permitted in the real number system:

I. **DENOMINATORS cannot equal zero;**

II. **RADICANDS of <u>even</u> radicals must be greater than or equal to <u>zero</u>.**

Based upon this, there are four main categories of domain problems to be solved in this course:

I. If there is a **denominator,** then: **Denominator $\neq$ 0.**

II. If there is an **<u>even</u> radical,** then: Set **radicand $\geq$ 0.**

III. If there is an **<u>even</u> radical/denominator:** Set **radicand > 0.**

IV. *<u>Usually</u>*, (for now, at least!) if there are **no denominators and <u>no</u> radicals** (this is the easiest case of all), then there are **no restrictions,** and the **domain is all real values.**

Here are the four categories of domain problems with examples of each type.

I. If there is a **denominator**, then: **Denominator ≠ 0.**

EXAMPLE: $Y = \frac{X - 2}{X^2 - 9}$. **Denominator:** $X^2-9 \neq 0$

$(X-3)(X+3) \neq 0$

$X \neq 3$ and $X \neq -3$

D: All X except $X = \pm 3$.

Notice that the numerator "X-2" is irrelevant to the problem!

II. If there is an **even radical**, then: Set **radicand ≥ 0.**

EXAMPLE: $Y = \sqrt{X - 4}$. **Radicand:** $X - 4 \geq 0$

$X \geq 4$

D: $[4, \infty)$

III. If there is an **even radical-denominator:** Set **radicand > 0.**

EXAMPLE: $Y = \frac{X - 2}{\sqrt{X - 4}}$. **Radicand:** $X - 4 > 0$

$X > 4$

D: $(4, \infty)$

Notice that the numerator factor "X-2" is not relevant!

IV. If there are **no denominators and no radicals**, then there are *usually* **no restrictions**, and the **domain is all real values.**

EXAMPLES: $Y = 4$ **D: All reals** or $(-\infty, \infty)$

$Y = 3X - 6$ **D: All reals** or $(-\infty, \infty)$

$Y = X^2 + 3X - 4$ **D: All reals** or $(-\infty, \infty)$.

EXERCISES. Find the domain for each of the following functions.

1. $Y = \frac{12}{X + 3}$ 2. $Y = \frac{2X}{X + 4}$ 3. $Y = \frac{X - 2}{X^2 - 16}$ 4. $Y = \frac{X + 3}{X^2 - 2X - 8}$

Denom ≠ 0

_____ ≠ 0

D: X ≠ _____

5. $Y = \sqrt{X + 6}$ 6. $Y = \sqrt{X - 6}$ 7. $Y = \sqrt{6 - X}$ 8. $Y = \sqrt{2X + 5}$

D: $X + 6 \geq 0$

$X \geq$ ___

D: [,)

9. $Y = \dfrac{X}{\sqrt{X + 6}}$ 10. $Y = \dfrac{X + 6}{\sqrt{X - 6}}$ 11. $Y = \dfrac{X - 2}{\sqrt{6 - X}}$ 12. $Y = \dfrac{3X - 12}{\sqrt{2X + 5}}$

13. $Y = X - 2$ 14. $Y = 3X + 12$ 15. $Y = X^2 - 4$ 16. $Y = 2$

17. $Y = \dfrac{X - 6}{X + 6}$ 18. $Y = \dfrac{X + 1}{\sqrt{2X - 6}}$ 19. $Y = 6 - X$ 20. $Y = \sqrt{12 - 2X}$

21. $Y = \sqrt{2X - 6}$ 22. $Y = \frac{x - 3}{\sqrt{2X + 16}}$ 23. $Y = \frac{X + 4}{X^2 - 4X}$ 24. $Y = 2X + 5$

25. $Y = \frac{X - 6}{X^2 + 9}$ 26. $Y = \frac{X}{X^2 - 16}$ 27. $Y = \frac{X - 3}{\sqrt{2 + X}}$ 28. $Y = \frac{2X + 5}{X^2-4X-12}$

Denom $\neq 0$

$X^2 + 9 \neq 0$

True for all real values of X.

D: (,)

29. $Y = \frac{4 + x}{X^2 - 64}$ 30. $Y = \frac{X^2 - 6X}{\sqrt{4 - X}}$ 31. $Y = \frac{X + 4}{X^2 + 25}$ 32. $Y = \frac{X^2 - 16}{X^2 - 25}$

33. $Y = \frac{X - 6}{\sqrt{X}}$ 34. $Y = X^2 - 16$ 35. $Y = \sqrt{6 + 9X}$ 36. $Y = \frac{X^2-6X+5}{X}$

In 37–56, sketch the graph with the graphing calculator, from which you can find the domain and range. Do you remember "quadratic inequalities," "BETWEENNESS," and "EXTREMES"?

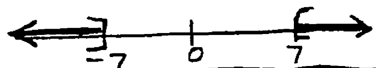
37. $Y = \sqrt{X^2-16}$

38. $Y = \sqrt{X^2 - 49}$

D: $x^2-49 \geq 0$ EXTREMES.

$(x-7)(x+7)=0$ Endpts.

$x=7,\ x=-7$



(,] ∪ [,)

39. $Y = \sqrt{16 - X^2}$

40. $Y = \sqrt{49 - X^2}$

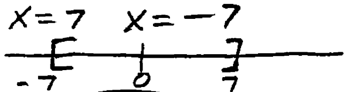
D: $49-x^2 \geq 0$

Divide each side by (-1)

$x^2-49 \leq 0$ BETWEEN

$(x-7)(x+7)=0$ Endpts.

$x=7 \quad x=-7$



[,]

41. $Y = \dfrac{X - 9}{\sqrt{X^2-25}}$

42. $Y = \dfrac{3X}{\sqrt{X^2-49}}$

43. $Y = \dfrac{3X}{\sqrt{49-X^2}}$

44. $Y = \dfrac{X - 2}{\sqrt{9-X^2}}$

45. $Y = \dfrac{X - 9}{\sqrt{X^2+25}}$

46. $Y = \dfrac{3X}{\sqrt{X^2 + 49}}$

47. $Y = \dfrac{3X}{\sqrt{9 - X}}$

48. $Y = \sqrt{X - 4}$

49. $Y = X^2 - 9$

50. $Y = X^2 + 9$

51. $Y = \dfrac{3X}{49 - X^2}$

52. $Y = \dfrac{X - 2}{\sqrt{9 + X^2}}$

53. $Y = \sqrt{X^2 - 5X - 6}$

54. $Y = \dfrac{X - 7}{\sqrt{X^2 - 5X - 6}}$

55. $Y = \sqrt{6 + 5X - X^2}$

56. $Y = \dfrac{3X}{X^2 - 2X - 15}$

57. $Y = X^2 - 2X - 15$

58. $Y = \sqrt{X^2 - 2X - 15}$

59. $Y = \dfrac{3X}{\sqrt{15 + 2X - X^2}}$

60. $Y = \dfrac{3X}{\sqrt{X^2 - 2X - 15}}$

Frequently the equation of the function is not given in the form **Y =** ______. In such cases, in order to find the **domain**, it is necessary to solve the equation for **Y**, as in **literal equations**, so as to write the equation in the form **Y =** ______.

61. $2X + 3Y = 6$

$3Y = -2X + 6$

$Y =$

No variables in denom,
no radicals. Therefore,
D: (,)

62. $7X + 4Y = 16$

63. $7X + XY = 14$

$XY =$ __________

$Y =$

Denominator $\neq 0$

D: ______________

64. $XY - 3X = 2$

65. $Y(X^2 - 4) = 12X$

Divide both sides by (X^2-4)

$Y =$ __________

D: ______________

66. $Y(X^2 - 25) = 12X$

67. $Y(X^2 + 4) = 12X$

68. $4X - 3Y = 12$

69. $XY - 5X = 10$

70. $5X - 5XY = 4$

71. **$XY - 5Y = X^2$**
(First factor the Y!)

72. **$XY + 5Y = X^2$**

73. **$X^2Y - 4Y = 6X$**

74. **$X^2Y - 16Y = 6X$**

75. **$X^2Y + 4Y = 6X$**

76. **$X^2Y + 16Y = 6X$**

77. **$XY = 2X - 4Y$**

$XY + 4Y = 2X$ (Get all "Y" terms on one side; get all "non-Y" terms on the other side.)

$Y(X + 4) = 2X$ (Factor out the "Y" common factor! Note: This is the key step--don't forget!)

$$Y = \frac{2X}{X + 4}$$

Denom $\neq 0$

______ $\neq 0$

D: $X \neq$ ______ (See #2)

78. **$XY = 3X + 5Y$**

79. **$XY - 3 = 3X - 6Y$**

80. $3XY + 4X = 6Y - 8$

81. $X^2Y = 9Y + 6X$

82. $X^2Y = 4XY + 6$

83. $X^2Y = 5XY - 6Y + 4X$

84. $X^2Y = 4XY + 12Y + 4X$

85. $X^2Y = 4XY - 6X + 5Y$

86. $Y^2 = 4$

$Y = \pm$ ____

D: ______________

87. $Y^2 = X$

$Y = \pm$ ____

____ ≥ 0

D: [,)

88. $Y^2 = X - 4$

D: [,)

89. $Y^2 = 4 - X$

90. $Y^2 = 4 + X$

91. $Y^2 = X^2 - 4$

92. $Y^2 = X^2 - 25$

93. $Y^2 = X^2 - 4X$

94. $Y^2 = X^2 + 4X$

95. $Y^2 = X^2 - 4X - 12$

96. $Y^2 = X^2 + 4X - 12$

97. $X^2Y^2 = 4Y^2 + 9X^2$

98. $X^2Y^2 = 4X^2 + 9Y^2$

99. $XY^2 = 4X^2 + 9Y^2$

100. $X^2Y^2 = 4X^2 - 9Y^2$

RANGE

Remember that the **domain** is the set of all possible (permissible) **X values (input)** , and the **range** is the set of all possible **Y values (output)**. To find the domain, it was necessary to **solve for Y in terms of X** in order to see what values of X would be allowed. Now, to find the range you must reverse the process. **To find the range, you must solve for X in terms of Y** in order to see what values of Y would be allowed.

EXERCISES: In each of the following exercises, find the range:

1. $2X + 3Y = 6$

   $2X = -3Y + 6$

   $X =$

   No variables in denom,
   no radicals. Therefore,
   R: (,)

2. $7X + 4Y = 16$

3. $XY + 7Y = 14$

   $XY =$ _________

   $X =$

   Denominator ≠ 0

   R: _____________

4. $XY - 3Y = 2$

5. $X(Y^2 - 4) = 12Y$

   Divide both sides by (Y^2-4)

   $X =$ _________

   R: _____________

6. $X(Y^2 - 25) = 12Y$

7. $XY - 5X = Y^2$

   (First factor the X!)

8. $XY + 5X = Y^2$

9. $XY - 4Y = 6X$

10. $XY - 16Y = -6X$

11. $XY^2 - 4X = 6Y$

12. $XY^2 - 16Y = 9X$

13. $XY^2 = 9Y + 16X$

14. $XY^2 = 4XY + 6$

15. $XY^2 = 5XY - 6X + 4Y$

16. $XY^2 = 4XY + 12X + 4Y$

17. $X^2 = 4$

$X = \pm$ ____

R: ________________

18. $X^2 = Y$

$X = \pm$ ______

______ ≥ 0

R: [,)

19. $X^2 = Y - 4$

R: [,)

20. $X^2 = 4 - Y$

21. $X^2 = -4 - Y$

22. $X^2 = 4 + Y$

23. $Y = X^2 - 16$

24. $Y = X^2 + 25$

25. $Y = -X^2 + 16$

26. $Y = -X^2 - 9$

27. $X^2 = Y^2 - 4$

28. $X^2 = Y^2 - 25$

29. $X^2 = Y^2 - 4Y$

30. $X^2 = Y^2 + 4Y$

31. $X^2 = Y^2 - 4Y - 12$

32. $X^2 = Y^2 + 4Y - 12$

33. $X^2Y^2 = 4Y^2 + 9X^2$

34. $X^2Y^2 = 4X^2 + 9Y^2$

When the equation involves **radical**, it is often necessary to square both sides of the equation in order to solve for X. However, **you must remember that squaring both sides of an equation does not guarantee equivalent equations.** For example, if you begin with the equation $\mathbf{Y = \sqrt{X}}$ **(See Fig. 1)**, and square both sides of the equation, you get the new equation $\mathbf{Y^2 = X}$ **(See Fig. 3)**, which is not the same. In the original equation $\mathbf{Y = \sqrt{X}}$, Y must be greater than or equal to 0. However, in the new equation $\mathbf{Y^2 = X}$, there is no restriction on the value of Y. So $\mathbf{Y = \sqrt{X}}$ and $\mathbf{Y^2 = X}$ are not the equivalent equations. As shown in the figures,

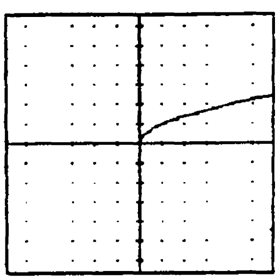


Fig. 1: $Y = \sqrt{X}$

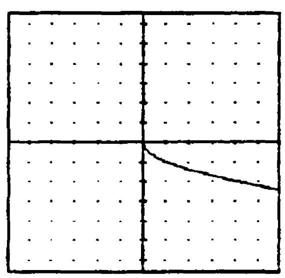


Fig. 2: $Y = -\sqrt{X}$

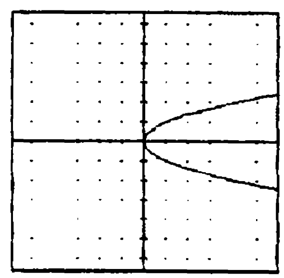


Fig. 3: $Y^2 = X$

the graph of $\mathbf{Y = \sqrt{X}}$ is the upper half of a parabola, $\mathbf{Y = -\sqrt{X}}$ is the lower half of the same parabola, and $\mathbf{Y^2 = X}$ is both halves of the parabola. Actually, $\mathbf{Y^2 = X}$ is equivalent to $\mathbf{Y = \pm\sqrt{X}}$. **In the process of finding the range, when radicals are involved, it is therefore necessary to notice restrictions on Y before squaring both sides, as demonstrated by the next exercise.**

In the following exercises, find the range. [NOTE: Try finding the range by sketching the graph with the graphing calculator!]

35. $Y = \sqrt{X + 6}$ Before squaring both sides, note restriction **$Y \geq 0$**,

(since Y equals a **positive square root**!)

$Y^2 = X + 6$ Square both sides, in order to solve for X.

$X = Y^2 - 6$ Subtract 6 from both sides.

Since there are no radicals and no denominators, it appears that there are no restrictions. However, remember that **$Y \geq 0$**.

R: [,)

36. $Y = \sqrt{X - 6}$ Before squaring both sides, note restriction _____.

37. $Y = -\sqrt{6 - X}$ Before squaring, note restriction **$Y \leq 0$**,

(since Y equals a **negative square root**!)

$Y^2 = 6 - X$ Square both sides, in order to solve for X.

$X = 6 - Y^2$ Add X and subtract Y^2 from both sides.

Since there are no radicals and no denominators, it appears that there are no restrictions. However, remember that $Y \leq 0$.

R: (,]

38. $Y = -\sqrt{X + 5}$

By now, perhaps you are tempted to conclude that if Y equals a positive square root, then the range is $[0, \infty)$ and that if Y equals a negative square root, then the range is $(-\infty, 0]$. Unfortunately, this is not always the case, since there may be additional restrictions, as the next exercises illustrate.

In each of the following, find the range. [NOTE: Try finding the range by sketching the graph with the graphing calculator!]

39. $Y = \sqrt{9 - X^2}$ Before squaring both sides, note restriction **Y≥0.**

$Y^2 = 9 - X^2$ Square both sides.

$X^2 = 9 - Y^2$ Add X^2 and subtract Y^2 from both sides.

$X = \pm\sqrt{9 - Y^2}$ Take square root of both sides to solve for X.

R: $9 - Y^2 \ge 0$

$Y^2 - 9 \le 0$ Divide both sides of inequality by (-1).

BETWEENNESS, with endpoints at Y = ± 3.

R: [-3, 3], with additional restriction that **Y≥0.**

R: [,]

40. $Y = \sqrt{16 - X^2}$

41. $Y = \sqrt{X^2 - 16}$

42. $Y = \sqrt{X^2 - 25}$

43. $Y = \sqrt{X^2 + 25}$

44. $Y = -\sqrt{9 - X^2}$

45. $Y = -\sqrt{16 - X^2}$

46. $Y = -\sqrt{X^2 - 16}$

47. $Y = -\sqrt{X^2 - 25}$

48. $Y = -\sqrt{X^2 + 25}$

49. $Y = -\sqrt{X^2 + 16}$

FUNCTIONS and RELATIONS

Each of the equations in the previous domain and range exercises represents a **relation.** In other words, in each equation there was a **relationship** between the variables **X** and **Y.** When for each value of X, there is a unique value of Y, the equation is defined to be a **function.** In other words, a **function** is a **relation** that has a special **uniqueness property.** What makes an equation not a function is the possibility of having two values of Y for a given X value. Therefore, whenever there is a **"Y = ± ____"** or a **"Y^2"** or **Y** raised to any **even power** involved in the equation, it is usually not a function. On the other hand, if the equation may be solved for Y and written in the form **"Y = ____"**, as in **functional notation "f(X) = ____"**, then it is a **function!**

One additional thought before the exercises, consider inequalities, such as **Y < X + 2.** Remember that, when graphed, inequalities, result in a **shaded area.** In any shaded area, for any given X value, there will be more than one, unique Y value. It is safe to conclude that shaded areas, and therefore inequalities, do not represent functions.

EXERCISES: Determine which of the following represent functions.

1. $Y = X^2$
2. $Y = X^2 + 4$
3. $X = Y^2 - 4$
4. $X = Y^2$
5. $Y = \sqrt{X}$
6. $Y = \sqrt{X - 6}$
7. $Y = \sqrt[3]{X}$
8. $Y = \sqrt[3]{X + 5}$
9. $Y = \pm\sqrt{X}$
10. $Y = \pm\sqrt[3]{X - 6}$
11. $Y = \pm\sqrt{X + 5}$
12. $Y = \sqrt[4]{X}$

13. $X^2 + Y^2 = 9$ 14. $X^2 - Y^2 = 9$ 15. $X^2 + Y = 9$

16. $X^3 + Y^3 = 9$ 17. $X^3 - Y^3 = 9$ 18. $X^2 - Y = 9$

19. $Y = 3X + 5$ 20. $4X - 12Y = 12$ 21. $2X + 3Y = 5$

22. $Y = -X - 3$ 23. $2X + Y^2 = 9$ 24. $2X^3 + Y^2 = 12$

25. $Y = 6$ 26. $Y = \pm 6$ 27. $X = 6$ 28. $X = -3$

29. $Y = \pm 3$ 30. $Y = -4$ 31. $Y = X^3 + 6X$

32. $Y = \frac{X^2 - 6X}{\sqrt{4 - X}}$ 33. $Y = \frac{X + 4}{X^2 + 25}$ 34. $Y = \frac{X^2 - 16}{X^2 - 25}$

35. $Y < X + 2$ 36. $3X - Y > 6$ 37. $3X - Y \geq 6$ 38. $Y \geq 3X$

FUNCTIONS, DOMAIN, RANGE

(SUBTITLE: GETTING IT ALL TOGETHER)

EXERCISES: **For each of the following, find the domain and range. Identify which are functions and which are not.**

1. $Y = X^2$

2. $X = Y^2$

3. $Y = XY + 4$

4. $4X = XY + 8$

5. $Y = 3X - 6$

6. $3X - 5Y = 10$

7. $Y = X^2 - 9$

8. $Y = X^2 + 4$

9. $X = Y^2 + 9$

10. $X = Y^2 - 4$

11. $Y = 9 - X^2$

12. $Y = 16 - X^2$

13. $X = 4 - Y^2$

14. $X = -Y^2 - 4$

[HINT: In 15-20, use the G.C. method--sketch using either the "Graphing Calculator" or recognize the "Graph of a Circle."]

15. $X^2 + Y^2 = 9$

16. $X^2 - Y^2 = 9$

17. $X^2 - Y^2 = -9$

18. $X^2 + Y^2 = 16$

19. $X^2 - Y^2 = 16$

20. $X^2 - Y^2 = -16$

21. $Y = \frac{12}{X + 3}$

22. $Y = \frac{2X}{X + 4}$

23. $Y = \frac{2X}{X - 3}$

24. $Y = \frac{6}{X - 4}$

25. $XY - 10X = 5$

26. $XY - 10Y = 5$

27. $XY = 2X - 3Y + 8$

28. $X = XY - 3Y + 6$

29. $X^2Y^2 = 4X^2 - 9Y^2$

30. $X^2Y^2 = 4Y^2 - 9X^2$

In 31-36, use the graphing calculator to sketch the graph. Then identify if it is a function and find domain and range.

31. $Y = \sqrt{16 - X^2}$

32. $Y = \sqrt{X^2 - 16}$

33. $Y = \pm \sqrt{X^2 - 25}$

34. $Y = \pm \sqrt{X^2 + 25}$

35. $Y = - \sqrt{9 - X}$

36. $Y = - \sqrt{X - 4}$

FUNCTIONS, DOMAIN, and RANGE
BY GRAPHING

Until now, the concepts of domain, range, and functions were presented from the perspective of **X,Y equations.** That is, given an equation in X and Y, you could determine the **domain** (the **set of all permissible X values**), the **range** (the **set of all resulting Y values**), and you could determine if it is a **function** (if **each Y value has a unique X value**). From the **graph** of the equation, it is also possible (even easier!) to determine the domain and range, and to determine if the equation represents a function. Consider the following two examples:

FUNCTION: $Y = X^2$

For any value of X, there is a unique Y value.

$X = 0 \rightarrow Y = 0^2 = 0$

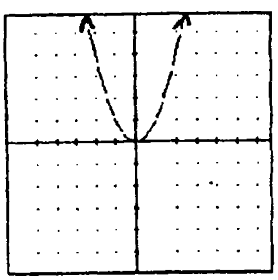
$X = 1 \rightarrow Y = 1^2 = 1$

$X = 2 \rightarrow Y = 2^2 = 4$

$X = 3 \rightarrow Y = 3^2 = 9$

$X = -1 \rightarrow Y = (-1)^2 = 1$

$X = -2 \rightarrow Y = (-2)^2 = 4$



From the graph, if no vertical line crosses the graph in more than one point, then **it is a function!**

NOT FUNCTION: $X = Y^2$

If you pick values of X, say

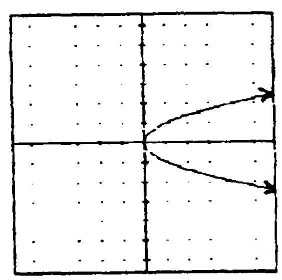
If $X = 0 \rightarrow Y^2 = 0$, so $Y = 0$

$X = 1 \rightarrow Y^2 = 1$, so $Y = \pm 1$

$X = 4 \rightarrow Y^2 = 4$, so $Y = \pm 2$

$X = 9 \rightarrow Y^2 = 9$, so $Y = \pm 3$

(Since $X=Y^2$, X cannot be negative)



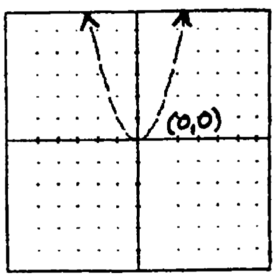
If any vertical line crosses the graph in **two or more points**, then **it is NOT a function!**

DEFINITION: A function is a set of points such that no two distinct points have the same X-coordinate.

RULE: If any vertical line crosses a graph in two or more points, then it is NOT a function!

Study the following examples to understand how to determine: **a) if the graph represents a function,** and how the **b) domain** and **c) range** can be determined from the graph.

EXAMPLE 1: $Y = X^2$

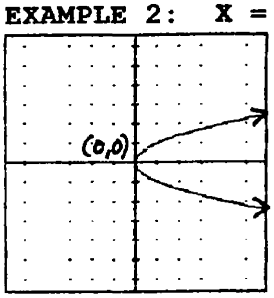


a) Function? Since no vertical line crosses the graph more than once, **it IS a function.**

b) Domain: Values of X extend all the way to the right and all the way to the left. Consider the equation $\mathbf{Y=X^2}$. Any value of X is permissible.
D: all reals or **$(-\infty, \infty)$**

c) Range: Values of Y are all on or above the X-axis. Considering the equation $\mathbf{Y=X^2}$, Y values are obtained from the squares of the X-values. Therefore,
R: $Y \geq 0$ or **$[0, \infty)$**

EXAMPLE 2: $X = Y^2$

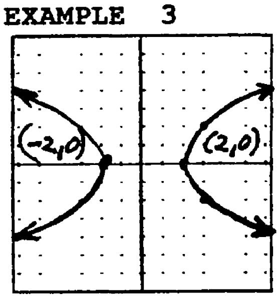


a) Function? **NOT a function,** since a vertical line crosses the graph in two points.

b) Domain: Values of X are on or to the right of the Y-axis.
D: $X \geq 0$ or **$[0, \infty)$**

c) Range: Values of Y extend all the way up and all the way down. Therefore, there are no restrictions on Y.
R: all reals or **$(-\infty, \infty)$**

EXAMPLE 3



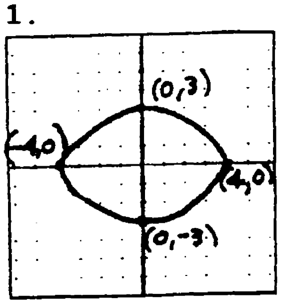
a) Function? **NOT a function,** by vertical line test.

b) Domain: X-values extend from X=2 to the right and from X=-2 to the left. No values of the graph are between -2 and 2.
D: $X \geq 2$ or $X \leq -2$, or
$(-\infty, -2] \cup [2, \infty)$

c) Range: Y-values go all the way up and all the way down. Therefore, no restrictions on Y.
R: all reals or **$(-\infty, \infty)$**

In the following exercises, from the graphs determine:
a) if a function b) Domain c) Range.

1.



a) Function? Can you draw a vertical line that crosses the graph twice? ______.

Circle one: Then it **(is, is not)** a function.

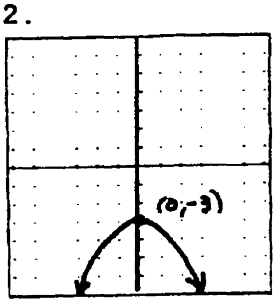
b) Domain: X values extend from ____ to ____.

D: Int. notation ______________.

c) Range: Y extends from _____ to _____.

R: Int. notation ______________.

2.



a) Function? Can you draw a vertical line that crosses the graph twice? ______.

Circle one: Then it **(is, is not)** a function.

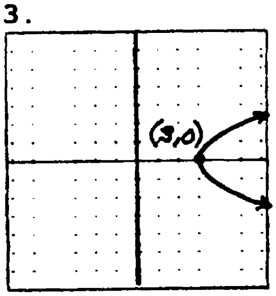
b) Domain: X values extend from ____ to ____.

D: Int. notation ______________.

c) Range: Y values are all at or below ____.

R: Int. notation **(- ∞,** ____**]**.

3.



a) F? ______

b) D: ________

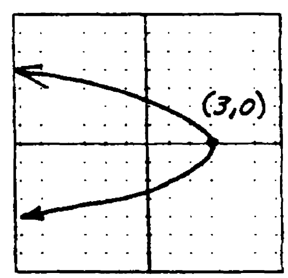
c) R: ________

4.

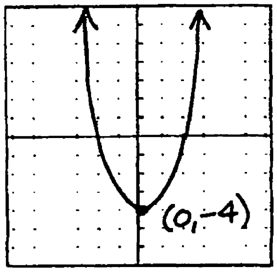
a) F? ______

b) D: ________

c) R: ________



5.



a) F? ______

b) D: ______

c) R: ______

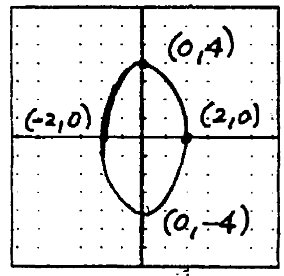
NOTE: Graph extends upward and right and left without bound!

6.

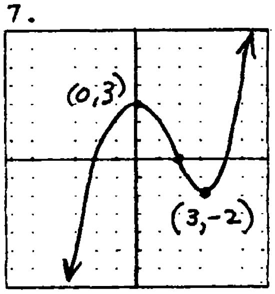
a) F? ______

b) D: ______

c) R: ______



7.



a) F? ______

b) D: ______

c) R: ______

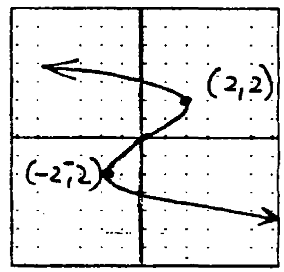
NOTE: Graph extends upward + right, downward + left, without bound.

8.

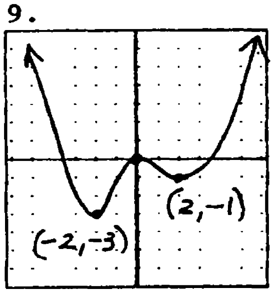
a) F? ______

b) D: ______

c) R: ______



9.



a) F? ______

b) D: ______

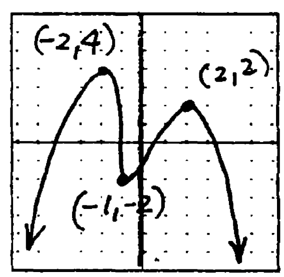
c) R: ______

10.

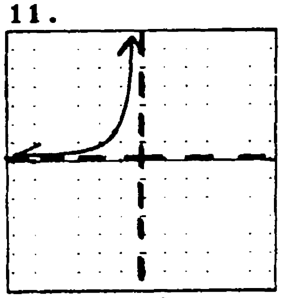
a) F? ______

b) D: ______

c) R: ______



11.



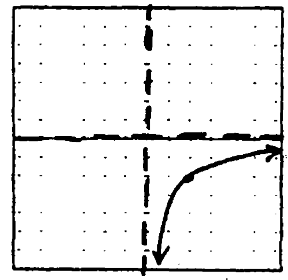
a) F? ______

b) D: (,)

c) R: (,)

Note: This graph never touches the x or y axis.

12.

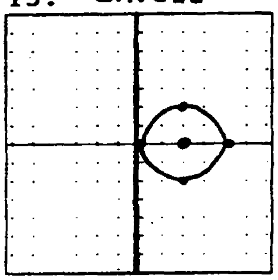


a) F? ______

b) D: ______

c) R: ______

13. *CIRCLE*



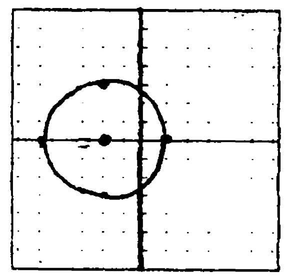
C= (2,0) r=2

a) F? ______

b) D: ______

c) R: ______

14. *CIRCLE*



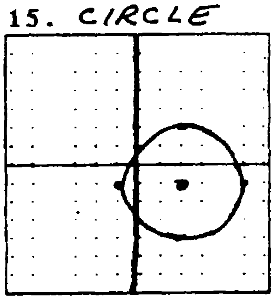
C = (-2,0) r = 3

a) F? ______

b) D: ______

c) R: ______

15. *CIRCLE*



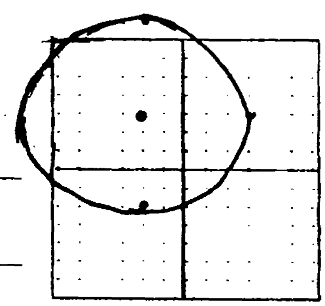
C=(2,-1) r=3

a) F? ______

b) D: ______

c) R: ______

16. *CIRCLE*



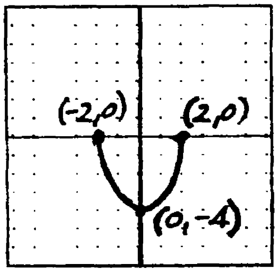
C(-2,3) r=5

a) F? ______

b) D: ______

c) R: ______

17.



NOTE: This graph stops!

a) F? _______

b) D: _________

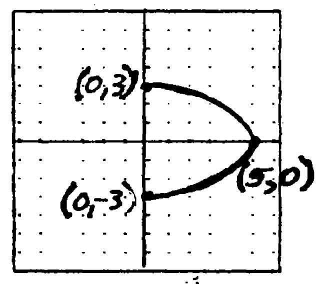
c) R: _________

18.

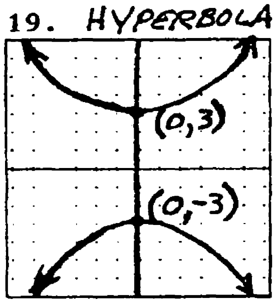
a) F? _______

b) D: _________

c) R: _________



19. HYPERBOLA



a) F? _______

b) D: _________

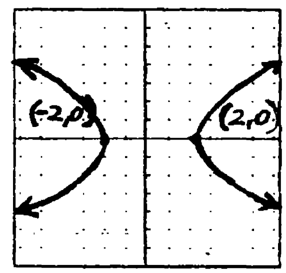
c) R: _________

20. HYPERBOLA

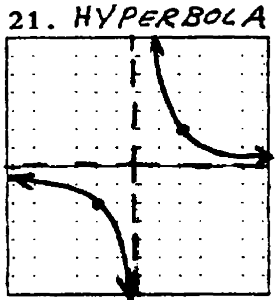
a) F? _______

b) D: _________

c) R: _________



21. HYPERBOLA



Never touches
x or y axis!

a) F? _______

b) D: _________

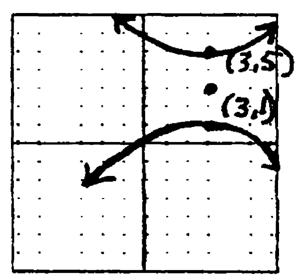
c) R: _________

22. HYPERBOLA

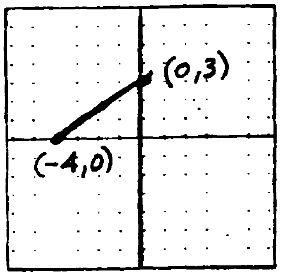
a) F? _______

b) D: _________

c) R: _________



23. LINE SEGMENT



a) F? ________

b) D: ________

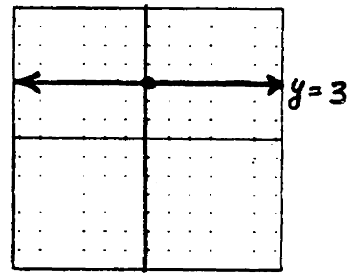
c) R: ________

24. HORIZONTAL LINE

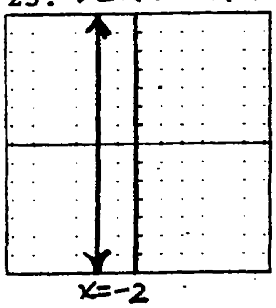
a) F? ________

b) D: ________

c) R: ________



25. VERTICAL LINE



a) F? ________

b) D: ________

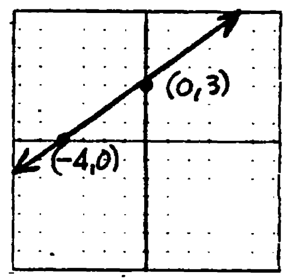
c) R: ________

26. LINE

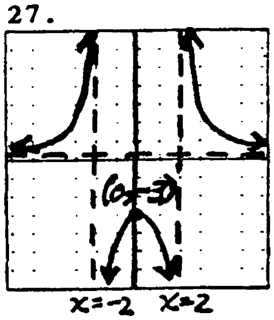
a) F? ________

b) D: ________

c) R: ________



27.



a) F? ________

b) D: ________

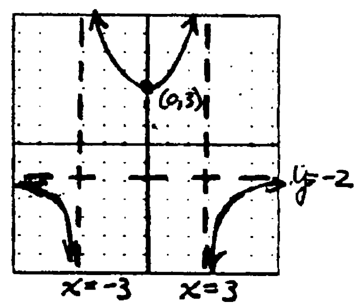
c) R: ________

28.

a) F? ________

b) D: ________

c) R: ________



NOTE: Graph never touches dotted lines (or asymptotes!)

ANSWERS 2.07

p. 262: 1a) 2; b) 8; c) 14; d) -7; e) 3$+2; f) 3*+2; g) 3(###)+2 h) 3(Junk)+2. 2a) 5; b) -1; c) -7; d) 14; e) -3$+5; f) -3*+5; g) -3(###)+5; h) -3(Junk)+5.

p. 265-273:

1. All X≠-3; **2.** All X≠-4; **3.** All X≠±4; **4.** All X≠-2,4
5. [-6,∞); **6.** [6,∞); **7.** (-∞,6]; **8.** [-2.5,∞); **9.** (-6,∞)
10. (6,∞); **11.** (-∞,6); **12.** (-2.5,∞); **13.** (-∞,∞);
14. (-∞,∞); **15.** (-∞,∞); **16.** (-∞,∞); **17.** All X≠-6;
18. (3,∞); **19.** (-∞,∞); **20.** (-∞, 6]; **21.** [3,∞) **22.** (-8,∞)
23. All X≠0,4; **24.** (-∞,∞); **25.** (-∞,∞); **26.** All X≠±4;
27. (-2,∞); **28.** All X≠-2,6; **29.** All X≠±8; **30.** (-∞,4);
31. (-∞,∞); **32.** All X≠±5; **33.** (0,∞); **34.** (-∞,∞);
35. [-2/3,∞); **36.** All X≠0; **37.** (-∞,-4]∪[4,∞);
38. (-∞,-7]∪[7,∞); **39.** [-4,4]; **40.** [-7,7];
41. (-∞,-5)∪(5,∞); **42.** (-∞,-7)∪(7,∞); **43.** (-7,7);
44. (-3,3); **45.** (-∞,∞); **46.** (-∞,∞); **47.** (-∞,9);
48. [4,∞); **49.** (-∞,∞); **50.** (-∞,∞); **51.** All X≠±7;
52. (-∞,∞); **53.** (-∞,-1]∪[6,∞); **54.** (-∞,-1)∪(6,∞);
55. [-1,6]; **56.** All X≠-3,5; **57.** (-∞,∞); **58.** (-∞,-3]∪[5,∞);
59. (-3,5); **60.** (-∞,-3)∪(5,∞); **61.** (-∞,∞); **62.** (-∞,∞);
63. All X≠0; **64.** All X≠0; **65.** All X≠±2; **66.** All X≠±5;
67. (-∞,∞); **68.** (-∞,∞); **69.** All X≠0; **70.** All X≠0;
71. All X≠5; **72.** All X≠-5; **73.** All X≠±2; **74.** All X≠±4;
75. (-∞,∞); **76.** (-∞,∞); **77.** All X≠-4; **78.** All X≠5;
79. All X≠-6; **80.** All X≠2; **81.** All X≠±3; **82.** All X≠0,4;
83. All X≠2,3; **84.** All X≠-2,6; **85.** All X≠-1,5; **86.** (-∞,∞);
87. [0,∞); **88.** [4,∞); **89.** (-∞,4]; **90.** [-4,∞);
91. (-∞,-2]∪[2,∞); **92.** (-∞,-5]∪[5,∞); **93.** (-∞,0]∪[4,∞);
94. (-∞,-4]∪[0,∞); **95.** (-∞,-2]∪[6,∞); **96.** (-∞,-6]∪[2,∞);
97. (-∞,-2)∪(2,∞); **98.** (-∞,-3)∪(3,∞); **99.** (9,∞);
100. (-∞,∞).

p.274-280:

1. (-∞,∞); **2.** (-∞,∞); **3.** All Y≠0; **4.** All Y≠0;
5. All Y≠±2; **6.** All Y≠±5; **7.** All Y≠5; **8.** All Y≠-5;
9. All Y≠6; **10.** All Y≠-6; **11.** All Y≠±2; **12.** All Y≠±3;
13. All Y≠±4; **14.** All Y≠0,4; **15.** All Y≠2,3; **16.** All Y≠-2,6;
17. (-∞,∞); **18.** [0,∞); **19.** [4,∞); **20.** (-∞,4]; **21.** (-∞,-4];
22. [-4,∞); **23.** [-16,∞); **24.** [25,∞); **25.** (-∞,16];
26. (-∞,-9]; **27.** (-∞,-2]∪[2,∞); **28.** (-∞,-5]∪[5,∞);
29. (-∞,0]∪[4,∞); **30.** (-∞,-4]∪[0,∞); **31.** (-∞,-2]∪[6,∞);
32. (-∞,-6]∪[2,∞); **33.** (-∞,-3)∪(3,∞); **34.** (-∞,-2)∪(2,∞);
35. [0,∞); **36.** [0,∞); **37.** (-∞,0]; **38.** (-∞,0]; **39.** [0,3];
40. [0,4]; **41.** [0,∞); **42.** [0,∞); **43.** [5,∞); **44.** [-3,0];
45. [-4,0]; **46.** (-∞,0]; **47.** (-∞,0]; **48.** (-∞,-5];
49. (-∞,-4].

p.281-282:

1.F; 2.F; 3.NF; 4.NF; 5.F; 6.F; 7.F; 8.F; 9.NF; 10.NF;
11.NF; 12.F; 13.NF; 14.NF ; 15.F; 16.F; 17.F; 18.F; 19.F;
20.F; 21.F; 22.F; 23.NF; 24.NF; 25.F; 26.NF; 27.NF;
28.NF; 29.NF; 30.F; 31.F; 32.F; 33.F; 34.F; 35.NF; 36.NF;
37.NF; 38.NF.

p.283-288:

1. D:(-∞,∞); R:[0,∞); F; 2. D:[0,∞); R:(-∞,∞); NF;
3. D:All X≠1; R:All Y≠0; F; 4. D:All X≠0; R:All Y≠4; F;
5. D:(-∞,∞); R:(-∞,∞); F; 6. D:(-∞,∞); R:(-∞,∞); F;
7. D:(-∞,∞); R:[-9,∞); F; 8. D:(-∞,∞); R:[4,∞); F;
9. D:[9,∞); R:(-∞,∞); NF; 10. D:[-4,∞); R:(-∞,∞); NF;
11. D:(-∞,∞); R:(-∞,9]; F; 12. D:(-∞,∞); R:(-∞,16]; F;
13. D:(-∞,4]; R:(-∞,∞); NF; 14. D:(-∞,-4]; R:(-∞,∞); NF;
15. D:[-3,3]; R:[-3,3]; NF;
16. D:(-∞,-3]∪[3,∞); R:(-∞,∞); NF;
17. D:(-∞,∞); R:(-∞,-3]∪[3,∞); NF;
18. D:[-4,4]; R:[-4,4]; NF;
19. D:(-∞,-4]∪[4,∞); R:(-∞,∞); NF;
20. D:(-∞,∞); R:(-∞,-4]∪[4,∞); NF;
21. D:All X≠-3; R:All Y≠0; F;
22. D:All X≠-4; R:All Y≠2; F;
23. D:All X≠3; R:All Y≠2; F;
24. D:All X≠4; R:All Y≠0; F;
25. D:All X≠0; R:All Y≠10; F;
26. D:All X≠10; R:All Y≠0; F;
27. D:All X≠-3; R:All Y≠2; F;
28. D:All X≠3; R:All Y≠1; F;
29. D:(-∞,∞); R:(-2,2); NF;
30. D:(-2,2); R:(-∞,∞); NF;
31. D:[-4,4]; R:[0,4]; F;
32. D:(-∞,-4]∪[4,∞); R:[0,∞); F;
33. D:(-∞,-5]∪[5,∞); R:(-∞,∞); NF;
34. D:(-∞,∞); R:(-∞,-5]∪[5,∞); NF;
35. D:(-∞,9]; R:(-∞,0]; F;
36. D:[4,∞); R:(-∞,0]; F.

p.291-295:

1. NF; D:[-4,4]; R:[-3,3]; 2. F; D:(-∞,∞); R:(-∞,-3];
3. NF; D:[3,∞); R: (-∞,∞); 4. NF; D:(-∞,3]; R: (-∞,∞);
5. F; D:(-∞,∞); R:[-4,∞); 6. NF; D:[-2,2]; R: [-4,4];
7. F; D:(-∞,∞); R:(-∞,∞); 8. NF; D:(-∞,∞); R: (-∞,∞);
9. F; D:(-∞,∞); R:[-3,∞); 10. F; D:(-∞,∞); R: (-∞,4];
11. F; D:(-∞,0); R:(0,∞); 12. F; D:(0,∞); R: (-∞,0);
13. NF; D:[0,4]; R:[-2,2]; 14. NF; D:[-5,1]; R: [-3,3];
15. NF; D:[-1,5]; R:[-4,2]; 16. NF; D:[-7,3]; R: [-2,8];
17. F; D:[-2,2]; R:[-4,0]; 18. NF; D:[0,5]; R: [-3,3];
19. NF; D: (-∞,∞); R: (-∞,-3]∪[3,∞);
20. NF; D: (-∞,-2]∪[2,∞); R: (-∞,∞);

ANSWERS 2.07 (Continued)

p.291-295:

21. F; D:All $X \neq 0$; R:All $Y \neq 0$;
22. NF; D: $(-\infty,\infty)$; R: $(-\infty,1]\cup[5,\infty)$;
23. F; D: $[-4,0]$; R: $[0,3]$;
24. F; D: $(-\infty,\infty)$; R: Y must be 3;
25. NF; D: X must be -2; R: $(-\infty,\infty)$;
26. F; D: $(-\infty,\infty)$; R: $(-\infty,\infty)$;
27. F; D:All $X \neq \pm 2$; R: $(-\infty,-3]\cup(0,\infty)$;
28. F; D:All $X \neq \pm 3$; R: $(-\infty,-2)\cup[3,\infty)$.

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE