2.02 *Graphing by Translation*

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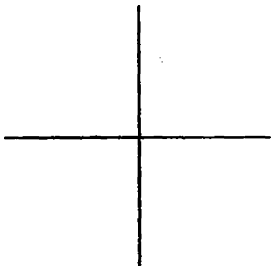
ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

While the emphasis in graphing thus far has been on **linear graphs**, by now you must certainly be aware that not all graphs are straight lines. Non-linear graphing begins in this section with simple point-plotting, followed by a search for insights and patterns to make the graphing easier and faster.

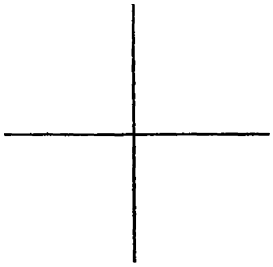
1. One of the simplest equations is the parabola **$Y = X^2$**. Complete the following table, and plot the points on the graph. The point (0,0) is called the **vertex** of the parabola.

X	Y
0	
1	
2	
3	
-1	
-2	
-3	



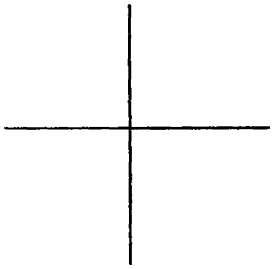
2. Complete the table and graph the equation **$Y = X^2 + 4$**.

X	Y
0	
1	
2	
-1	
-2	



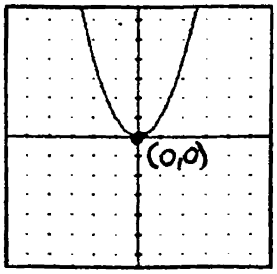
3. Next complete the table and graph the equation **$Y = X^2 - 4$**.

X	Y
0	
1	
2	
3	
-1	
-2	
-3	

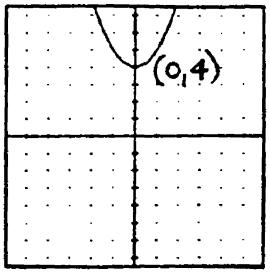


The correct graphs for these equations and the **vertices** (plural for vertex) are given and discussed below.

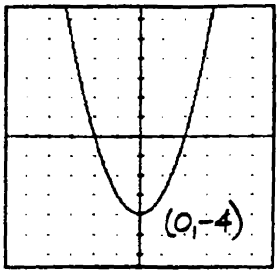
1. $Y = X^2$



2. $Y = X^2 + 4$



3. $Y = X^2 - 4$

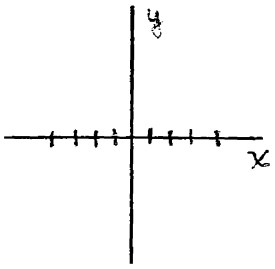


How do the graphs of $Y = X^2 + 4$ and $Y = X^2 - 4$ compare to the graph of $Y = X^2$? Did you notice that the "+4" just moves the graph up 4 units, and the "-4" moves the graph down 4 units? Except for the position of the graphs, these are all really the "same" graph.

Now complete the tables and graph the equations:

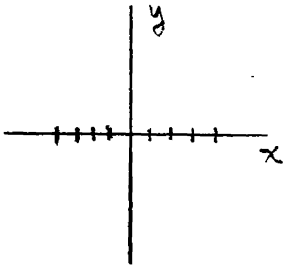
4. $Y = (X-2)^2$

X	Y
0	
1	
2	
3	
4	



5. $Y = (X+2)^2$

X	Y
-4	
-3	
-2	
-1	
0	
1	



Before turning the page to see the correct graphs of these equations, compare the graphs of $Y = (X-2)^2$ and $Y = (X+2)^2$ to the graph of $Y = X^2$. Can you speculate about the effect of the "X-2" and the "X+2" on the graphs of these equations?

6. Complete the following statements:

a. The graph $Y = X^2 + 4$ is shifted _____.

b. The graph $Y = X^2 - 4$ is shifted _____.

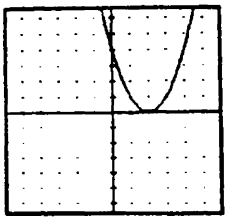
c. The graph $Y = (X-2)^2$ is shifted _____.

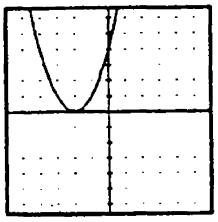
d. The graph $Y = (X+2)^2$ is shifted _____.

e. The graph $Y = (X-2)^2 + 4$ is shifted _____ and _____.

f. The graph $Y = (X+2)^2 - 4$ is shifted _____ and _____.

The correct graphs of $Y = (X-2)^2$ and $Y = (X+2)^2$ are as follows:





The principles you have discovered thus far, which involve shifting a known graph either up, down, right, left, or any combination of these, are called **translations** of the graphs. Using these principles, graph each of the following without plotting any points except the vertices:

7. $Y = X^2 - 2$

8. $Y = X^2 + 3$

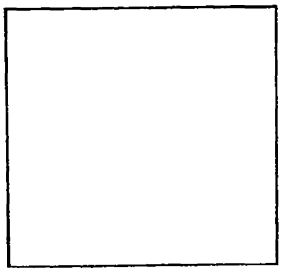
9. $Y = (X+3)^2$

10. $Y = (X-1)^2$

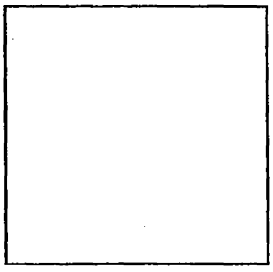
11. $Y = X^2 + 1$

12. $Y = (X+1)^2$

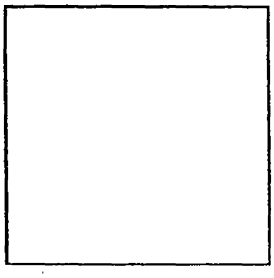
13. $Y = (X-2)^2$



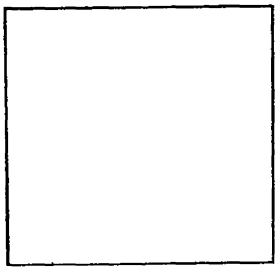
14. $Y = (X+2)^2 - 4$



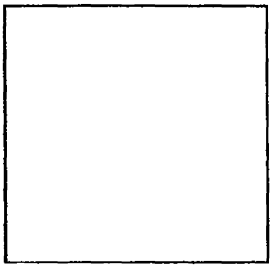
15. $Y = (X+2)^2 + 4$



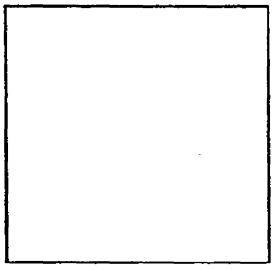
16. $Y = (X-2)^2 - 4$



17. $Y = (X+4)^2 - 2$

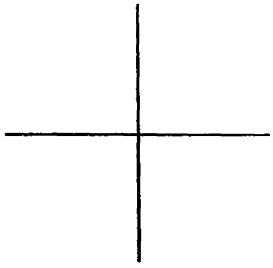


18. $Y = (X-4)^2 + 2$



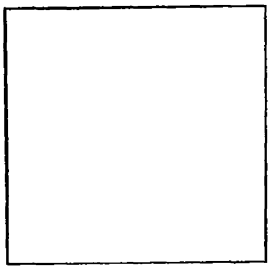
19. What do you think the graph would look like if the coefficient of X^2 is negative? (The negative coefficient is not squared!) Complete the table for $Y = -X^2$ and then draw the graph.

X	Y

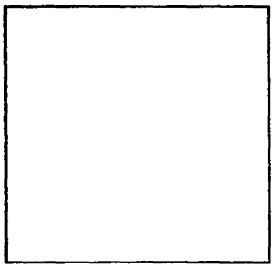


Graph the following parabolas by translating the graph of $Y=-X^2$ up, down, right, left, or appropriate combination thereof.

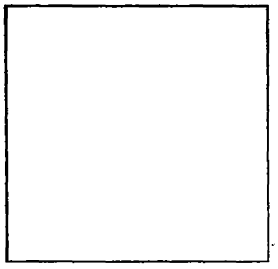
20. $Y = -X^2 + 4$



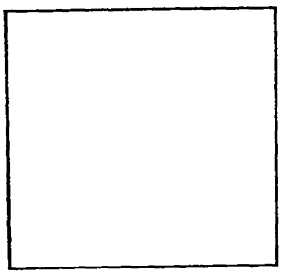
21. $Y = -X^2 + 2$



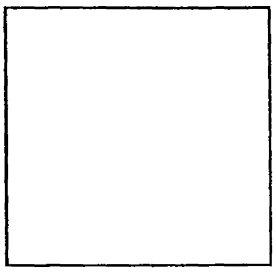
22. $Y = -X^2 - 4$



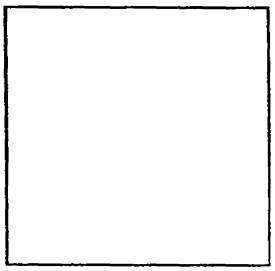
23. $Y = -(X+3)^2$



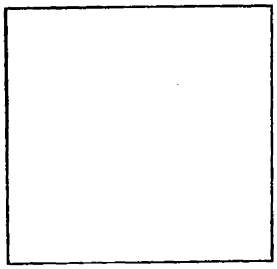
24. $Y = -(X-1)^2$



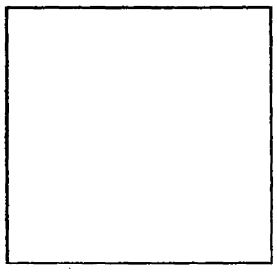
25. $Y = -(X-2)^2$



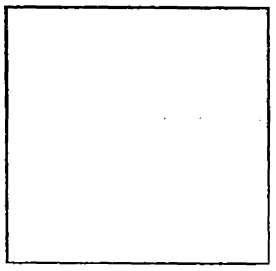
26. $Y = -(X+1)^2$



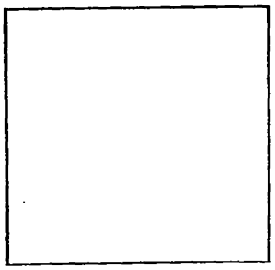
27. $Y = -(X-2)^2 + 4$



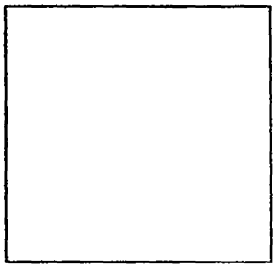
28. $Y = -(X+2)^2 - 4$



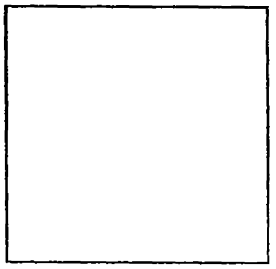
29. $Y = -(X+2)^2 + 4$



30. $Y = -(X-2)^2 - 4$



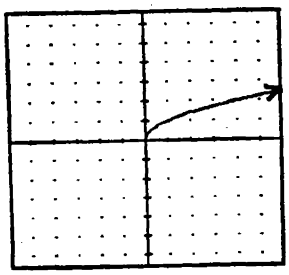
31. $Y = -(X+4)^2 - 2$



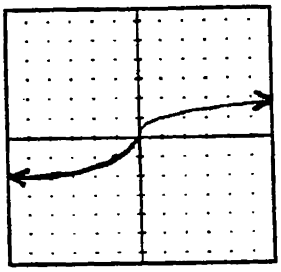
32. In general, the equation $Y = (X-h)^2 + k$ represents a **a)**_____ that opens **b)**_____, with vertex at **c)**_____.
$Y = -(X-h)^2 + k$ opens **d)**_____, with vertex at **e)**_____.

The technique of graphing by translation may be applied to any type of graph. For example, given the graphs of $Y = \sqrt{X}$, $Y = \sqrt[3]{X}$, and $Y = |X|$, use these graphs to sketch each of the following:

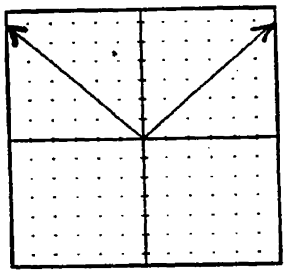
$Y = \sqrt{X}$



$Y = \sqrt[3]{X}$



$Y = |X|$



33. $Y = \sqrt{X} + 2$

34. $Y = \sqrt{X + 2}$

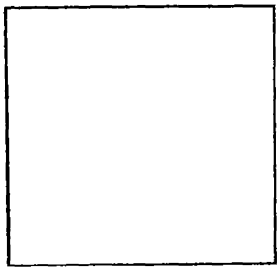
35. $Y = -\sqrt{X}$

36. $Y = \sqrt[3]{X} - 2$

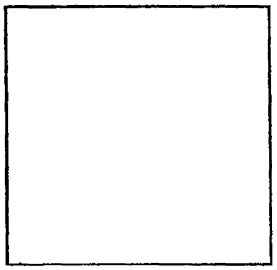
37. $Y = \sqrt[3]{X - 2}$

38. $Y = -\sqrt[3]{X}$

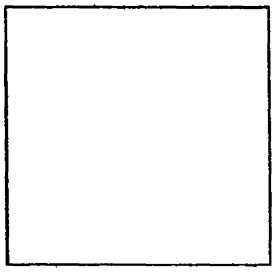
39. $Y = |X + 2|$



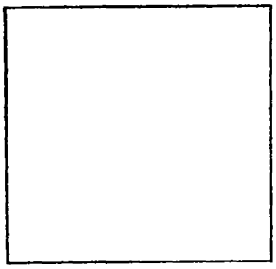
40. $Y = |X + 2| + 2$



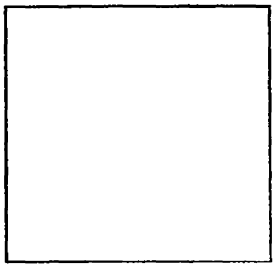
41. $Y = -|X - 2|$



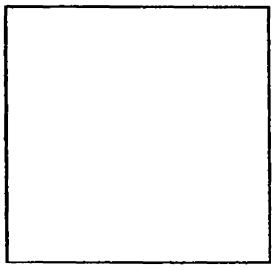
42. $Y = -\sqrt{X - 2}$



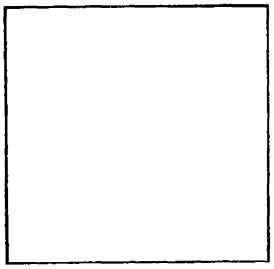
43. $Y = -\sqrt[3]{X - 2}$



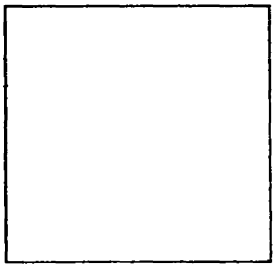
44. $Y = -|X| + 2$



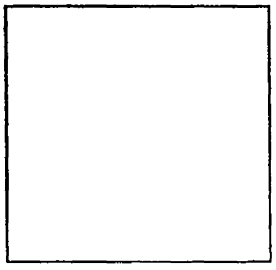
45. $Y = -\sqrt{X - 2} + 2$



46. $Y = -\sqrt[3]{X - 2} + 2$

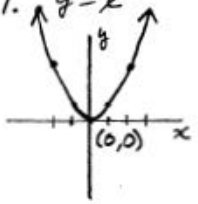


47. $Y = -|X - 2| + 2$

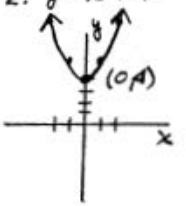


2.02 p. 212–218:

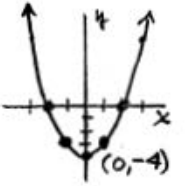
1. $y = x^2$



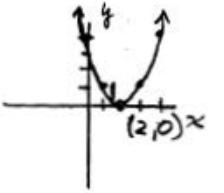
2. $y = x^2 + 4$



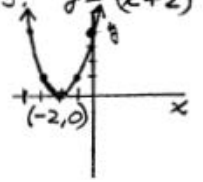
3. $y = x^2 - 4$



4. $y = (x-2)^2$

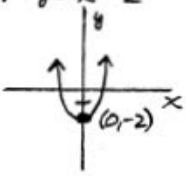


5. $y = (x+2)^2$

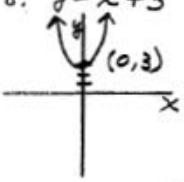


6a) Up 4
b) Down 4
c) Right 2
d) Left 2
e) Up 4, right 2
f) Down 4, left 2

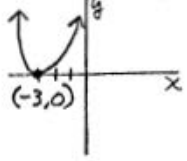
7. $y = x^2 - 2$



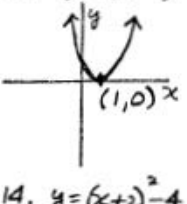
8. $y = x^2 + 3$



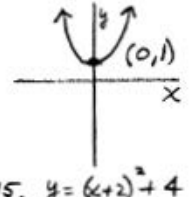
9. $y = (x+3)^2$



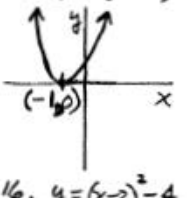
10. $y = (x-1)^2$



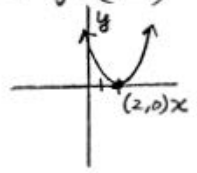
11. $y = x^2 + 1$



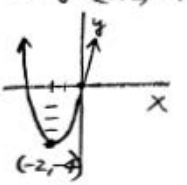
12. $y = (x+1)^2$



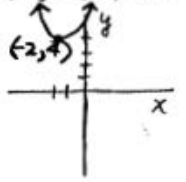
13. $y = (x-2)^2$



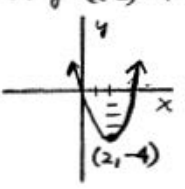
14. $y = (x+2)^2 - 4$



15. $y = (x+2)^2 + 4$



16. $y = (x-2)^2 - 4$



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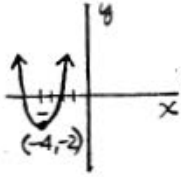
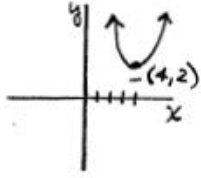
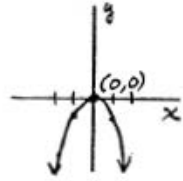
17. $y=(x+4)^2-2$



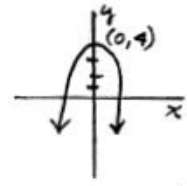
18. $y=(x-4)^2+2$



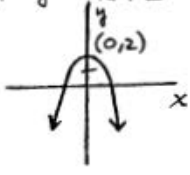
19. $y=-x^2$



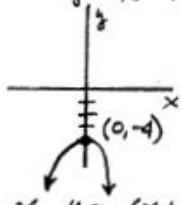
20. $y=-x^2+4$



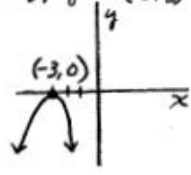
21. $y=-x^2+2$



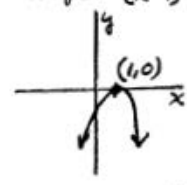
22. $y=-x^2-4$



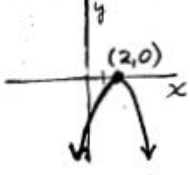
23. $y=-(x+3)^2$



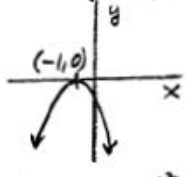
24. $y=-(x-1)^2$



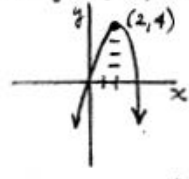
25. $y=-(x-2)^2$



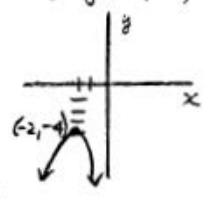
26. $y=-(x+1)^2$



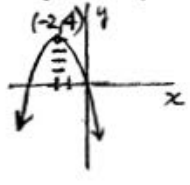
27. $y=-(x-2)^2+4$



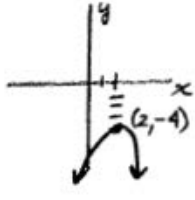
28. $y=-(x+2)^2-4$



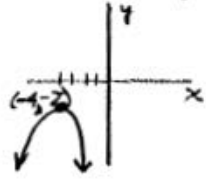
29. $y=-(x+2)^2+4$



30. $y=-(x-2)^2-4$



31. $y=-(x+4)^2-2$



32 a) Parabola
b) up
c) (h,k)
d) down
e) (h,k)

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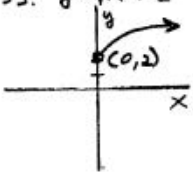
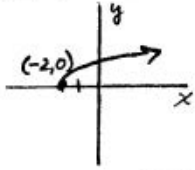
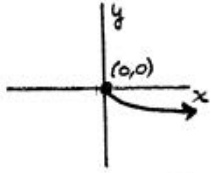
33. $y = \sqrt{x} + 2$



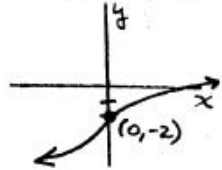
34. $y = \sqrt{x+2}$



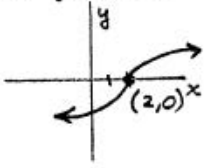
35. $y = -\sqrt{x}$



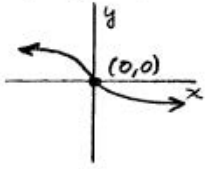
36. $y = \sqrt[3]{x} - 2$



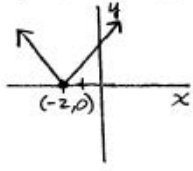
37. $y = \sqrt[3]{x-2}$



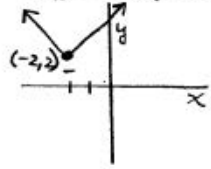
38. $y = -\sqrt[3]{x}$



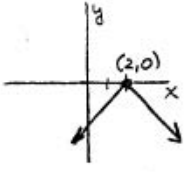
39. $y = |x+2|$



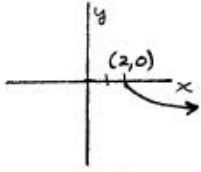
40. $y = |x+2| + 2$



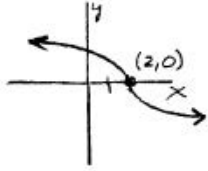
41. $y = -|x-2|$



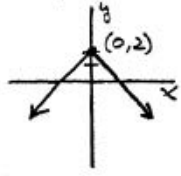
42. $y = -\sqrt{x-2}$



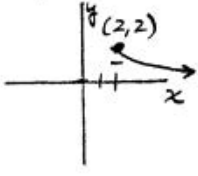
43. $y = -\sqrt[3]{x-2}$



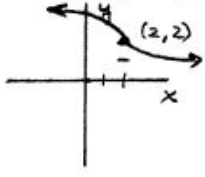
44. $y = -|x| + 2$



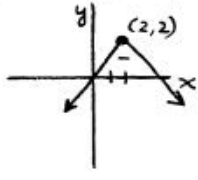
45. $y = -\sqrt{x-2} + 2$



46. $y = -\sqrt[3]{x-2} + 2$



47. $y = -|x-2| + 2$



Dr. Robert J. Rapalje

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE