3.02 *Polynomial Expressions and Equations*

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ANSWERS TO ALL EXERCISES ARE INCLUDED AT THE END OF THIS PAGE

Since the beginning of your experience in algebra, you have worked with polynomial expressions and equations. Perhaps this is a good time to clarify the terminology. A **polynomial expression** can sometimes by **simplified;** sometimes it can be **factored;** and sometimes it can be **expanded.** A polynomial expression is not an equation, and therefore you cannot solve an expression. On the other hand, **polynomial equations** are equations (sometimes linear, sometimes quadratic, sometimes more than quadratic)--the problem is to **solve** the **equations.** Solutions to equations are called **roots** of the equations. The next section involves **polynomial functions**--you will learn to **graph** the **polynomial functions** by finding the **X-intercepts** or **"zeros"** of the functions.

Summary

Type	Examples	Action to be taken
Polynomial Expression	$3X + 5(X-8)$ $(2X - 3)^2$ $X^4 - 5X^2 + 4$	**Simplify, expand, or factor the expression.**
Polynomial Equation	$3X + 5(X-8) = 0$ $(2X - 3)^2 = 0$ $X^4 - 5X^2 + 4 = 0$	**Solve (or find the roots of) the equation.**
Polynomial Function	$Y = 3X + 5$ $Y = (2X - 3)^2$ $Y = X^4 - 5X^2 + 4$	**Graph the function by finding the zeros of the function.**

General Form

Polynomial Expression	$a_nX^n + a_{n-1}X^{n-1} + a_{n-2}X^{n-2} + \ldots a_0$
Polynomial Equation	$a_nX^n + a_{n-1}X^{n-1} + a_{n-2}X^{n-2} + \ldots a_0 = 0$
Polynomial Function	$Y = a_nX^n + a_{n-1}X^{n-1} + a_{n-2}X^{n-2} + \ldots a_0.$

In 1-12, factor the polynomial expressions completely, using real numbers:

1. $X^4 - 5X^2 + 4$

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2. $X^4 - 13X^2 + 36$

3. $X^4 - 5X^2 - 36$

4. $X^4 + 8X^2 - 9$

5. $X^4 - 16$

6. $X^4 - 81$

7. $X^3 + 2X^2 + 4X + 8$

(Factor by grouping)

8. $X^3 - 2X^2 + 4X - 8$

9. $X^3 - 2X^2 - 4X + 8$

10. $X^3 - 2X^2 - 16X + 32$

11. $X^4 + 2X^3 + 8X + 16$

12. $X^4 + 2X^3 - 8X - 16$

In 13-28, find all real roots of the following equations (that is, solve the polynomial equations using real numbers).

13. $X^4 - 13X^2 + 36 = 0$

14. $X^4 - 29X^2 + 100 = 0$

15. $X^4 + 13X^2 + 36 = 0$

16. $X^4 = 8X^2 + 9$

17. $X^3 + 4X = 2X^2 + 8$

18. $X^3 + 2X^2 + 4X + 8 = 0$

19. $X^3 + 12 = 4X + 3X^2$

20. $X^3 + 5X^2 = 9X + 45$

21. $X^5 - 81X = 0$

22. $X^5 - X = 0$

23. $X^5 - 5X^3 + 4X = 0$

24. $X^5 - 10X^3 + 9X = 0$

25. $X^3 - 4X^2 - 3X = 0$

26. $X^3 + 2X^2 - 6X = 0$

The quadratic formula or completing the square may be necessary!

27. $X^3 - 4X^2 + 5X = 0$

28. $X^3 + 4X^2 + 13X = 0$

In 29-36, solve the polynomial equations using complex numbers. Use the quadratic formula or completing the square as necessary.

29. $X^4 = 8X^2 + 9$
(See #16)

30. $X^3 + 4X = 2X^2 + 8$
(See #17)

31. $X^5 - X = 0$
(See #22)

32. $X^4 + 13X^2 + 36 = 0$
(See #15)

33. $X^3 - 4X^2 + 5X = 0$
(See #27)

34. $X^3 + 4X^2 + 13X = 0$
(See #28)

35. $X^3 + 4X^2 + 6X = 0$

36. $X^3 - 4X^2 + 8X = 0$

Frequently, an equation has a **multiple root**, as illustrated by the equation $X^2 - 6X + 9 = 0$. Factoring this gives a double factor $(X - 3)^2 = 0$, since $(X - 3)$ is to second power. Therefore, $X = 3$ is said to be a **double root.** The root $X=3$ is said to be of **multiplicity 2.**

In 37-44, give all roots and multiplicities.

37. $X^5 - 6X^4 + 9X^3 = 0$

$X^3(X^2 - 6X + 9) = 0$

$X^3 (X - 3)^2 = 0$

X=___(mult ___); X=___(mult ___)

38. $X^5 + 8X^4 + 16X^3 = 0$

39. $X^3 - 8X^2 + 16X = 0$

40. $X^6 + 10X^5 + 25X^4 = 0$

41. $X^4 = 5X^3 + 6X^2$

42. $X^6 = 2X^5 - X^4$

43. $X^7 = 2X^5 - X^3$

44. $X^7 + 16X^3 = 8X^5$

In the previous exercises, you were given the equation and asked to solve the equation or to find the roots of the equation. Now, what if the problem is turned around? What if you are given the roots of the equations, and the problem is to find the equation? This is like reversing the process of solving a

quadratic equation by factoring (or completing the square).

Consider the equation: $X^2 + X - 6 = 0$

$(X + 3)(X - 2) = 0$

$(X+3) = 0 \; ; \; (X-2) = 0$

$X = -3 \; ; \; X = 2$

Now, find an equation whose solutions (roots) are X=-3 and X=2.

You begin with this: $X = -3 \; ; \; X = 2$

$(X+3) = 0 \; ; \; (X-2) = 0$

$(X + 3)(X - 2) = 0$

And you end up with this: $X^2 + X - 6 = 0$

In 45-60, find an equation whose solutions (roots) are:

45. $X = 3; \; X = -2$

46. $X = -7; \; X = 4$

47. $X = -2; \; X = -5$

48. $X = -4; \; X = -10$

49. $X = 2 \pm \sqrt{3}$ [Isolate radical]

$X - 2 = \pm \sqrt{3}$ [Square both sides]

$(X - 2)^2 = (\pm \sqrt{3})^2$

$X^2 - 4X + 4 = 3$

$X^2 - 4X + 1 = 0$

50. $X = -2 \pm \sqrt{5}$

51. $X = 2 \pm i$ [Subtract 2]

$X - 2 = \pm i$ [Square both sides]

$(X - 2)^2 = (\pm i)^2$

$X^2 - 4X + 4 = i^2$

$X^2 - 4X + 4 =$ ____

[Remember that $i^2 = -1$!!]

Final Answer: _____

52. $X = -2 \pm 3i$

53. $X = -3 \pm 4i$

54. $X = 4 \pm 2i$

[Note: In 55 - 60, a somewhat "factored form" is acceptable!]

55. $X = 3;\ X = 2 \pm 3i$

56. $X = 2;\ X = -1 \pm 5i$

57. $x = -2;\ x = 2 \pm i\sqrt{2}$

58. $x = -3;\ x = 3 \pm i\sqrt{2}$

59. $x = 3;\ x = -2;\ x = 4 \pm i$

60. $x = 2;\ x = -5;\ x = -3 \pm 5i$

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1. $(x-2)(x+2)(x-1)(x+1)$ 2. $(x-3)(x+3)(x-2)(x+2)$
3. $(x-3)(x+3)(x^2+4)$ 4. $(x-1)(x+1)(x^2+9)$ 5. $(x-2)(x+2)(x^2+4)$
6. $(x-3)(x+3)(x^2+9)$ 7. $(x+2)(x^2+4)$ 8. $(x-2)(x^2+4)$
9. $(x-2)^2(x+2)$ 10. $(x-2)(x-4)(x+4)$ 11. $(x+2)^2(x^2-2x+4)$
12. $(x+2)(x-2)(x^2+2x+4)$ 13. $x=\pm 2, \pm 3$ 14. $x=\pm 2, \pm 5$
15. No real solution 16. $x=\pm 3$ 17. $x=2$ 18. $x=-2$
19. $x=\pm 2, 3$ 20. $x=\pm 3, -5$ 21. $x=0, \pm 3$ 22. $x=0, \pm 1$
23. $x=0, \pm 1, \pm 2$ 24. $x=0, \pm 1, \pm 3$ 25. $x=0, 2\pm\sqrt{7}$
26. $x=0, -1\pm\sqrt{7}$ 27. $x=0$ 28. $x=0$ 29. $x=\pm 3, \pm i$
30. $x=2, \pm 2i$ 31. $x=0, \pm 1, \pm i$ 32. $x=\pm 2i, \pm 3i$
33. $x=0, 2\pm i$ 34. $x=0, -2\pm 3i$ 35. $x=0, -2\pm i\sqrt{2}$
36. $x=0, 2\pm 2i$ 37. $x=0$ (mult 3); $x=3$ (mult 2)
38. $x=0$ (mult 3); $x=-4$ (mult 2) 39. $x=0$; $x=4$ (mult 2)
40. $x=0$ (mult 4); $x=-5$ (mult 2) 41. $x=0$ (mult 2); $x=-1, 6$
42. $x=0$ (mult 4); $x=1$ (mult 2)
43. $x=0$ (mult 3); $x=1$ (mult 2); $x=-1$ (mult 2)
44. $x=0$ (mult 3); $x=2$ (mult 2); $x=-2$ (mult 2)
45. $x^2-x-6=0$ 46. $x^2+3x-28=0$ 47. $x^2+7x+10=0$
48. $x^2+14x+40=0$ 49. $x^2-4x+1=0$ 50. $x^2+4x-1=0$
51. $x^2-4x+5=0$ 52. $x^2+4x+13=0$ 53. $x^2+6x+25=0$
54. $x^2-8x+20=0$ 55. $(x-3)(x^2-4x+13)=0$
56. $(x-2)(x^2+2x+26)=0$ 57. $(x+2)(x^2-4x+6)=0$
58. $(x+3)(x^2-6x+11)=0$ 59. $(x^2-x-6)(x^2-8x+17=0$
60. $(x^2+3x-10)(x^2+6x+34)=0$.

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